Lab Class S:II

By November 12th 2014, solutions for the following exercises have to be submitted: 1b, 2, 3a-c, 4, 5, 7.

Exercise 1 : Computational complexity theory [Review]

Review the terminology associated with asymptotic computational complexity and Big O notation, then answer the following questions.

(a) Given two functions $f, g : \mathbf{N} \to \mathbf{R}^+$, what is meant by the notation $f(n) = O(g(n))$? Give a precise definition.

(b) With f, g defined as above, what is meant by the notation $f(n) = \Omega(g(n))$?

(c) Which of the following statements are true?

- ☐ $2n = O(n)$
- ☐ $n^2 = \Omega(n)$
- ☐ $n^2 = O\left(n \log^2 n\right)$
- ☐ $n^2 = \Omega\left(n^2\right)$
- ☐ $n \log n = O\left(n^2\right)$
- ☐ $3^n = O(2^n)$
- ☐ $2^{2^n} = O\left(2^{2^n}\right)$

Exercise 2 : Computational complexity theory [Review]

(a) What are the computational model and the resource limit for decision problems in $\mathbf{NP}$?

(b) What are the characteristics of an $\mathbf{NP}$-hard problem?

(c) What are the characteristics of an $\mathbf{NP}$-complete problem?

(d) Let there be the formal grammar $G = (V, \Sigma, R, \langle EXPR \rangle)$. With $\Sigma = \{a, +, \times, (,)\}$, $V = \{\langle EXPR \rangle, \langle TERM \rangle, \langle FACTOR \rangle\}$ and the following production rules R:

$$
\begin{array}{rcl}
\langle EXPR \rangle & \to & \langle EXPR \rangle + \langle TERM \rangle \\
\langle EXPR \rangle & \to & \langle TERM \rangle \\
\langle TERM \rangle & \to & \langle TERM \rangle \times \langle FACTOR \rangle \\
\langle TERM \rangle & \to & \langle FACTOR \rangle \\
\langle FACTOR \rangle & \to & (\langle EXPR \rangle) \\
\langle FACTOR \rangle & \to & a
\end{array}
$$

Of what type is G with respect to the Chomsky hierarchy (`http://en.wikipedia.org/wiki/Chomsky_hierarchy`)? Justify your answer.

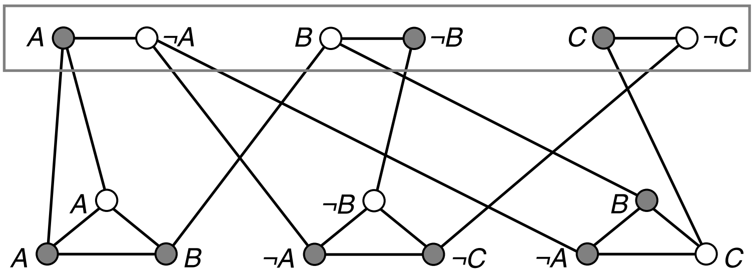
Exercise 3 : Computational complexity theory Review

(a) Briefly state the 3-SAT problem. (What is an instance? What is the decision?)

(b) Briefly state the vertex cover problem. (What is an instance? What is the decision?)

(c) What steps are necessary to show that the 3-SAT problem can be reduced to the vertex cover problem with $k = v + 2 \cdot c$? k is the maximum size of the vertex cover, v is the number of variables in the formula and c the number of clauses. Only list and describe the steps on an abstract level, but perform no proofs.

(d) Reduce the 3-satisfiability problem to the vertex cover problem (with $k = v + 2 \cdot c$). Hint: the figure below shows how the satisfiability problem $(A \vee B) \wedge (\neg A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee C)$ is solved as a vertex cover (grey nodes) of a graph. The grey nodes within the box give a solution that satisfies the expression.



Exercise 4 : Concepts

Give definitions of the following terms in the context of search problems.

(a) Heuristic.

(b) Database, operators, and control strategy.

(c) Code or encoding (give at least one example).

Exercise 5 : Concepts

Does the encoding the 8-Puzzle problem given in the unit on state-space representation meet the requirements for a systematic search? If so, is this a reasonable encoding?

Exercise 6 : Concepts

What is the principle of *Least Commitment*?

Exercise 7 : State space search

Picture the following situation: Brazilian rain forest, sweltering heat, mosquitoes and poisonous snakes. Three cannibals and three missionaries have to cross a river full of crocodiles on the way to the mission station. The only boat carries no more than two people. The missionaries are worried they might end up as breakfast to the cannibals if there are more cannibals than missionaries on a bank of the river at any given time. How can everyone cross the river without anybody getting eaten?

Note that the persons in the boat count as if they were on the respective river bank and two people can board the boat at the same time. The boat can not change river sides on its own, but can be moved by a single person (missionary or cannibal).

(a) Formulate the above problem as a search problem. What is the database (the encoding of a—possibly partial—configuration), what are the operators and what is the control strategy?

(b) Implement an exhaustive search, which simply enumerates all solutions, in Python or Java.

(c) Draw (a partial) state-space graph and mark in it the solution found by your implementation.

Exercise 8 : State space search

Consider the following problem:

A farmer with a wolf, a goat and a cabbage stands at the bank of a river that they need to cross. There is a boat, large enough to carry the farmer and one further object, such that only one of the three can be taken across the river at a time. The goat will eat the cabbage, and the wolf will eat the goat, if either pair is left unsupervised at one of the river banks.

Is it possible to transport all three to the other river bank? What is the minimum number of trips across the river needed to achieve this?

[attributed to Alcuin of York, 735–804]

Formulate this problem as a search problem. What is the database, i.e., the coding of (sets of) candidate solutions? What are the operators? What is the control strategy?

Exercise 9 : Concepts

In order to deepen your understanding of problems and problem solving, read the article at
http://en.wikibooks.org/wiki/Cognitive_Psychology_and_Cognitive_Neuroscience/Problem_Solving_from_an_Evolutionary_Perspective

Exercise 10 : State space search

You need to send out all the objects listed in the following table in postal packages.

Object	A	B	C	D	E	F	G	H	I	J
Weight (kg)	1	4	6	7	7	9	11	13	15	16

One package can weigh at most 20 kilograms. The shipping costs per package are 9,90 EUR. Your goal is to pack all items into as few packages as possible.

(a) You can represent the problem as a state-space graph (OR graph) or a problem-reduction graph (AND-OR graph). Which problem representation is better suited to this task? Justify your choice briefly.

(b) What information is represented by the nodes in the graph? What operations are represented by the edges?

(c) What are the characteristics of solution nodes in your chosen representation, what are dead ends?

(d) How do you determine the costs associated with the edges in your representation?

(e) Given a partial solution, how would you estimate the residual cost?