

## 4: SPARK: Language & Tools for Static Verification

This section provides

- ▶ a quick intro for the
  - ▶ SPARK language and the
  - ▶ SPARK toolset
- ▶ and a couple of simple examples.

**Read:** Sections 1–5 of the SPARK 2014 User's Guide

<https://docs.adacore.com/spark2014-docs/html/ug/>,

# What is “SPARK”

ask Wikipedia

...

## Computer Science:

**Cisco Spark (application)** , a collaboration application and platform now part of the Webex Teams application

**Spark (application)** , a mobile email application for iOS devices by Readdle

**SPARK (programming language)**

**Spark (software)** , a web application framework.

**Spark (cellular automaton)** , a type of pattern in Conway's Game of Life and related rules.

**Spark (XMPP client)** , an instant messaging client.

**Apache Spark** , a cluster computing framework

...

## → SPARK (programming language)

from Wikipedia, again

**SPARK** is a formally defined computer programming language based on the Ada programming language, intended for the development of high integrity software used in systems where predictable and highly reliable operation is essential. It facilitates the development of applications that demand safety, security, or business integrity.

The entry isn't wrong but incomplete. SPARK is both

- ▶ a programming language (actually, a subset of Ada)
- ▶ and a toolset to support static verification.

Without the toolset, nobody would care about the language.

# History

- ▶ Pre-Ada: SPADE: **S**outhampton **P**rogram **A**nalysis **D**evelopment **E**nvironment (for Pascal)
- ▶ Ada: SPARK **S**PADE **A**da **K**e**R**nel
  - ▶ Following the revisions of Ada: SPARK 83, SPARK 95, SPARK 05
- ▶ Ada 2012 / SPARK 2014: a very significant change (→ next slide)

# From SPARK 05 to SPARK 2014

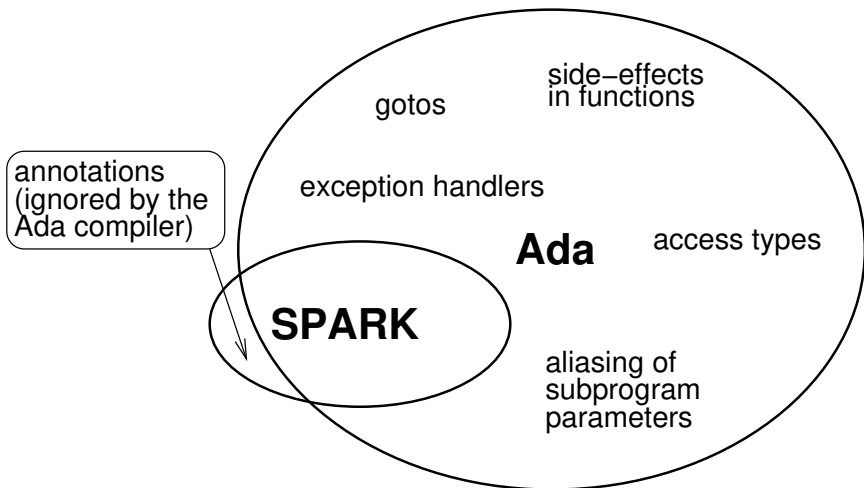
## SPARK 05:

```
procedure Swap(X, Y: in out Integer);  
--# derives X from Y \& Y from X;  
--# post (X = Y~) and (Y = X~);
```

## SPARK 2014:

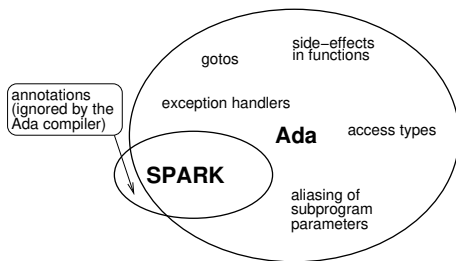
```
procedure Swap (X : in out Integer;  
                Y : in out Integer) with  
  Global => null,  
  Depends => (X => Y, Y => X),  
  Post => ((X = Y'Old) and (Y=X'Old));
```

