

Computer Animation

8-Collisions

SS 15

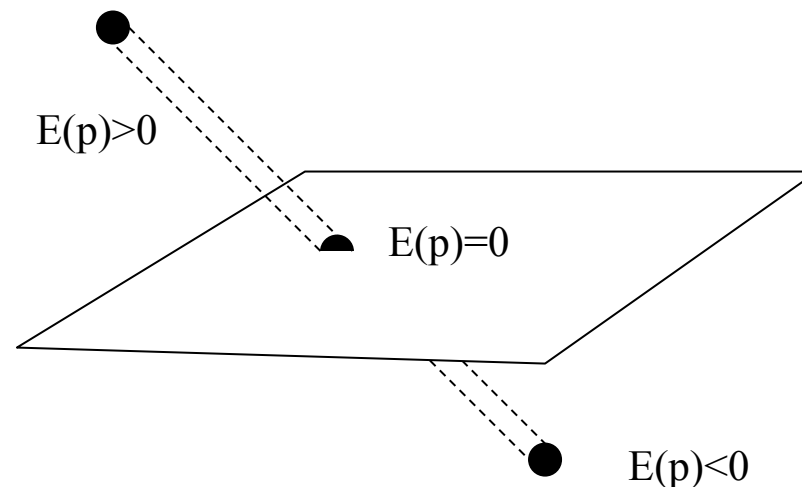
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Collisions

- When objects start to move, they actually collide
- Two issues must be addressed:
 - Detecting collision
 - Computing appropriate response
- Detecting collision: two main approaches
 - Penalty method: calculate the reaction after collision has occurred
 - when more particles involved, assume they collided at same instant
 - Imprecise but often acceptable
 - Back up time to first instant of collision and compute appropriate response
 - By heavy no of collisions, quite time consuming
- Computing the appropriate response to collision (depends on physics and distribution of mass of the object)
 - Kinematic response
 - Penalty method: introduce a nonphysical force to restore non penetration but compute it at time of collision
 - Calculation of impulse force

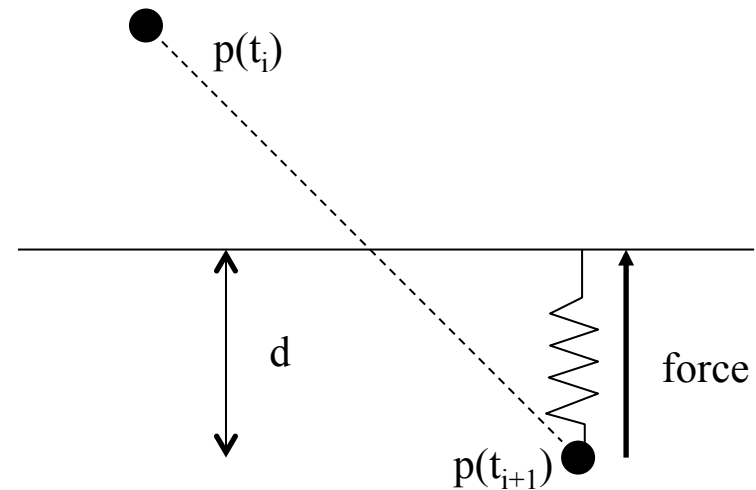
Kinematic response

- A simple case is a particle moving at constant velocity and impacting a plane
 - Questions:
 - When is the impact?
 - How does it bounce off?
 - Use plane equation
 $E(p): ax+by+cz+d$
 - If normals correct, then
 - If $E(p)=0$ then p plane point
 - If $E(p)>0$ then p above plane
 - If $E(p)<0$ then below plane
 - The particle moves with equations:
 $p(t_i)=p(t_{i-1})+t \cdot v_{ave}(t)$
- When $E(p(t_i))$ switches to ≤ 0 then we had a collision
 - Now the component of the velocity parallel to the normal to the plane is negated
 - Some damping factor N is added
$$v(t_{i+1})=v(t_i)-v(t_i)N-kv(t_i)N$$
$$=v(t_i)-(1+k)v(t_i)N$$



