By January 21st, 2015 solutions for the following exercises have to be submitted: 1, 2, 3, 4, 7.

Exercise 1 : Uniform-cost search as a special case of A*
Give a proof sketch for the following statement from the lecture notes:
Uniform-cost search is a special case of A*, where $h = 0$.

Exercise 2
Let $G = \langle V, E \rangle$ be a search graph (OR graph), and let $G$ denote the set of all pointer paths (from $s$ to nodes on the OPEN list) that can be created during A* search. Give an upper bound on the size $|G|$ of this set with respect to the number of nodes $|V|$.

Exercise 3
Complete step 2 of the proof for Pearl Lemma 1 / Nilsson Result 2:
Let $G$ be a search graph with $Prop(G)$. Show by induction that during A* search using some admissible heuristic $h$, each optimal solution path $P^*_s \gamma \in P^*_s \Gamma$ contains a shallowest OPEN node at any point in time before termination of A*.

Exercise 4
Under which conditions is the CLOSED list unnecessary for A* search?

Exercise 5
A negative-cost cycle is a cycle in a search graph whose sum of edge weights is negative. Why are negative-cost cycles problematic for A* search?

Exercise 6
Define the term “monotonicity” in the context of heuristic cost estimation functions.
For a search problem of your choice, give examples for
(a) monotone
(b) non-monotone, but admissible
estimation functions.
Exercise 7: Implementing A* Search

Consider the 8-puzzle problem. Your task is to reach the goal state $\gamma$ with as few moves as possible.

\[
\begin{array}{ccc}
6 & 4 & 7 \\
8 & 5 & 3 \\
3 & 2 & 1 \\
\end{array} 
\]
\[
\begin{array}{ccc}
6 & 7 & 4 \\
8 & 5 & 3 \\
4 & 5 & 6 \\
\end{array} 
\]
\[
\begin{array}{ccc}
6 & 7 & 4 \\
3 & 2 & 1 \\
7 & 8 & 1 \\
\end{array} 
\]

$s_1$ $s_2$ $\gamma$

You are given 3 heuristics to estimate the number of remaining moves:

- $h_0 \equiv 0$.
- $h_1 =$ Number of the 8 tiles which are at a different position than in the goal state.
- $h_2 =$ Sum of the Manhattan distances from each of the 8 tiles to its position in the goal state.

(a) How many different states exist for the 8-Puzzle?

(b) Is the goal state $\gamma$ reachable from every state? (You can use the World Wide Web to answer this question as long as you cite your sources.)

(c) Does the search graph $G$ fulfill $Prop(G)$?

(d) Which of the heuristics $h_0$, $h_1$ and $h_2$ are optimistic, i.e. never overestimate the remaining cost?

(e) Implement an A* search for the 8-Puzzle problem using $h_0$. For both start states $s_1$ and $s_2$, give the number of necessary moves (if the search succeeds) as well as the number of expanded nodes during search (if it succeeds or fails). Hint: for all 3 heuristics, a CLOSED state will never be reopened.

(f) How will the result change if you use $h_1$ instead of $h_0$? Why? Verify your thoughts with your implementation (give the number of expanded nodes when using $h_1$).

(g) How will the result change if you use $h_2$ instead of $h_1$? Why? Verify your thoughts with your implementation (give the number of expanded nodes when using $h_2$).