Fundamentals of Imaging
Light and Radiometry

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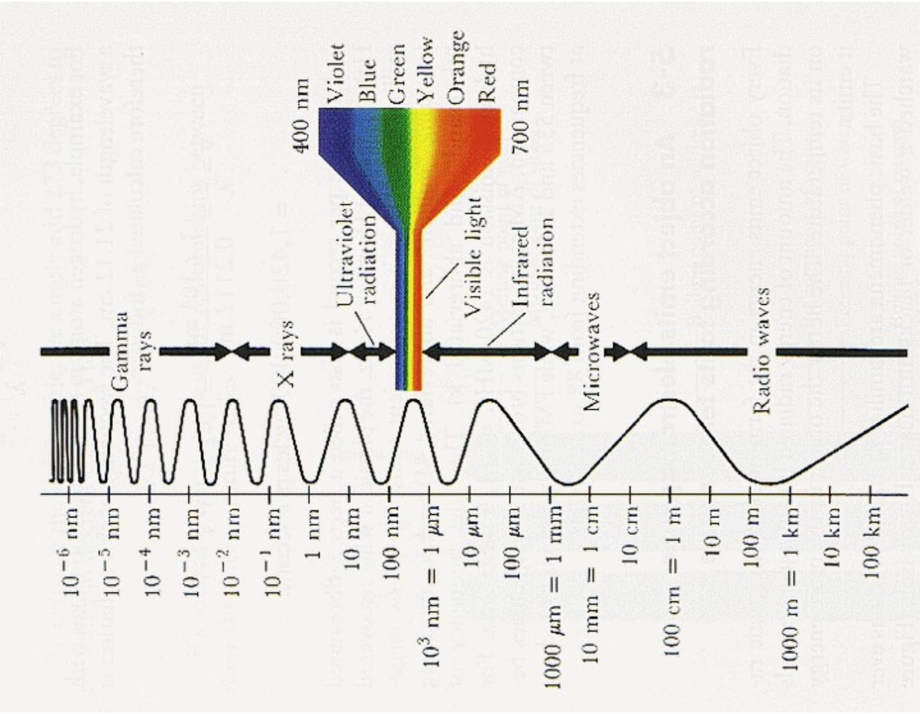
Today’s lesson

- Today’s themes:
  - Light, its definition
  - Characteristics of light: coherence and polarization
- How to measure light: radiometry
  - Spectrometry
  - Perfect diffusors
  - The radiant theorem
  - How light reflection is measured through a blackbody

Light

- What is light?
  - A full explaination of light can be found in Quantum ElectroDynamics
  - Very abstract and difficult to understand
- We don’t want to get to that depth in this course
- However, we can calculate and predict its behaviour from phenomena that we can observe:
  - Light can be seen as electromagnetic waves between the frequencies of 4×10^{14} (740nm) and 7.8×10^{14} Hz (380nm).
  - Light carries energy in discrete amounts
  - Light can be seen as photons, has linear momentum, and therefore exerces some force on the surfaces it reaches
  - Light of the same frequency has different polarizations, which can be filtered by some materials (polarizers)



Light

- There are therefore 2 models for light:
  - Geometrical optics:
    light behaves as rays
  - Wave model:
    based on Maxwell's equations for electromagnetic waves
- Quantum theory uses two objects to describe a physical system:
  - The operator for the physical variables (e.g. the electric field intensity)
  - Schrödinger's wave equation ψ
    - complex function
    - product with its complex conjugate $\psi\psi^*$, gives the probability of finding photons at a point in space and time
- Schrödinger's wave equation is NOT the electromagnetic wave described by Maxwell:
  - For classical phenomena, like interference, the time-averaged Poynting vector calculated from Maxwell's equations

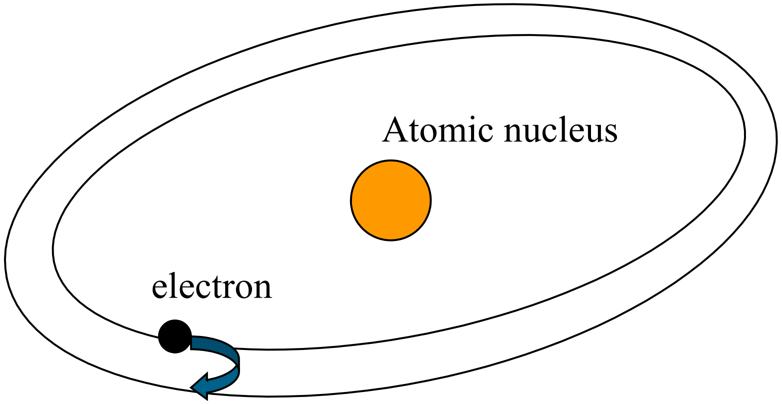
    $$\langle \mathbf{E} \times \mathbf{H} \rangle [\mathrm{Wm}^{-2}]$$

