# Pattern formation in natural convection flows

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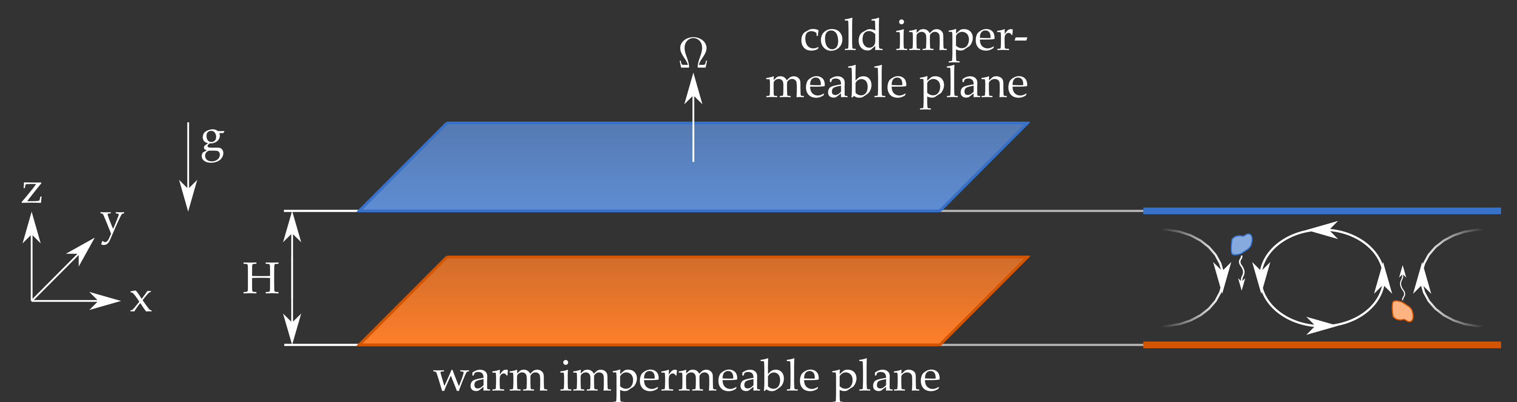
## Motivation

Turbulent convection, the essential mechanism by which heat is transported in natural flows, manifests often in a hierarchy of structures and flow patterns. Illustrations for this phenomenon can be found throughout our solar system - from clusters of clouds over the warm oceans in the tropcis on Earth up to giant storm systems in the atmosphere of the big gas planet Jupiter. One of the most prominent astrophysical examples for such hierarchies is the solar convection zone in the outer 30% of the Sun, in which granules (small convection cells) and supergranules (significantly larger patterns) represent important building blocks. However, the origin of the larger structures is still unsolved.




A basic understanding of physical causes and connections of our observations is crucial for their modelling. This fundamental research aims to provide contributions to these open questions by studying an idealised setup, see above.

## Method

Rayleigh-Bénard convection in the Oberbeck-Boussinesq approximation is considered as the simplest paradigm for research of convection and thus of thermally driven turbulence. Consequently, it represents a manageable experimental setting for natural convection flows. The basic idea is to transfer heat across a fluid layer that is confined between two horizontal planes as sketched on the left. Note that the bottom plane is heated and the top plane is cooled, generating an unstable stratification. Under the action of gravity, buoyancy sets in and might eventually drive a flow.

In this work the focus is on applying a *constant heat flux* at both horizontal planes, the latter of which are assumed to be stress-free. The lateral boundaries shall obey periodicity. Here, this dynamical system is studied via direct numerical simulations using the spectral element method Nek5000.