Penalty method

- Here we construct a reaction to the collision
 - A spring with zero rest length is attached at the instant of collision
 - The closest point on the surface to the penetrating point is used as attachment point
 - The spring obeys Hooke's law: $F = -kd$
 - The approach needs to assign arbitrary masses and constant, and therefore is not ideal
- Moreover, for fast moving points it might take a few steps to push back the obj
 - For polyhedra, it might also generate torque



Polyhedras colliding

- Shape can be complicated for complex objects
- Thus, collisions can be tested before on bounding boxes
- Or by adding hierarchical bounding boxes
- Testing a point to be inside a polyhedron is not easy
- But for a polyhedron one needs to test all vertices for the two objects
- And each point has to be tested against all the planes of the faces of the polyhedron
- This works only for convex polyhedra
- For concave polyhedra, one can use a similar method to the point in polygon test
- Construct a semi-infinite ray from the point towards the polyhedron, and check no of intersections
 - If they are even, then the point is outside
 - If they are odd, then it lies inside
- Of course correct counting double points has to be done
- In some cases, for solids of simple shape and moving with an easy movement, the volume of it can be swept along its trajectory

Impulse force of collision

- To do accurate computations, time has to be backed to the instant of collision
- Then the exact reaction can be computed
- If a collision appeared between t_i and t_{i+1} , then
 - recursive bisection of the time step between these two timepoints will eventually yield the exact time of the impact
 - Alternatively, a linear approximation of the velocity can be used to simplify the calculations
- At the time of the impact, the normal component of the point velocity can be modified to reflect the bounce
- This normal can be multiplied by a scalar to model the degree of elasticity of the impact
- This scalar is called coefficient of restitution

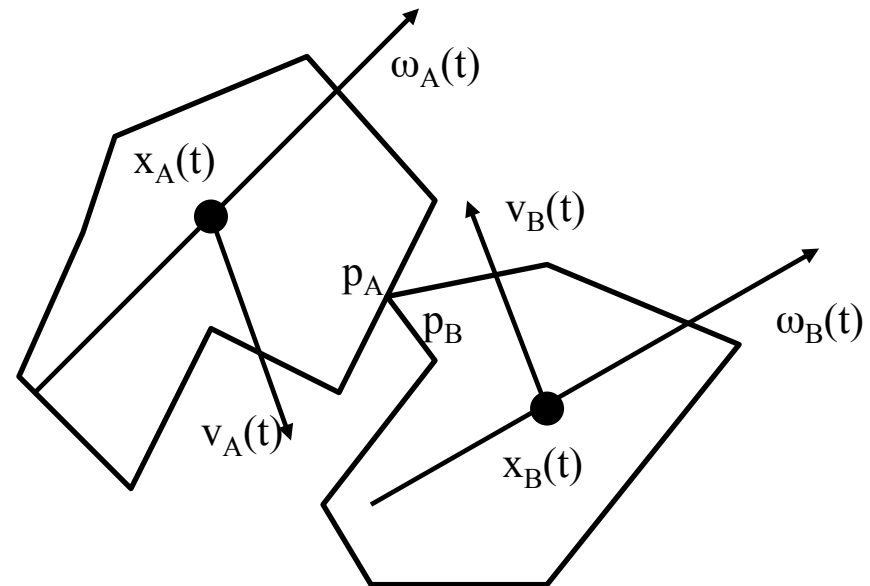
Impulse forces

- Once the simulation is backed up to the time of the collision, the reaction can be computed
- By working back from the desired change in velocity, the required change in momentum can be deduced
- This equation uses the a new term, the impulse, expressed in units of momentum
 $J = F\Delta t = Ma\Delta t = M\Delta v = \Delta(Mv) = \Delta P$
- J can be seen as a large force acting in a short time interval
- This allows computing the new momentum
- To characterize elasticity, the coeff. of restitution, ε is computed ($0 \leq \varepsilon \leq 1$)
- The velocities along the normal before and after the impact are related by $v_{rel}^+ = -\varepsilon v_{rel}^-$

Impulse forces

- Assume that the collisions of the two objects A and B has been detected at t
- Each obj Ob has position of mass center $x_{Ob}(t)$, lin. velocity $v_{Ob}(t)$ and ang. velocity $\omega_{Ob}(t)$
- At the point of intersection, the normal to the surface of contact is determined (note, it can be a surface, but also a point)
- Let r_A and r_B be the relative positions of the contact points WRT the center of mass
- Relative velocities of the contact points WRT center of mass and the velocities of the contact points are computed as