# The Relationship between SPARK and Ada



# No Access Types! No Gotos!

- ▶ access types
  - ▶ worry about memory safety
  - ▶ extremely difficult to analyze (heap = one huge array)
- ▶ goto
  - ▶ bad style: “goto considered harmful”
  - ▶ difficult to analyze (especially when going backward)



# Why no side-effects in functions?

- ▶ 

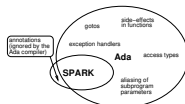
```
X := 10;  
Y := Fun(0);  
Put(X);
```

 — *This prints 0. But why?*
- ▶ 

```
function Fun(N: Integer) return Integer is  
begin  
  X := N mod 2; — OK in Ada, prohibited in SPARK.  
  return N;  
end;
```
- ▶ 

```
X := 10;  
Z := Fun(1) + Fun(2);  
Put(X);
```

 — *Does this print 0 or 1?*
- ▶ Both 0 and 1 possible: order of expression evaluation not defined.





# Why no aliasing of subprogram parameters?

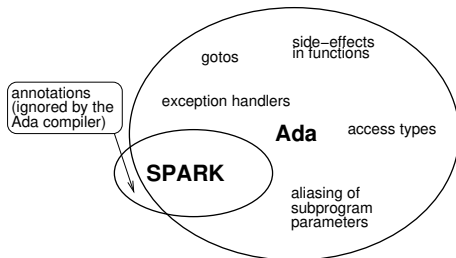
What is that, at all?

- ▶ **type** Math\_Int is ...; — *unbounded integers*  
**procedure** Divide(X, Y: Math\_Int;  
                  Result, Remainder **out** Math\_Int);  
...  
Divide (A, B, A, A); — *OK in Ada, prohibited in SPARK.*
- ▶ The semantic of calling “Divide(A,B,A,A)” is undefined in Ada:
  - ▶ Subprograms can copy their parameters forward and backward. Then, the result of calling “Divide (A, B, A, A);” should be either Result or Remainder – or perhaps a mixture of both.
  - ▶ Subprograms can use pointers to their parameters. Then, after calling “Divide (A, B, A, A);” the value of A is almost impossible to predict.



# Why no exception handlers?

- ▶ complex control flow difficult to analyze (as discussed before)
- ▶ also:
  - ▶ early programming languages did not have exceptions and did crash unexpectedly (e.g., “division by 0”) (→ next slide)
  - ▶ exceptions as an approach to “tame” crashes (improved reliability)
  - ▶ SPARK: static analysis to prove that nothing unexpected happens (improves reliability more) (→ next slide +1)



# Early Languages (Here: C)

from the textbook:

```
fptr = fopen("~/comments.txt",  
            "a");  
// open for append  
  
if (fptr == NULL)  
{  
    error_handling()  
};  
  
if ((100*done) / full > 50) {  
    fprintf(fptr, "> 50 %'");  
    // store comment in file  
    // hope that full is never 0!  
}  
  
fclose(fptr);
```

lazy programmers' time bomb

```
fptr = fopen("~/comments.txt",  
            "a");  
// open for append  
  
// forgot the error handling!  
  
if ((100*done) / full > 50) {  
    fprintf(fptr, "> 50 %'");  
    // store comment in file  
}  
  
// if fopen above failed:  
// crash — here we come!  
// (same if full=0)  
  
fclose(fptr);
```

# The Same in SPARK

```
procedure Append(File: File_of_Item; It: Item) with  
  Pre => Valid(File);
```

textbook-like:

```
File := Open("~/comments.txt",  
            Append_Mode);  
  
if Valid(File) then  
  if (100*Done)/Full > 50 then  
    Append(File, Comment);  
  end if;  
  Close(File)  
else  
  Error_Handling;  
end if;
```

(Must prove  $\text{Full} > 0$ .)

SPARK catches the time-bomb:

```
File := Open("~/comments.txt",  
            Append_Mode);  
  
if (100*Done)/Full > 50 then  
  Append(File, Comment);  
end if;  
— SPARK proof fails:  
—   "cannot prove  
—   precondition!"  
  
Close(File)
```

(Even if  $\text{Full} > 0$  proven.)

