Lab Class S:V-2

By January 21st, 2015 solutions for the following exercises have to be submitted: 1, 2, 3, 4, 7.

Exercise 1 : Uniform-cost search as a special case of A*

Give a proof sketch for the following statement from the lecture notes:

Uniform-cost search is a special case of A*, where $h = 0$.

Exercise 2

Let $G = \langle V, E \rangle$ be a search graph (OR graph), and let $\mathcal{G}$ denote the set of all pointer paths (from s to nodes on the OPEN list) that can be created during A* search. Give an upper bound on the size $|\mathcal{G}|$ of this set with respect to the number of nodes $|V|$.

Exercise 3

Complete step 2 of the proof for Pearl Lemma 1 / Nilsson Result 2:

Let G be a search graph with $Prop(G)$. Show by induction that during A* search using some admissible heuristic h, each optimal solution path $P^*_{s-\gamma} \in \mathbf{P}^*_{s-\Gamma}$ contains a shallowest OPEN node at any point in time before termination of A*.

Exercise 4

Under which conditions is the CLOSED list unnecessary for A* search?

Exercise 5

A negative-cost cycle is a cycle in a search graph whose sum of edge weights is negative. Why are negative-cost cycles problematic for A* search?

Exercise 6

Define the term “monotonicity” in the context of heuristic cost estimation functions.

For a search problem of your choice, give examples for

(a) monotone

(b) non-monotone, but admissible

estimation functions.

Exercise 7 : Implementing A* Search

Consider the 8-puzzle problem. Your task is to reach the goal state γ with as few moves as possible.

s_1:

6	4	7
8	5	
3	2	1

s_2:

6	7	4
8	5	
3	2	1

γ:

1	2	3
4	5	6
7	8	

You are given 3 heuristics to estimate the number of remaining moves:

- $h_0 \equiv 0$.
- $h_1 =$ Number of the 8 tiles which are at a different position than in the goal state.
- $h_2 =$ Sum of the Manhattan distances from each of the 8 tiles to its position in the goal state.

(a) How many different states exist for the 8-Puzzle?

(b) Is the goal state γ reachable from every state? (You can use the World Wide Web to answer this question as long as you cite your sources.)

(c) Does the search graph G fulfill $Prop(G)$?

(d) Which of the heuristics h_0, h_1 and h_2 are optimistic, i.e. never overestimate the remaining cost?

(e) Implement an A* search for the 8-Puzzle problem using h_0. For both start states s_1 and s_2, give the number of necessary moves (if the search succeeds) as well as the number of expanded nodes during search (if it succeeds or fails). Hint: for all 3 heuristics, a CLOSED state will never be reopened.

(f) How will the result change if you use h_1 instead of h_0? Why? Verify your thoughts with your implementation (give the number of expanded nodes when using h_1).

(g) How will the result change if you use h_2 instead of h_1? Why? Verify your thoughts with your implementation (give the number of expanded nodes when using h_2).