Computer Animation
4-Motion Control
SS 15

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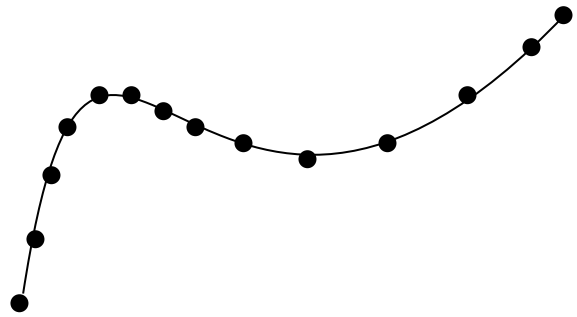
Controlling motion along curves

- We all know now how to control the shape of the curve
- To an animator, it is equally important to know the speed at which a curve is traced by increasing parametric steps
- Obviously, since motion curves are of higher order, this relation is not straightforward
- Equal parameter intervals do not lead to arcs of equal length on the curve
- That is, speed is different at different points of the curve
- This can be overcome through a reparametri-zation of the curve



Curve length

- There are different methods to compute such a reparametrization
- One can create a table of values so as to establish a relationship between arc length and parameter values
- In the first two method, one creates a table of values to establish the relationship between parametric value and approximate arc length
- Once the table is built, one can use the table to approximate values of the parameter at steps of equal length along the curve

Curve length

- The first method supersamples the curve, and then uses summed linear ´distance to compute the approximate arc length
- The second method uses Gauss quadrature to numerically estimate the arc length
- Both methods can use adaptive subdivision to control the error
- The third method analytically computes arc length. Unfortunately, it is not always possible to do so for all curves.

Computing arc length

- To specify how fast the object moves in the environment, animators might want to specify the time at which positions along the curve are reached.
- This in general would be position and frame pairs.
- Or, maybe, the animator might want to specify velocities

- For example:
  - start at position A
  - accellerate till frame 20
  - move at constant speed till frame 35
  - Decelerate slowly till frame 60 and end at position B
- It is clear what we want: be able to control not only the curve (*space function*), but also the relationship between position and time (*distance-time function*).
- The distance we are traveling along the curve is called the *arc length* and will be denoted by *s*.

Computing arc length

- Suppose that we are moving along the curve

  $$P(u)=U^TMB$$

- The relation between parameter and arc length is not linear.
- When a unit change in parameter results in a unit change in curve length the curve is said to be *parametrized by arc length*
- How do I establish the relationship between parameter and arc length?
- What we want is to kow the function s=G(u) which computes the length of the curve from it starting point for all values of the paramenter u
- If we have G, then knowing G^{-1} allows us to compute the parameter values corresponding to a certain length

Arc length: analytic approach

- Obviously, the length of a curve between parameter values u_1 and u_2 is

  $$s=\int_{[u1,u2]}|dP/du|du,$$

  where

  $$dP/du= ((dx(u)/du), (dy(u)/du), (dz(u),du))$$

  and

  $$|dP/du|=SQRT(dx(u)/du)^2+ (dy(u)/du)^2+ dx(z)/du)^2)$$

- For a cubic curve
  $P(u)=\underline{a}u^3+\underline{b}u^2+\underline{c}u+\underline{d}$
  this will mean that the derivative of one of the 3 eq with respect to u is
  $dx(u)/du=3a_xu^2+2b_xu+c_x$
  and under the SQRT one would have a curve of 4th degree
  $\underline{A}u^4+\underline{B}u^3+\underline{C}u^2+\underline{D}u+\underline{E}$
- With a bit of computations one can compute then A,B,C,D

Arc length: Estimating through forward differences

- Suppose we have P(u).
- One can compute a table of the distance of P(u) from the point P(0) at regular intervals:

  P(0), P(Δu), P(2Δu),..,P(1)

  that is, containing

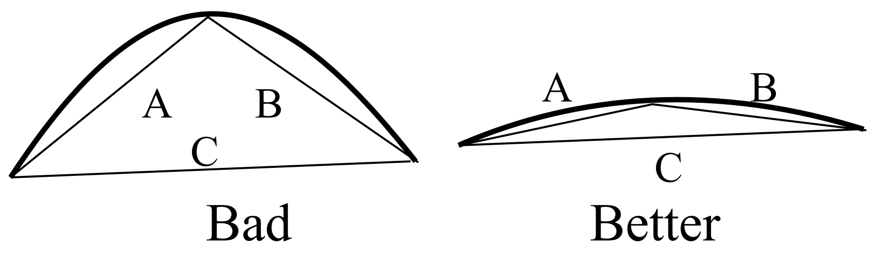
  P((i+1)Δu)-P(i Δu)

- One can interpolate these values first order (or higher order) to estimate the length of a segment in image space
- Conversely, one can use similar methods to deduce from the right hand column the corresponding value of u
- Main problem with this approach is controlling the error

0	|P(Δu)-P(0)|=G(Δu)
Δu	G(ΔU) +|P(2Δu)-P(Δu)|=G(2 Δu)
2 Δu	G(2ΔU) +|P(3Δu)-P(2Δu)|=G(3 Δu)
...	...