# Mixing Ada and SPARK

```
pragma SPARK_Mode(on);
package X is
  type T is ...;

  procedure P(...)
  with
    Pre    => ...,
    Post   => ...;

  function F(...) return ...
  with
    Pre    => ...,
    Post   => ...;
end X;
```

```
pragma Spark_Mode(Off);
package body X is
  — SPARK doesn't check body

  procedure P(...) is
  begin
    ...
  end P;

  function F(...) return ... is
  begin
    ...
  end F;
end X;
```

- ▶ SPARK will check compilation units **withing** X
- ▶ **assuming** (without proof!) the body of X keeps its promises

# The SPARK toolset

1. SPARK checks conformity with SPARK rules and restrictions (no gotos no side-effects in functions, no exception handlers, no access types, no aliasing, ...).
2. SPARK performs flow analysis. It generates information about
  - ▶ global variables,
  - ▶ and dependencies.

If flow specifications are given (**Global, Depends**), SPARK verifies that the implementation conforms with the specification.

If no flow specifications are given, SPARK derives the flow information from the implementation.

3. And SPARK performs the functional analysis: SPARK
  - ▶ proves that no exceptions are raised,
  - ▶ proves that all calls to subprogram satisfy the subprograms' preconditions,
  - ▶ proves the postcondition (if given) (*partial correctness*),
  - ▶ and proves the "loop variants" (if given) (*termination*).

For the proofs, SPARK assumes that the precondition holds.

# Subprogram Contracts

beyond Ada 2012

```
procedure Decr_Abs (X : in out Integer) with  
  Pre => X /= 0,  
  Post => ( if X'Old < 0 then X = X'Old + 1  
            elsif X'Old > 0 then X = X'Old - 1);  
            — else the result is unspecified
```

```
procedure Decr_Abs(X: in out Integer) with Contract_Cases  
=> (X < 0 => X = X'Old + 1,  
     X > 0 => X = X'Old - 1,  
     X = 0 => True — can do anything if X = 0,  
                  — but must specify this case, else  
                  — "contract cases might not be complete"  
);
```

- ▶ sometimes easier to read or maintain
- ▶ must be disjoint and complete (SPARK checks this)

# To use SPARK, you should have a gnat project file

Here is one for homework problems: p.gpr

```
project P is
  for Source_Dirs use (".");

  package Compiler is
    for Default_Switches ("Ada")
      use ("-gnat12", "-gnat13");
    end Compiler;

  package Prove is
    for Switches use
      ("—report=fail",
       "—proof=progressive",
       "—warnings=continue",
       "—timeout=5");
    end Prove;

end P;
```



# What does the project file do?

```
project P is
  for Source_Dirs use (".");

  package Compiler is
    for Default_Switches ("Ada")
      use ("-gnat12", "-gnat013");
    end Compiler;

  ...
```

- ▶ check/compile in the current directory
- ▶ `-gnat12`: use Ada 2012
- ▶ (`-gnat013`: later in this chapter)

## What does the project file do? (2)

```
package Prove is
  for Switches use
    ("--report=fail",
     "--proof=progressive",
     "--warnings=continue",
     "--timeout=5");
  end Prove;
end P;
```

- ▶ `--report=fail`: only report failures to prove checks (no success stories, no statistics)
- ▶ `--proof=progressive`: start with one formula per check; split into paths when needed
- ▶ `--timeout=5`: after 5 seconds, prover gives up (default: 1 sec.)

Also: `--mode=flow` and `--mode=prove` (default: `--mode=all`).

## 4.1: First Examples

spec (data flow, precondition):

```
1 procedure What
2   (A: in Integer;
3    B,C: in out Integer;
4    D: out Integer) with
6   Global =>
7     null ,
9   Depends =>
10    (B => null;
11     C => (A,B,C) ,
12     D => A) ,
14  Pre =>
15    A+1 in Integer and
16    (B+C) in Integer;
```

implementation:

```
1 procedure What
2   (A: in Integer;
3    B,C: in out Integer;
4    D: out Integer) is
5   T: Integer := A;
6 begin
7   if T < 1 then
8     T := 1;
9   end if;
10  D := T+1;
11  C := B+C;
12  while C < 0 loop
13    C := C + T;
14  end loop;
15  B := 1;
16 end What;
```