$$\nabla \cdot \vec{u} = 0$$

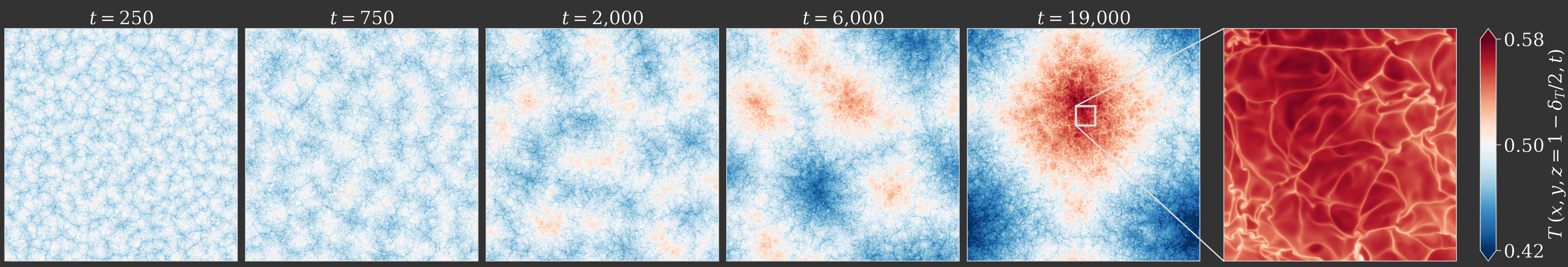
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} + \frac{1}{\mathrm{Ro}}\vec{e}_z \times \vec{u} = -\nabla p + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}}\nabla^2 \vec{u} + T\vec{e}_z$$

$$\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)T = \frac{1}{\sqrt{\mathrm{Ra}\,\mathrm{Pr}}}\nabla^2 T$$

$$\mathrm{Pr} := \frac{\nu}{\kappa} = 1 \qquad \mathrm{Ra} := \frac{\alpha g \beta H^4}{\nu\kappa} \qquad \mathrm{Ro} := \frac{U_f}{2\Omega H} \qquad \Gamma := \frac{L}{H} = 60$$

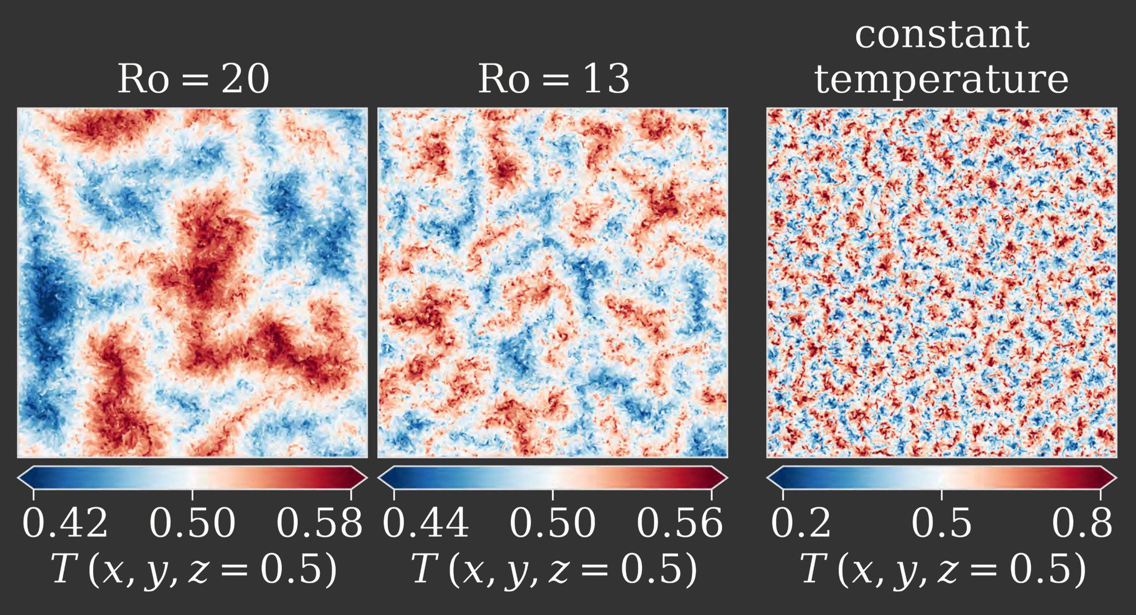
## Pattern formation and the hierarchy of flow structures

In case of well-chosen boundary conditions (see above) and *without the action of rotation*, the establishment of a hierarchy of flow structures can be observed despite fully developed turbulence. Here, the supergranule forms as a result of the gradual aggregation of individual granules. This effect can be found for all accessible Rayleigh numbers Ra (thermal drive) and witnessed from both the temperature, as well as the time-averaged velocity field. Visualisation: $\mathrm{Ra} \approx 7.7 \times 10^7$, $\mathrm{Ro} = \infty$