Adaptive forward differences

- Since the relations between the variation of the parameter and the length of the curve is non-linear, the method of the last slide has problems when there is a big error
  - i.e. When the polyline implicitly used to estimate the parameter values inbetwen table points is far from the actual curve
- This can be improved by computing the value of the midpoint of each interval between the table points.
  - if the sum of the sides A+B of the triangle is too different in length from the line joining the interval extre-mes C (over a threshold value), the midpoint is added to the list



Numerical meth.: Gaussian quadrature

- Another approach to computing length is bases on numerics
- Computing the length of the curve implies computing the integral of the curve length
- Gaussian quadrature uses unevenly spaced intervals to achieve the greatest accuracy
- Gaussian quadrature computes

$$s=\int_{[0,1]} f(u)du \sim \Sigma_i w_i f(u_i)$$

- Since Gaussian quadr. is usually defined in the interval [0,1], one has to reparametrize at first the original interval [a,b] we are considering
- This is achieved by using the new parameter t such that

$$t=(2u-a-b)/(b-a)$$

- Do not forget to apply the usual integral rules for changing parameters, that is adding the factor of parameter substitution to the integral

Adaptive Gaussian quadrature

- If the curve derivative varies very fast in some areas, and less fast in other areas, the gaussian quadrature will either undersample part of the curve, or oversample

- In this case, a similar adaptive method to the one presented before can be used:
  - One subdivides intervals in half,
  - each half is evaluated using gaussian quadrature
  - The sum of the two halves is compared to the result of the whole interval.
  - If the difference is greater than a certain threshold, then the two halves are added to the sample points

Finding u given s

- Suppose one wants to find the value of the parameter u at a given arc length s from the point $R(u_1)$
- This equals to solving the equation
  $s\text{-}LEN(u_1,u)=0$
- Arc length is monotonic, so such a solution is unique as long as dR(u)/du is not 0

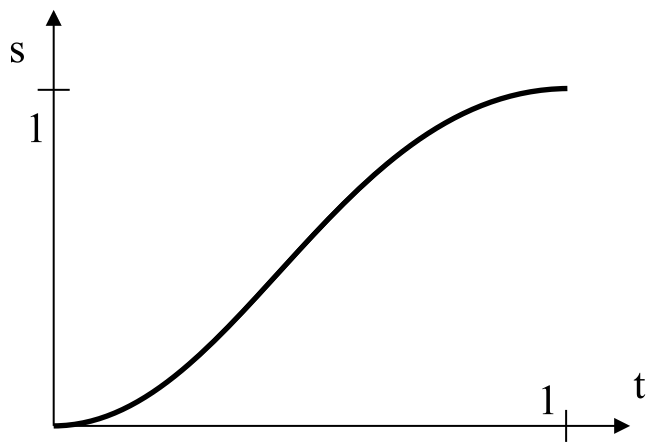
- Newton-Raphson integration can be used: generate the sequence $\{p_n\}$

  $$p_n=p_{n-1}-f(p_{n-1})/f'(p_{n-1})$$

  where
  - f is $s\text{-}LEN(u_1,P_{n-1})=0$ and can be evaluated at p_{n-1} using techniques of last slide
  - f´ is dP/du evaluated at p_{n-1}
- This eliminates the need for quadrature, and is faster
- But can have two problems:
  - Some p_k might not be on the curve, thus also p_{k+1},p_{k+2},.. will not
  - When the derivative approaches 0 we divide by zero
- Use subdivision instead