## Example: Linear Search

The basic specification for linear search is the following:

**in:** Array A, value V (no precondition)

**out:** Index I with  $A(I)=V$  (this is the postcondition)

```
type Arr is array (Positive range <>) of Item;
```

```
A: Arr(First , Last) := Read_Array(From_Somewhere);
```

```
procedure Lin_Search (Value: Item; Result: out Positive) with  
Post => A(Result)=Value;
```

Anything wrong with this specification? Yes, it is incomplete:

1. What shall we output if no such I exists?
  - 1.1 Raise an exception, if no J with  $A(J)=V$  exists.
  - 1.2 New precondition: Require the existence of J with  $A(J)=V$ .
  - 1.3 Define special value “none” not in the range of A.  
Output I=“none” if no J with  $A(J)=V$  exists.
2. If there are several such indices I, which will we output?

# Fixing Problem 1.

1.1 Raise an exception: not permitted in SPARK.

```
procedure Lin_Search (Value: Item; Result: out Positive) with  
  Pre => (for some I in A'Range => A(I)=Value),  
  Post => A(Result)=Value;
```

1.2 Require the existence of an index J with A(J)=V.

```
No_Such_Index: Natural := 0; — no positive value
```

```
procedure Lin_Search (Value: Item; Result: out Natural) with  
  Post =>  
    (if (for all I in A'Range => A(I) /= Value) then  
      Result = No_Such_Index  
    else A(Result)=Value);
```

1.3 Define special value “none” to handle the case that no such index exists. (I prefer this!)

## Dealing with problem 2.

2. If there are several such indices  $I$ , which will we output?

2.1 The first index  $I$  with  $A(I)=V$ .

2.2 The last index  $I$  with  $A(I)=V$ .

2.3 The first prime index  $I$  with  $A(I)=V$ ,  
and the last index  $I$  with  $A(I)=V$  if no such prime index exists.

⋮ ...

2.\* We don't care. (I prefer this!)

# First Implementation

```
procedure Lin_Search (Value: Item; Result: out Positive) is  
begin  
    Result := A' First;  
    A(Result) := Value;  
end Lin_Search;
```

This matches the contract! But it is hardly what you would expect.  
Oh, and what happens when A is empty?

# Revised Specification

## global flow specification

```
No_Such_Index: Natural := 0; — no positive value
```

```
procedure Lin_Search (Value: Item; Result: out Natural) with  
  Global => in A, — read from global A,  
               — don't write to any gloabl  
  
  Post =>  
  ( if ( for all I in A'Range => A(I) /= Value ) then  
    Result = No_Such_Index  
  else A(Result) = Value );
```

The flow annotation “**Global** => A” makes sure that

1. A is the only global variable used, and
2. A is read from, but never written to.



# Revised Specification

rewrite `Lin_Search` as a function

```
function Lin_Search (A: Arr; Value: Item) return Natural with  
Post =>  
  ( if ( for all I in A'Range => A(I) /= Value ) then  
    Lin_Search ' Result = No_Such_Index  
  else A(Lin_Search ' Result) = Value );
```

Why did we specify `Lin_Search` as a **procedure** with a **Global** flow annotation at all?

→ Because I needed an example for writing a **Global** flow annotation.

For the problem at hand, the **function** is by far better!

## Second Implementation

```
function Linear_Search (A: Arr; Value: Item) Natural is  
  Idx: Positive := A'First;  
begin  
  while Idx <= A'Last loop  
    if A(Idx) = Value then  
      return Idx;  
    end if;  
  end loop;  
  return No_Such_Index;  
end Linear_Search;
```

- ▶ This implementation would pass static verification.
- ▶ Nevertheless, it hardly ever does, what we want it to do!
- ▶ Why? (Surprised?)