## Limiting the aggregation by rotation

This gradual supergranule aggregation continues without additional physical mechanisms until the numerical domain size is reached. However, applying only *weak rotation* around the vertical axis limits this process effectively, allowing finally for a comparability with natural flows. Visualisation: $\mathrm{Ra} \approx 3.9 \times 10^6$

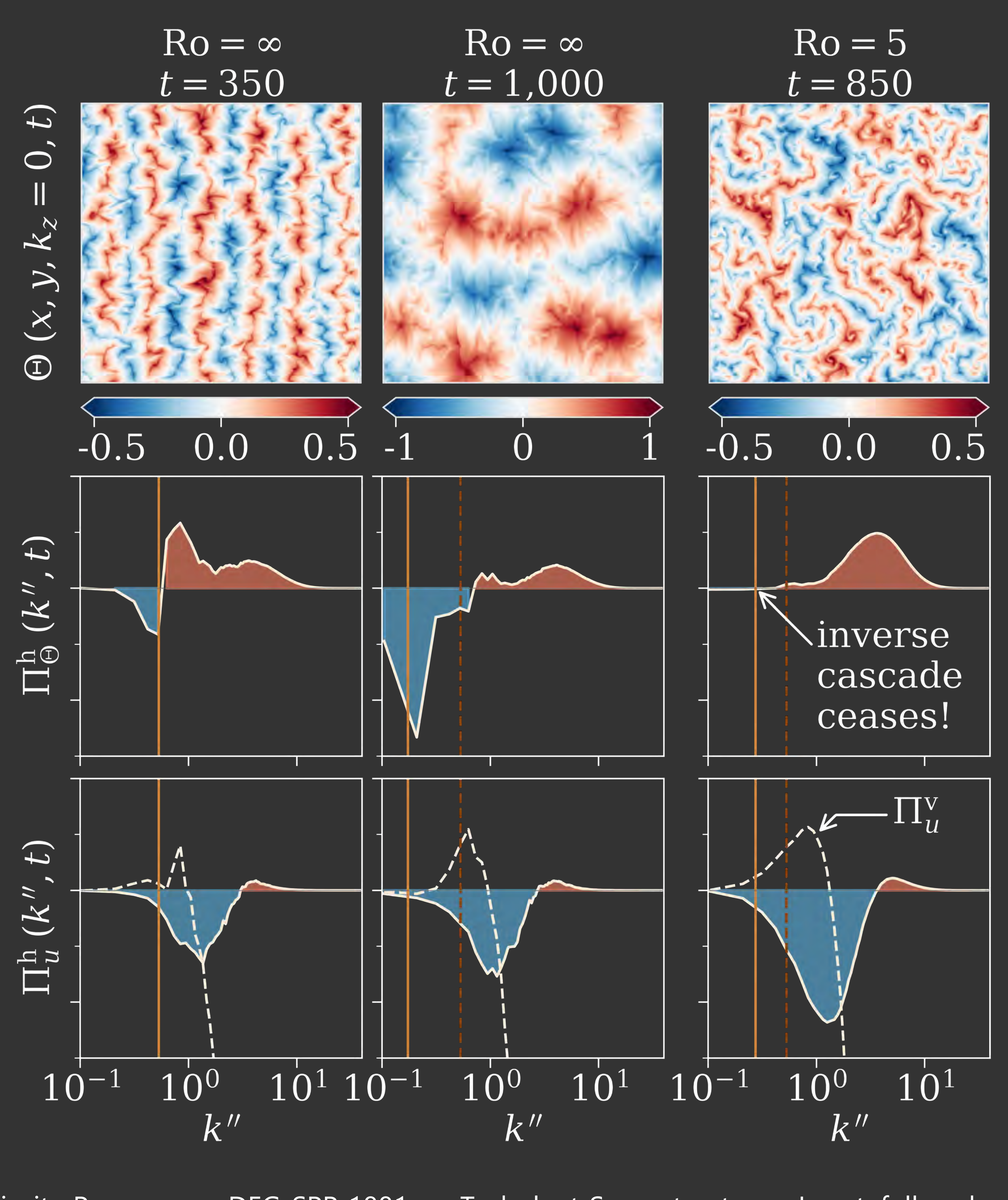



Neither the formation of such a hierarchy, nor confining the pattern size effectively under weak rotation is possible in case of classical thermal boundary conditions - see the right panel.

