Search Algorithms January 23, 2015

# Lab Class S:VI

By February 4th, 2015 solutions for the following exercises have to be submitted: 1, 2, 3, 5, 7,

#### Exercise 1: Heuristics

Let n be a node in a search space graph that is explored by  $A^*$  using heuristic h.

- (a) When can the value of h(n) change during A\*-search?
- (b) When can the value of g(n) change during A\*-search?
- (c) When can the value of g(n) change during A\*-search with admissible h?
- (d) When can the value of g(n) change during A\*-search with monotone h?

## Exercise 2: Admissibility

In the lecture, we have proven that the algorithm  $A^*$  is admissible when using an admissible heuristic. Does this also apply to the algorithm Z?

Justify your answer.

# Exercise 3: Monotonicity

Consider the search problem of finding the shortest sequence of knight moves between two given squares on a chessboard, as discussed in the lecture. Let h be a heuristic that estimates the number of moves remaining for a given board square represented by node n.

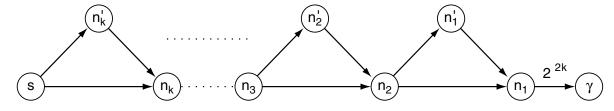
$$h(n) = \frac{d_M(n, \gamma)}{3}$$

where  $d_M(n_1, n_2)$  is the Manhattan distance between the board squares corresponding to nodes  $n_1$  and  $n_2$ .

- (a) Define the term "monotonicity" in the context of heuristic cost estimation functions.
- (b) Show that h is both admissible and monotone.
- (c) For the same search problem, find a heuristic that is admissible, but not monotone.

## Exercise 4: Reopening

Consider the following search space graph:



Add edge cost values and h-values for the nodes, such that the number of reopenings done by algorithm  $A^*$  is exponential in k.

#### Exercise 5: Relaxed Models

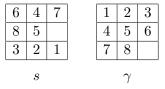
Consider the general approach to weighted node evaluation, where  $f_w(n) = (1 - w) \cdot g(n) + w \cdot h(n)$ . Under which conditions does it make sense to use a weighting where w is close to 1, disregarding the current path cost g?

## Exercise 6: Relaxed Models

The algorithm  $A_{\varepsilon}^*$  uses two heuristic functions h and  $h_F$ . What is the advantage of using  $h \neq h_F$ ?

# Exercise 7: Implementing DWA\* Search

Consider the 8-puzzle problem. Your task is to reach the goal state  $\gamma$  with as few moves as possible.



- (a) Consider the case that  $\gamma$  would not be reachable from a start state. Would DWA\* expand the same number of nodes as A\* until it fails?
- (b) Consider the case N=1,  $\epsilon\approx 1$  and  $h\approx h^*$ . Would you expect that DWA\* performs more, less, or the same amount of node expansions as A\*?
- (c) Consider the case  $N \to \infty$ ,  $\epsilon \approx 1$  and  $h \approx h^*$ . Would you expect that DWA\* performs more, less, or the same amount of node expansions as A\*?
- (d) Implement a DWA\* search for the 8-Puzzle problem using  $h_2$ , the sum of the Manhattan distances from each of the 8 tiles to its position in the goal state. What changes do you need to make to your A\* implementation? You can also use the A\* example implementation from the lecture's web page as a base.
- (e) Run your implementation with  $N \in \{5, 10, 15, \dots, 50\}$  and  $\epsilon \in \{\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2\}$ . Use s as the start and  $\gamma$  as the goal state. Show the number of node expansions performed by DWA\* for each of the 50 pairs of N and  $\epsilon$ . You can use a table or graphs.
- (f) For the same parameters as above, show the number of moves of the solution found by DWA\*,  $g(\gamma)$ , for each pair of N and  $\epsilon$ .
- (g) For what parameter setting does DWA\* terminate with an optimal solution with the smallest amount of node expansions? How many node expansions are these?
- (h) For what parameter setting does DWA\* terminate with a solution with the smallest amount of node expansions? How many node expansions are these?

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