## Third Implementation: Loop Variant

```
function Linear_Search (A: Arr; Value: Item) Natural is
  Idx: Positive := A'First;
begin
  while Idx <= A'Last loop
    pragma Loop_Variant(Decreases => A'Last-Idx);
    — this loop does not run forever!
    if A(Idx) = Value then
      return Idx;
    end if;
    Idx := Idx + 1;
  end loop;
  return No_Such_Index;
end Linear_Search;
```

- ▶ This fixes the main problem from the previous code fragment.
- ▶ It still poor design.
- ▶ And in certain cases, it might raise a Constraint\_Error. (When?)

## Fourth Implementation: For-Loop

```
function Lin_Search (A: Arr; Value: Item) return Natural is  
begin  
  for I in A'Range loop  
    if A(I) = Value then  
      return I;  
    end if;  
  end loop;  
  return No_Such_Index;  
end Lin_Search;
```

A **for**-loop in Ada always terminates (no need for a loop variant).

But for the correctness proof, we still need a **loop invariant**:

- ▶ it must hold in first iteration,
- ▶ **if** it did hold in the previous iteration,  
 **then** it also holds in the current one, and
- ▶ it should allow us to prove the (missing part of the) postcondition.

(→Black- or whiteboard!)

## 4.2: Example: Seven Swaps

- ▶ task: swap two integer variables
- ▶ provide six flawed implementations and
- ▶ use `gnatprove` from the command line to find flaws
- ▶ our main program “test.adb” looks like this:

```
with Swap_It; use Swap_It;

procedure Test is
  X, Y: Integer;
begin
  X := 0; Y := 1;
  Swap(X, Y);
  if X = Y then
    raise Program_Error;
  end if;
end Test;
```

# First Swap

```
package Swap_It is
  procedure Swap (X : in out Integer;
                 Y : in out Integer) with
    Global => null,
    Depends => (X => Y, Y => X);
end Swap_It;
```

```
package body Swap_It is
  procedure Swap (X : in out Integer;
                 Y : in out Integer) is
  begin
    X := Y;
    Y := X;
  end Swap;
end Swap_It;
```

# Analyzing First Swap

```
> gnatprove -Pswapper --mode=flow
Phase 1 of 2: frame condition computation ...
Phase 2 of 2: translation to intermediate language ...
swap_it.ads:2:20: ineffective import "X"
swap_it.ads:5:11: "X" missing from null dependency
swap_it.ads:5:31: "Y" depends on "Y"
swap_it.ads:5:31: "Y" does not depend on "X"
```

**Flaw Found!**

## Second Swap

```
procedure Swap (X : in out Integer ;  
                Y : in out Integer) is  
begin  
    X := X + Y ;  
    Y := X - Y ;  
    X := X - Y ;  
end Swap ;
```

That looks alright – the additions and subtractions seem to cancel out.



# Analyzing Second Swap

```
> gnatprove -Pswapper --mode=flow
Phase 1 of 2: frame condition computation ...
Phase 2 of 2: translation to intermediate language ...
swap_it.ads:5:23: "X" depends on "X"
swap_it.ads:5:31: "Y" depends on "Y"
```

Wait a minute: “ $X := X + Y - Y$ ” cancels out, but the result is  $X$ , not  $Y$ !

# Third Swap

```
procedure Swap (X : in out Integer ;
                Y : in out Integer)
is
begin
  X := X + Y;
  Y := X - Y; — = (X'Old + Y) - Y = X'Old
  X := X - Y; — = (X'Old + Y'Old) - X'Old = Y'Old
end Swap;
```

This time, the program logic appears to be OK (see comments)

## Third Swap: gnatprove is not convinced

```
> gnatprove -Pswapper --mode=flow
Phase 1 of 2: frame condition computation ...
Phase 2 of 2: translation to intermediate language ...
swap_it.ads:5:23: "X" depends on "X"
swap_it.ads:5:31: "Y" depends on "Y"
```

This doesn't mean our program is wrong. It means, `gnatprove` cannot verify it is correct!

Flow analysis doesn't "know" the nature of operations, such as additions and subtractions cancelling out!

Is our program correct?