These observations exhibit interesting analogies to analytically accessible results concerning the instability of flow structures, the latter of which can be studied through a *leading Lyapunov vector analysis* even in the turbulent regime. For the rotating case, both the Lyapunov and the energy transfer analysis (see in the column to the right) indicate the ceasing aggregation process.

## Supergranules as result of inverse cascades

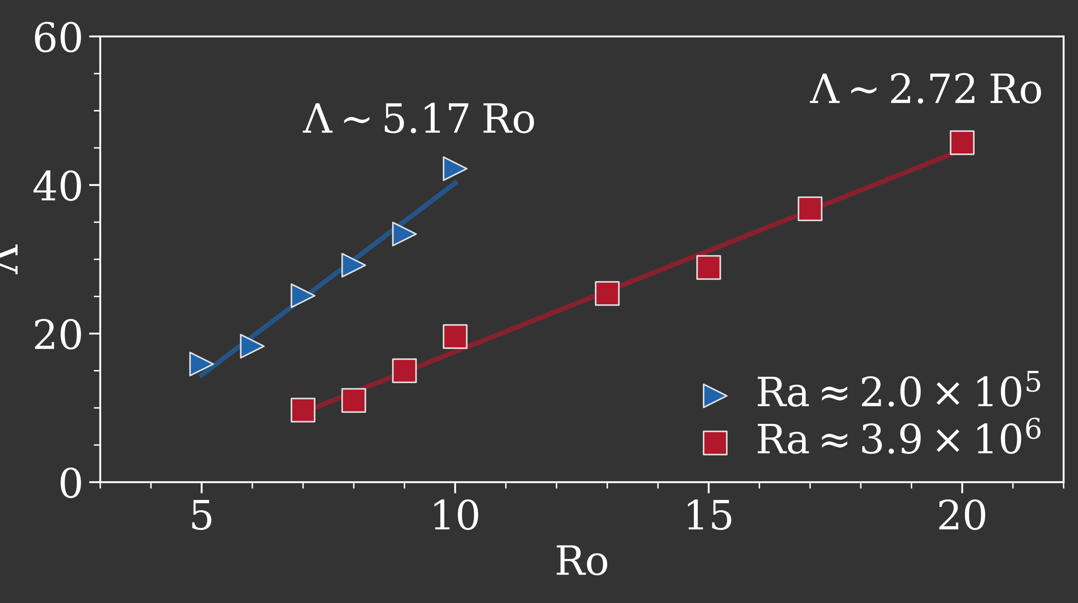
By studying the modal energy transfer it can be found that the supergranule aggregation is caused by *inverse cascades* for both the turbulent kinetic energy and the thermal variance. These cascades establish dominantly among the subset of two-dimensional modes (with $k_z = 0$) within the three-dimensional flow.
Visualisation: $\mathrm{Ra} \approx 1.0 \times 10^4$




## Scaling of the supergranules

In the weakly rotating regime, the supergranule size can be observed to scale *linearly* with the Rossby number Ro (weakness of rotation). This conclusion holds independently of the Rayleigh number and can be explained based on the Rossby number's definition:

$$\mathrm{Ro} = \frac{\tau_\Omega}{\tau_f} = \frac{\text{time scale of rotation processes}}{\text{time scale of buoyancy processes}}$$




## Selected publications

P. P. Vieweg, J. D. Scheel, J. Schumacher, *Supergranule aggregation for constant heat flux-driven turbulent convection*, Phys. Rev. Research **3**, 013231 (2021)

P. P. Vieweg, C. Schneide, K. Padberg-Gehle, J. Schumacher, *Lagrangian heat transport in three-dimensional convection*, Phys. Rev. Fluids **6**, L041501 (2021)

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