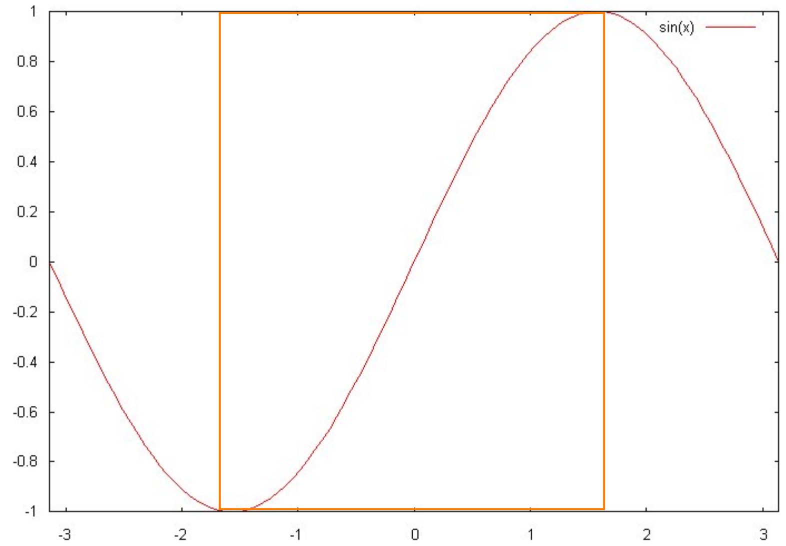
Speed control

- On a arc-length parametrized curve, it is possible to control speed
- Simplest (and dullest) control: constant speed (equal space *s* in equal time *t*)
- Easiest speed control is *ease-in*/ *ease-out*:
  - From standstill, accelerate until maximum speed
  - Decelerates and stop
- Speed along a curve can be controlled by varying arc length at something else than a linear function of *t*.
- The speed variations are seeable in the distance-time curve, which plots the space traversed *s* against the time *t*.
- Here is an example of a distance-time curve for ease-in



Speed control: ease in/ease out

- There are different ways of mathematically achieve ease in/ ease out
- The first one is to use the sinus between $-\pi/2$ and $\pi/2$ and scaling the parameter to cover [0,1]
- $S(t)=(1/2)(\sin(\pi t-\pi/2)+1)$
- This curve can be split and joined with a straight line (take care of continuity at the splits) to add a period of constant speed



Speed control: constant acceleration

- The computational cost of the sinus function is high.
- A better method is to use physics for the calculations: s=vt, and v=at
- This obtains a parabolic ease-in function thus $s=at^2$
- Similarly for deceleration one can use a constant (limited) deceleration until the object stops
- To describe the distance-time function of such a movement the following equations are used

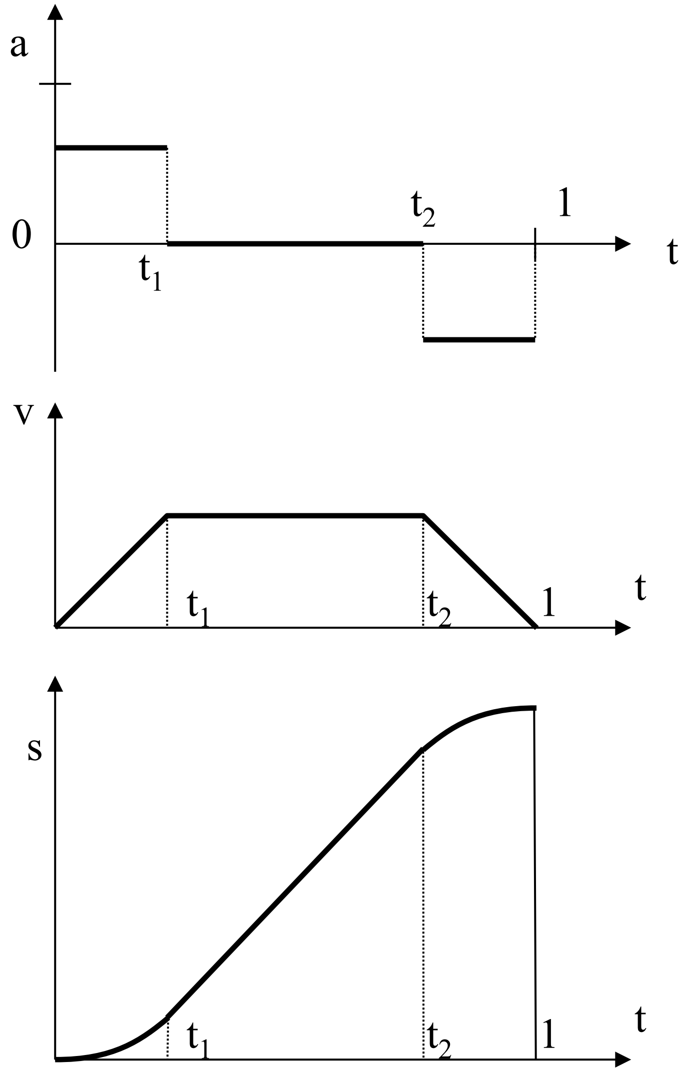
- In formulas:

$d=½t^2/2t_1$ $0<t<t_1$
$d=½v_0t_1+v_0(t-t_1)$ $t_1<t<t_2$
$d=½v_0t_1+v_0(t-t_1) + (v_0-½(v_0(t-t_2)/1-t_2)(t-t_2)$ $t_2<t<1$

- Whereby v_0 is the velocity when acceleration ends

Speed control: constant acceleration

- $a=a_0$ $\quad 0<t<t_1$
  $a=0$ $\quad t_1<t<t_2$
  $a=-a_0$ $\quad t_2<t<1$
- $v=v_0 t/t_1$ $\quad 0<t<t_1$
  $v=v_0$ $\quad t_1<t<t_2$
  $a=v_0(1-(t-t_2)/(1-t_2)$ $t_2<t<1$
- The formulas look really complicated, but there are different ways to plot this to make it understandable