## Third Swap: A Real Flaw

```
> gnatprove -Pswapper --mode=prove
Phase 1 of 3: frame condition computation ...
Phase 2 of 3: translation to intermediate language ...
...
Phase 3 of 3: generation and proof of VCs ...
analyzing Swap3.Swap, 3 checks
swap_it.adb:7:14: overflow check not proved
swap_it.adb:8:14: overflow check not proved
swap_it.adb:9:14: overflow check not proved
```

Ouch! Additions and subtractions can overflow!

# Swap Number Four

```
procedure Swap (X : in out Integer;  
                Y : in out Integer)  
is  
    T: Integer := X + 1;  
begin  
    X := Y;  
    Y := T;  
end Swap;
```

This is clearly wrong, and may overflow!

## Fourth Swap

```
> gnatprove -Pswapper --mode=flow
Phase 1 of 2: frame condition computation ...
Phase 2 of 2: translation to intermediate language ...
```

So the information flow appears to be OK, now.

```
> gnatprove -Pswapper --mode=prove
...
swap_it.adb:6:23: overflow check not proved
```

But  $X + 1$  can overflow.

# Swap Five: Fix the Overflow

```
procedure Swap (X : in out Integer;  
                Y : in out Integer) with  
  Global => null,  
  Depends => (X => Y, Y => X),  
  Pre      => X < Integer'Last;
```

```
> gnatprove -Pswapper --mode=flow
```

```
Phase 1 of 2: frame condition computation ...
```

```
Phase 2 of 2: translation to intermediate language ...
```

```
...
```

```
> gnatprove -Pswapper --mode=prove
```

```
Phase 1 of 3: frame condition computation ...
```

```
Phase 2 of 3: translation to intermediate language ...
```

```
...
```

```
Phase 3 of 3: generation and proof of VCs ...
```

```
analyzing Swap_It.Swap, 1 checks
```

# Why does gnatprove not find the flaw?

```
procedure Swap (X : in out Integer ;  
                Y : in out Integer) is  
  — same as swap four  
  T: Integer := X + 1;  
begin  
  X := Y;  
  Y := T;  
end Swap;
```

If  $X = 3$  and  $Y = 13$ , what does `Swap(X, Y)` provide?

Why doesn't gnatprove find the flaw?



## Swap Six: Specify, what you really want!

```
procedure Swap (X : in out Integer;  
               Y : in out Integer) with  
  Global      => null ,  
  Depends    => (X => Y, Y => X) ,  
  Pre        => X < Integer ' Last ,  
  Post       => ((X = Y' Old) and (Y=X' Old));
```

(The implementation is the same as for swap four and five.)

Will `gnatprove` tell us what is wrong?

## Swap Six

```
> gnatprove -Pswapper --mode=flow
```

```
Phase 1 of 2: frame condition computation ...
```

```
Phase 2 of 2: translation to intermediate language ...
```

```
...
```

```
> gnatprove -Pswapper --mode=prove
```

```
Phase 1 of 3: frame condition computation ...
```

```
Phase 2 of 3: translation to intermediate language ...
```

```
'''
```

```
Phase 3 of 3: generation and proof of VCs ...
```

```
analyzing Swap_It.Swap, 2 checks
```

```
swap_it.ads:7:22: postcondition not proved
```

```
analyzing precondition for Swap_It.Swap, 0 checks
```

**Postcondition not proved!**

# Swap Seven: All is well, Finally!

```
procedure Swap (X : in out Integer;  
                Y : in out Integer) with  
  Global    => null ,  
  Depends  => (X => Y, Y => X),  
  Post     => ((X = Y'Old) and (Y=X'Old));
```

```
procedure Swap (X : in out Integer;  
                Y : in out Integer) is  
  T: Integer := X;  
begin  
  X := Y;  
  Y := T;  
end Swap;
```

Confirmed by `gnatprove`: This is correct!