- $r_A = p_A - x_A(t)$
 $r_B = p_B - x_B(t)$
 $v_{rel} = (p_A^\circ(t) - p_B^\circ(t))$
 $p_A^\circ(t) = v_A(t) + \omega_A(t) \times r_A$
 $p_B^\circ(t) = v_B(t) + \omega_B(t) \times r_B$



Impulse forces

- Linear and angular velocities of the objects before the collision

$v_{ob}^- \omega_{ob}^-$ are updated $v_{ob}^+ \omega_{ob}^+$

$$v_A^+ = v_A^- + jn/M_A$$

$$v_B^+ = v_B^- + jn/M_B$$

$$\omega_A^+ = \omega_A^- + I_A^{-1}(t)(r_A \times j \cdot n)$$

$$\omega_B^+ = \omega_B^- + I_B^{-1}(t)(r_B \times j \cdot n)$$

where the impulse J is a vector quantity in the direction of the normal

$$J = j \cdot n$$

- To find the impulse, the difference between the velocities of the contact points after collision in the direction of the normal to the surface of collision is formed

$$v_{rel}^+ = n \cdot (p_A^+(t) - p_B^+(t))$$

$$v_{rel}^+ = n \cdot (v_A^+(t) + \omega_A(t) \times r_A - v_B(t) + \omega_B(t) \times r_B)$$

- Substituting previous equations one obtains

$$j = \frac{-((1 + \epsilon) \cdot v_{rel}^+)}{\frac{1}{M_A} + \frac{1}{M_B} + n \cdot (I_A^{-1}(t)(r_A \times n)) \times r_A + (I_B^{-1}(t)(r_B \times n)) \times r_B}$$

- Contact between two objects is defined by the point on each involved and the normal to the surface of contact
- If the collision occurs, the eq. above is used to compute the magnitude of the impulse
- The impulse is then used to scale the contact normal, and update linear and angular momenta

Friction

- An object resting on another one has a resting contact with it
- This applies a force due to gravity which applies to both objects and can be decomposed along the directions parallel F_{Pa} to the resting surface and F_N perpendicular to it
- The static friction force is proportional to F_N :
$$F_s = \mu_s F_N$$
- Once the object is moving, there is a kinetic friction taking place. This friction creates a force, opposite to the direction of travel, and again proportional to the normal
$$F_k = \mu_k F_N$$

Resting contact

- It is difficult to compute forces due to the resting contact
- For each contact point, there is a force normal to the surface of contact
- All these forces have to be computed for all objects involved in resting contact
- For each contact point, a torque is also generated on it.
- If bodies have to rest, all those forces and torques have to be zero
- Solutions to this problem include quadratic programming, and are beyond the scope of this course

Constraints

- One problem occurring in animation is the fact that variables are not free.
- Constraints are usually set on objects and limit the field of the independent variables.
- There are two types of constraints:
 - hard constraints: strictly enforced
 - soft constraints: the system only attempts to satisfy them

Flexible objects

- To simulate elastic objects, Spring-mass-damper model is most used approach
- Springs: work with Hooke 's law: the force applied is $F_{i,j} = -F_{j,i} = k_s(d_{i,j}(t) - \text{len}_{i,j})v_{i,j}$ where
 - d_{ij} distance between the two points
 - len_{ij} rest length of the spring
 - k_s spring constant
 - v_{ij} unit vector from point i to point j
- The flexible model is modelled as a net of points with mass and springs and dampers between them
- A damper can impart a force in the direction opposite to the velocity of the spring length and proportional to that velocity $F^d_i = -k_d v_i(t)$
- One can also introduce angular dampers and springs between faces
- Additional internal springs have often to be added to add stability to the system



Virtual springs

- Induce forces that do not directly model physical elements
- For example, in the penalty method
- Sometimes one can use a proportional derivative controller which controls that a certain variable and speed is close to the desired value
- For example, this is used to keep the object close to the desired speed
- A virtual spring is added to keep things as desired

Energy minimization

- One can use energy to control the motion of the objects
- Energy constraints can be used to pin objects together, to restore the shape of an object, to minimize the curvature of a path or trajectory
- Energy constraints induce restoring forces on the system

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