General distance-time functions

- Many interesting aspects come up when allowing the user to control motion
- The more influence a user is given, the more problems come up
- Suppose the user defines some velocities at some points:
  - The rest of the velocity curve has to be fitted to these „fixed“ values
  - Sometimes leading to unwanted effects (reverse velocity to fit the time contraints)
- More intuitive is to control on the space-time curve
  - This because it allows to control velocities as a tangent, and to adapt the rest of the curve accordingly
- Motion control often requires specifying positions at specific times
  - The motion is specified as a series of constraints at a specific time, formally, a t-uple $<t_i,s_i,v_i,a_i,...>$
  - higher order approximation is needed for smooth movement

Curve fitting

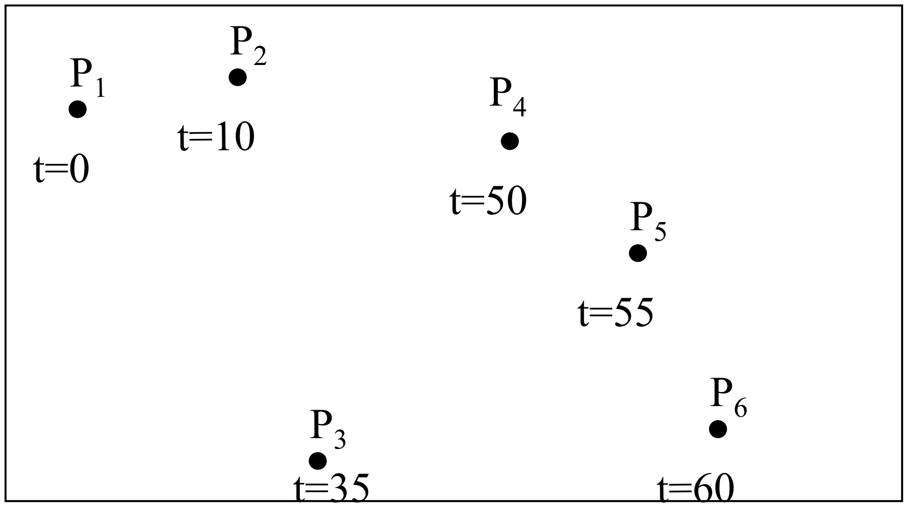
- If the animator specifies certain constraints then the time parametrized curve can be computed using these constraints as control points
- Suppose contraints are of the form (P_i, t_i) $(i=1,\ldots,j)$
- It only requires to compute the curve passing through these points, i.e.

  $$P(t) = \Sigma_{1,n} B_i N_{i,k}(t)$$

  with $2 \leq k \leq n+1 \leq j$
- In matrix form $P=NB$
- Inverting this equation leads to find the control point values for the curve

Curve Fitting to position-time pairs

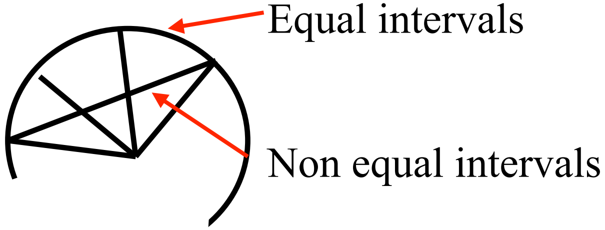
- Suppose the user gives the following positions and the corresponding times
- One can fit a B-spline curve to the values (P_i,t_i) $(i=1,\ldots j)$:
  - That is, take the general eq. of B-splines and make it pass through points
  - Find corresp. control points.



- Computing the curve passing through these points means computing $P(t)=\Sigma_{1,\ldots,n} B_i N_{i,k}(t)$ with $2 \leq k \leq n+1 \leq j$
- In matrix form P=NB,
- Inverting this equation leads to find the control point values for the curve: $B=N^{-1}P$
- This is done through the pseudoinverse:
  $P=NB$
  $N^TP=N^TNB$
  $[N^TN]^{-1}N^TP=B$
- Remember the tradeoff: the higher the order, the higher the wiggling

Interpolation of quaternion rotations

- A major reason for choosing quaternions is that they can be easily interpolated
- Quaternion form can be interpolated to produce good intermediate orientations
- This does not work easily with direct interpolation
- Unit quaternions are used to represent orientation, and can be seen as point of on the unit sphere in 4-dimensional space
- To interpolate between two unit quaternions, one can linearly interpolate
- But this will not produce constant speed rotation, because a path on a sphere is not the same as a path on a plane (which is what linear interpol. follows)
- Equal speed interpolations can be computed by interpolating directly on the path on the sphere



Interpolation of quaternion rotations

- The problem (of course) is how to do that
- Remember: $q=[s,v]$ and $-q=[-s,-v]$ represent the same orientation
- So interpolation from q_1 to q_2 can be also carried between q_1 and $-q_2$.
- The difference is that one path will be longer
- The shorter one is the one distinguished by the smallest angle