## 4.3: Example: Math

for a moment, remove `-gnat013` from the switches

```
project Adder is
  for Source_Dirs use (".");

  package Compiler is
    for Default_Switches ("Ada") use ("-gnat12");
                                     --, "-gnat013");
  end Compiler;

  package Prove is
    for Switches use
      ("--report=fail",
       "--proof=progressive",
       "--warnings=continue",
       "--timeout=5");
    end Prove;

end P;
```

# Specification and Implementation

```
pragma Spark_Mode (On);  
package Add is  
  
    function Sum(X: Integer; Y: Integer) return Integer with  
        Pre  => X + Y in Integer ,  
        Post => Sum' Result = X + Y;  
  
end Add;
```

```
pragma Spark_Mode (On);  
package body Add is  
  
    function Sum(X: Integer; Y: Integer) return Integer is  
    begin  
        return X + Y;  
    end Sum;  
  
end Add;
```

# Flow Check gnatprove

```
> gnatprove -Padder --mode=flow  
Phase 1 of 2: frame condition computation ...  
Phase 2 of 2: translation to intermediate language ...
```

No surprise: No errors!

# Proving with gnatprove

```
> gnatprove -Padder --mode=prove
```

```
...
```

```
add.ads:5:17: warning: overflow check might fail
```

- ▶ `gnatprove` could not disprove an overflow
- ▶ but the problem is in the precondition, not in the implementation

```
pragma Spark_Mode (On);  
package Add is
```

```
    function Sum(X: Integer; Y: Integer) return Integer with
```

```
        Pre => X + Y in Integer ,
```

```
        Post => Sum' Result = X + Y;
```

```
end Add;
```

# The Source of the Problem

```
function Sum(X: Integer; Y: Integer) return Integer with  
  Pre => X + Y in Integer,  
  Post => Sum' Result = X + Y;
```

Catch 22: Before we could possibly check “X+Y in Integer”, we’d have to add X and Y, and if they are not in Integer, this would overflow.

Possible fix: Rewrite the “offending” precondition:

```
function Sum(X: Integer; Y: Integer) return Integer with  
  Pre => ( if X > 0 then  
            Integer 'Last-X >= Y  
          else  
            Integer 'First-X <= Y);  
  Post => Sum' Result = X + Y;
```



# A Different Fix

... without rewriting the precondition

-gnato13:

Tell `gnat` and `gnatprove` to assume **unbounded mathematical integers** in assertions (including pre- and postconditions).

```
for Default_Switches ("Ada") use ("-gnat12", "-gnato13");
```

```
> gnatprove -Padder
```

```
Phase 1 of 3: frame condition computation ...
```

```
Phase 2 of 3: analysis and translation to intermediate
```

```
Phase 3 of 3: generation and proof of VCs ...
```

```
analyzing Add, 0 checks
```

```
analyzing Add.Sum, 2 checks
```

# Spec of a Simple Math Package

```
pragma Spark_Mode(On);  
package Math is  
  
  function SQRT(X: Natural) return Natural with  
    Contract_Cases =>  
      ( X <= 1 => SQRT' Result = X,  
        X > 1  => SQRT' Result > 1 and then  
          X/SQRT' Result <= SQRT' Result and then  
            X/(SQRT' Result-1) > (SQRT' Result-1)  );  
  
  function Log_2(X: Positive) return Natural with  
    Post => 2**Log_2' Result <= X and then  
            2**Log_2' Result > X/2);  
  
end Math;
```

Observe that the postcondition of SQRT is written as a complete set of disjoint Contract\_Cases.

# Implementation of SQRT

```
function SQRT(X: Natural) return Natural is
  R: Positive;
begin
  if X <= 2 then
    return X;
  else
    R := 2;
    while X/R > R loop
      pragma Loop_Variant(Increases => R);
      R := R + 1;
    end loop;
    return R;
  end if;
end SQRT;
```

# Implementation of SQRT

```
function SQRT(X: Natural) return Natural is
  R: Positive;
begin
  if X <= 2 then
    return X;
  else
    R := 2;
    while X/R > R loop
      pragma Loop_Variant(Increases => R);
      R := R + 1;
    end loop;
    return R;
  end if;
end SQRT;
```

- ▶ To prove program termination, we did provide a **loop variant**.
- ▶ In the case of SQRT, we have been exceptionally lucky: SPARK did automatically find a **loop invariant**, with which it managed to prove the postcondition.
- ▶ Most of the time, we must supply the **loop invariant** manually.
- ▶ To understand more about choosing **loop variants**, **loop invariants**, and how to get the proof done, we will have to introduce some theory.