- One can compute the cosine of the angle between q_1 and q_2:

$$\cos\theta = q_1 \cdot q_2 = s_1 \cdot s_2 + v_1 \cdot v_2$$

- If it is positive, then shortest path is from q_1 to q_2
- Else shortest path is from q_1 to $-q_2$

Interpolation of quaternion rotations

- So, the spherical linear interpolation (SLERP) between q_1 and q_2 with parameter uÎ[0,1] is

  $$\mathrm{SLERP}(q_1,q_2,u)=((\sin((1-u)\theta))/\sin\theta))q_1+ (\sin(u\theta))/\sin\theta q_2$$

- Note that this does not generate a unit quaternion, so one has to normalize the result
- Notice that in the case u=1/2, SLERP is easy to compute except for a scaling factor
- Finally notice that if a chain of SLERPs is performed, it will perform similarly to linear interpolation (i.e. with rough changes)
- Higher order interpolations, based on Bezier curves, have been developed, but are beyond the purpose of this lesson

Following a path

- Animating an object to move along a path is quite natural and common
- Not only following the path is needed: also moving the orientation
- Typically, one would have a local coordinate system associated with the object
- Let the coordinates be (u,v,w), and suppose they are right handed
- Suppose the origin of the coordinate system follows the curve P(s), and that the movement of P(s) is specified
- Call POS the current position
- One can view the u,v,w coordinates as a view vector, an up vector and a vector perpendicular to u and v
- This is similar to camera definition in Computer Graphics

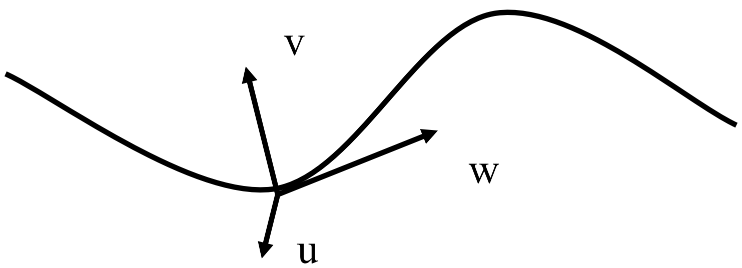
Following a path: Frenet Frame

- The orientation of the camera system can be made dependent from the properties of the curve P(s)
- A Frenet frame is given by the following axes definitions
  - w follows the tangent of the curve (its first derivative P´(s))
  - v is orthogonal to w and in the direction of the second order derivative (P´´(s))
  - u is the cross product of w and v
- In symbols:

  $w=P'(s)$

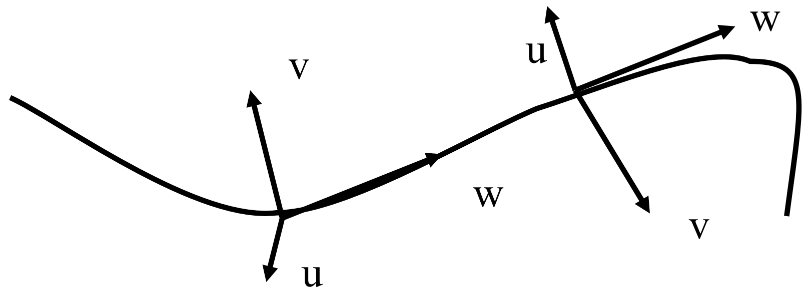
  $u=(P'(s) \times P''(s)$

  $v=w \times u$



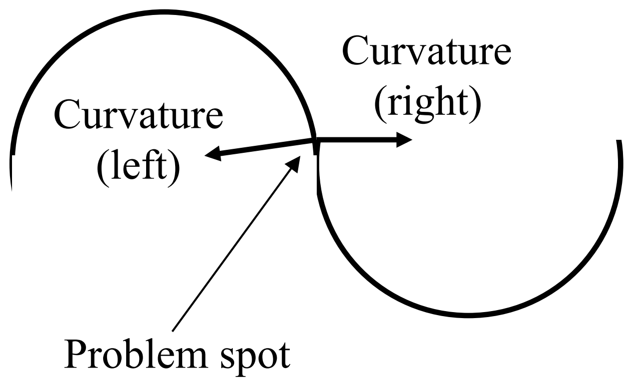
Following a path: Frenet Frame

- Frenet frames are quite nice, but bear some flaws
- When the curve has no curvature, its second order derivative is zero. Here the Frenet frame is undefined
  - This problem can be solved by interpolating the Frenet frames at the start and end of the rectilineal trait
  - Since the tangent vector must be the same at the extremities, it is only a rotation that has to be interpolated



Following a path: Frenet Frame

- A more complicated problem occurs at discontinuities in the curvature vector
- For example, when the path follows first a circle, and then a second circle
- At the problem point, the curvature will switch to pointing from one circle center to the other one
- Here, the Frenet frame is defined everywhere but is discontinuous
- Here, the object will rotate wildly along the path with „instant switches“



Following a path: Frenet Frame

- The worst problem is that the path following is not so natural:
  - when we view at something, we we do not look along the tangent
  - When we move, we anticipate curves
- Similar effect to your car light not following the road
- Also, one might want to make the object bend towards the interior to „anticipate the force“
- .... or, opposite, to let it bend out to give the effect of a force acting on the object

Camera Path Following: Center of Interest

- A more natural way of specifying the orientation of a camera is to use the center of interest (COI)
  - One can view towards a fixed point
  - Or alternatively the center of an object
- Good method for a camera circling some arena of action
- The center of interest is specified, and so the view vector w=COI-POS
- This leaves one degree of freedom in camera specification
- One simple way is to set the view vector v as viewing „up“, i.e. perpendicular to w and lying in the *wy* plane

  w=COI-POS

  $u = w \times y$

  $v = u \times w$
- This works quite well for a camera moving along a path and focussing to a single object.
- When it gets very close to the object, this results in drastic changes (fly-near effect)
- This is not always bad!!!

Camera Path Following: Center of Interest

- There are variations to specifying a fixed point
- One can for example specify various points on the camera path itself
- The up vector
  - is usually specified as lying in the *wy* plane
- But one can also allow the user to input
  - Either a tilting value with respect to the default up vector
  - Or the up vector on a whole

- Following a points on the path is relatively easy:
  - If P(s) describes the position on the curve, then P(s+δs), with δs >0, specifies its position in the future
  - It is advisable to choose points at equidistances on the curve, so as to make changes not that noticeable
  - Alternatively, one can take the baricenter of some future points to avoid too much hopping
- The real flaw of this method is the fact that camera views look jerky

Camera Path Following: Center of Interest

- A better method is to use instead of some function of the position path, a different function altogether for the POI
- Let P(s) be the curve of the camera path, and C(s) the curve of the COI (obviously the animator specifies this)
- Similarly, and up vector path must be specified U(s), so that the general up direction is U(s)-P(s)
- The resulting coordinates for the camera will then become

$$w=C(s)-P(s)$$
$$u=w \times (U(s)-P(s))$$
$$v=u \times w$$

- This gives maximum control, but is also difficult to control.
- An easy way of specifying C(s) is to use fixed positions, with ease-in/ease-out moves between the different fixed points

Smoothing paths

- There are several ways to smooth a path if it has been generated by a sample process, such as a motion capturing system
- This path acquisition method is getting more and more frequent and inexpensive
- However, data here can be prone to noise or imprecision, depending on the input method

Courtesy Animazoo Ltd.

Smoothing paths: linear interpolation

- The simplest way of smoothing the data is to average neighbouring data point.
- Suppose we have the chain of points $\{P_i\}_{i=0,N}$
- In the simplest form, one averages P_i as the average itself and of P_{i-1} and P_{i+1}.

$$P'_i = \frac{P_i + \frac{P_{i-1} + P_{i+1}}{2}}{2} = \frac{1}{4}P_{i-1} + \frac{1}{2}P_i + \frac{1}{4}P_{i+1}$$

– Obviously, here the „spikes“ are flattened, so applying this method many times makes little sense

Smoothing paths: cubic interpolation

- A second method use the four adjacent points $P_{i-2}, P_{i-1}, P_{i+1}, P_{i+2}$ on either side to fit a cubic curve that is then evaluated at the midpoint.
- This midpoint is averaged with the original point to obtain the smoothed point
- Remembering that a 3rd order curve was $P(u)=au^3+bu^2+cu+d$

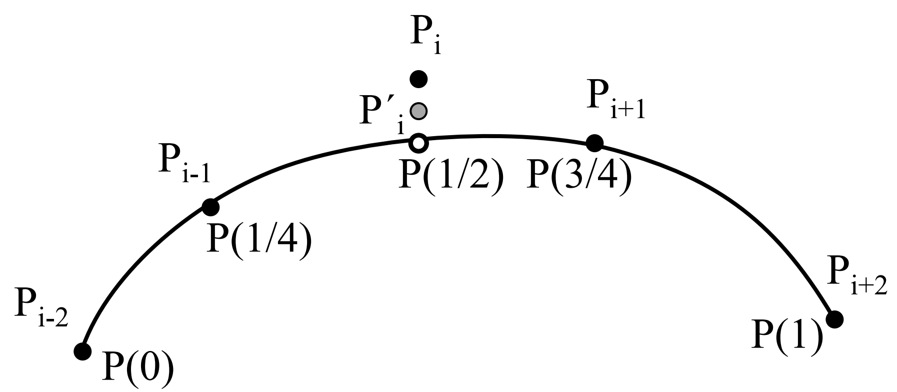
- One obtains

  $P_{i-2}=P(0)=d$

  $P_{i-1}=P(1/4)=$
  $a(1/64)+b(1/16)+c/4+d$

  $P_{i+1}=P(3/4)=$
  $a(27/64)+b(9/16)+3c/4+d$

  $P_{i+2}=P(1)=a+b+c+d$



Smoothing paths: cubic interpolation

- For the last points, a parabolic arc can be computed to fit the second and forelast points
- Notice that here the curve will be of the form au^2+bu+c , and the equation turns into

$$P'_1=P_2+1/3(P_0-P_3)$$

and similarly for the last three points

Smoothing paths: convolution kernels

- If the data can be viewed as a data function $y_i=f(x_i)$ then convolution can be used to smooth the data
- Convolution with the convolution kernel g(u) defined in the interval
  [-s,s] is in fact computing

  $$P(x)= \int_{[-s,s]} f(x+u)\, g(u)\, du$$

- The resulting integral can be computed directly or approximated by discrete means

Smoothing paths: B-spline approximation

- If the path does not necessarily have to pass through the sample points, one can use approximation methods we saw before
- Particularly B-splines are well adapted for the defining a path tacked from real data

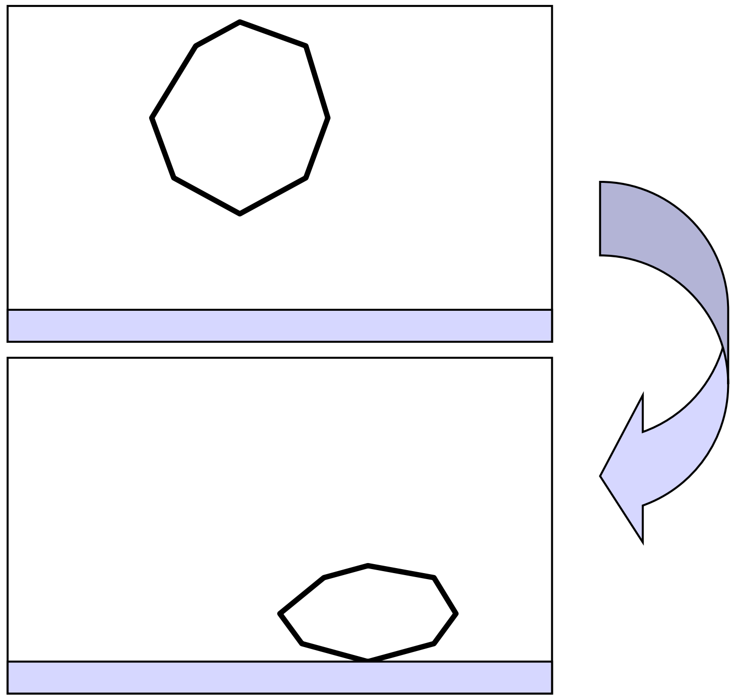
Path along a surface

- If an object needs to follow a surface when it moves, then a path on the surface itself has to be found
- If we know start and endpoints, then this is simple:
  - trace a plane „perpendicular“ to the surface
  - Compute the intersection plane-surface
- Alternatively, other methods can be used, for example if one wants to follow the „valleys“ on the surface
- Here „greedy“ methods can be used, or methods that compute the normal to the surface and follow it

Keyframe systems

- Early computer animation systems were keyframe systems
- Most were 2D too, and implemented keyframe animations made by hand
- In computer animation a key frame is a variable set by the user at specific timepoints
- The system interpolates intermediate frames from the key frames

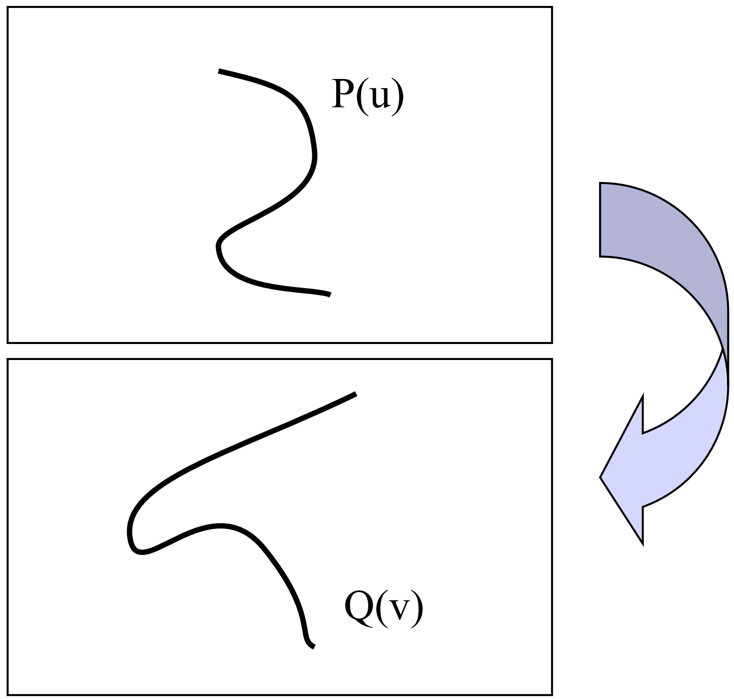
- The interpolation is quite straightforward if the shapes to be interpolated have the same number of controlling points



Keyframe systems

- In this case, linear interpolation can be used to produce the inbetween frames
- However, this is not the general case
- The general problem is: given two curves in 2D, how do I transform them into each other?
- If both curves are of the same type (eg Bezier of 3rd degree) then one can interpolate between control points
- Another method is to use interpolating functions to generate the same numbers of points on both lines, and then interpolate these points

- However, this does not allow sufficient control



Keyframe systems

- Reeves proposed a method based on surface patch technology to solve the problem of interpolating a curve in time
- Basically, one defines a patch in 3D to join the curves and allow the time parameter to be interpolated
- Sample points are taken on the patch to define the intermediate curves (=curves at inbetweens)

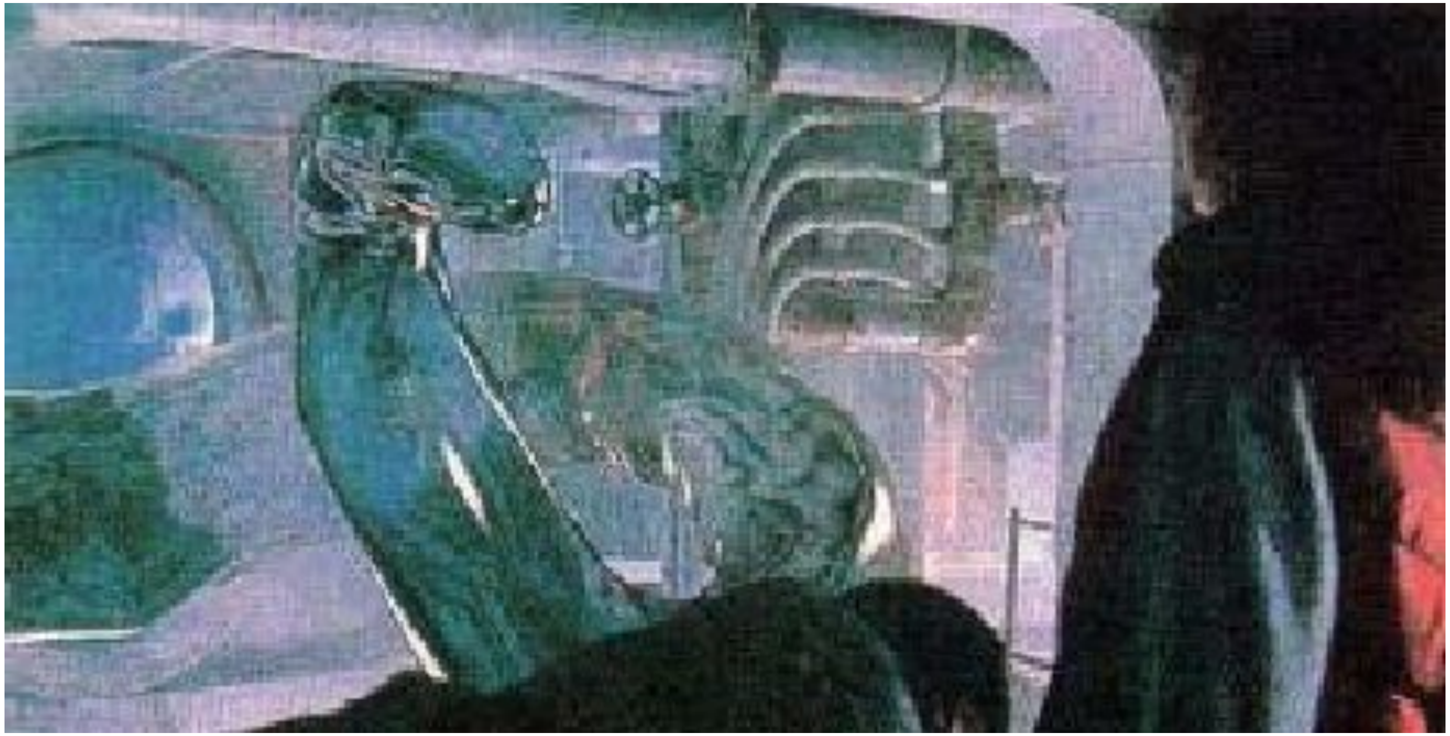
Animation languages

- In recent times, scripting languages have been developed to support animation systems
- Most animation languages are not easy to understand, and are close to hardcore programming
- A typical animation language is Renderman, or Alias/wavefront ‘s MEL
- Their big advantage is control

Animation languages

- Some effort has been put to accomodate unskilled artistic animators without scripting capabilities
- Simpler scripting languages such as ANIMA II have been developed
- Recently, actor based languages have appeared
- This is a novel approach but still at its infancy
- The idea is to have objects (=actors) and the instantiation of their variables representing the moving parameters
- Finally, the development of avatars has generated the need for some form of interaction with the animated models.

End



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