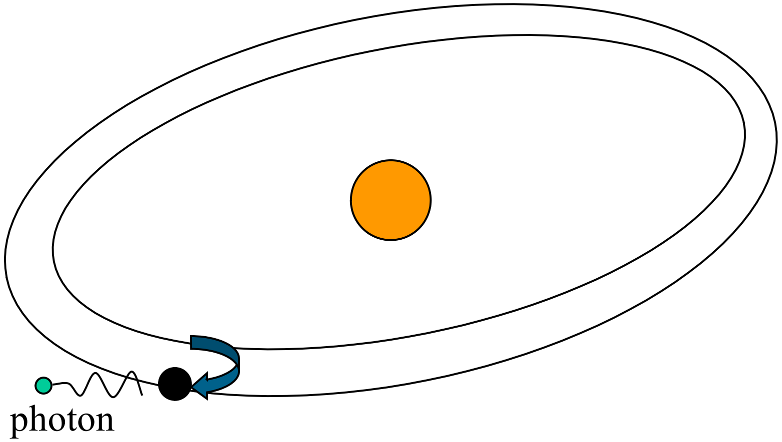
    predicts the number of photons per unit time per unit area at that point in space, as calculated by quantum electrodynamics.
- In this course we'll mostly consider light as electromagnetic waves

Light emission

- When an electron of molecules makes the transition from one level of energy to a lower one a photon is emitted
- The time it takes for this transition is very short…
- …and short is the length of the wave train of emitted light (10^{-8}s)

- Remember the wave propagation equations:
  - For a sinus wave, $\lambda=v/f$
  - Thus, for light
    $\lambda=c/f$
    where c is the speed of light



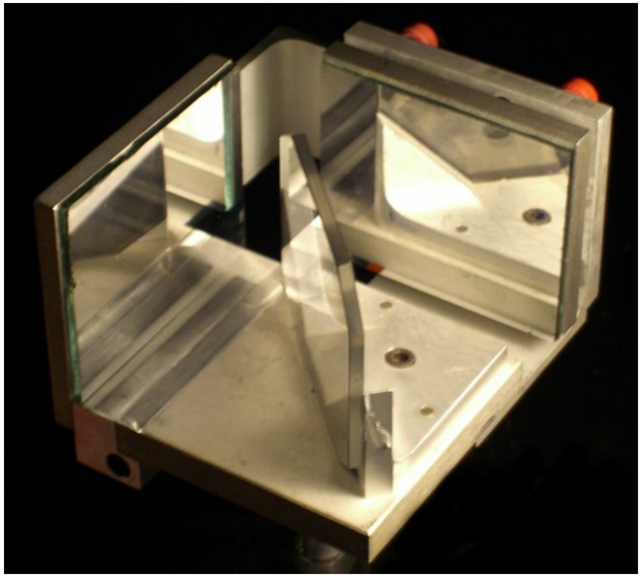


Light coherence

- The electromagnetic field at two points in space-time can fluctuate completely independently
  - In this case, they are said to be incoherent.
- If the fluctuations are not completely independent they are said to be partially or completely coherent.
- The degree of coherence can be measured through statistical correlation
- There are two special cases of coherence which have been studied through simple experiments
  - Temporal coherence: here field fluctuation is measured at the same location in space
  - Spatial coherence: here field fluctuation is measured at the same instant

Temporal coherence

- Michelson’s interferometer: a half reflecting mirror splits the light
  - Some of the light (the refracted one through the mirror) will take a shorter path than the reflected light
  - The light paths are then joined again, but having travelled different distances their phase differs
  - Interference fringes appear at the detector



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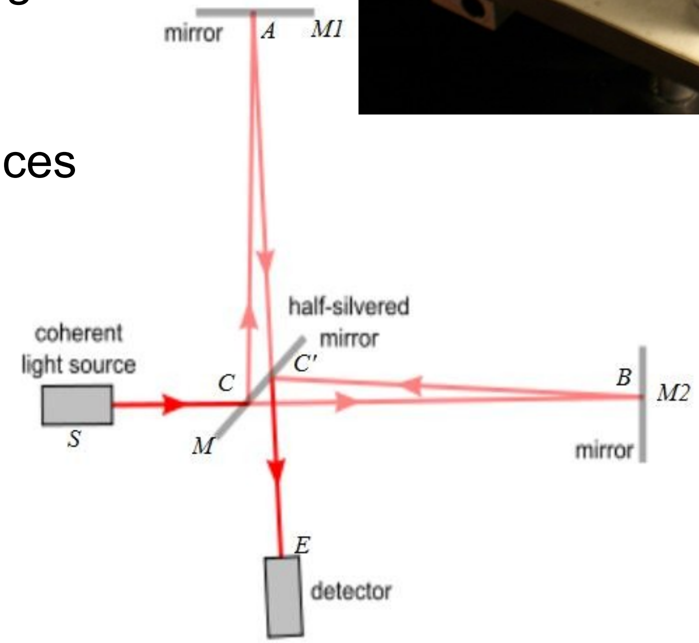
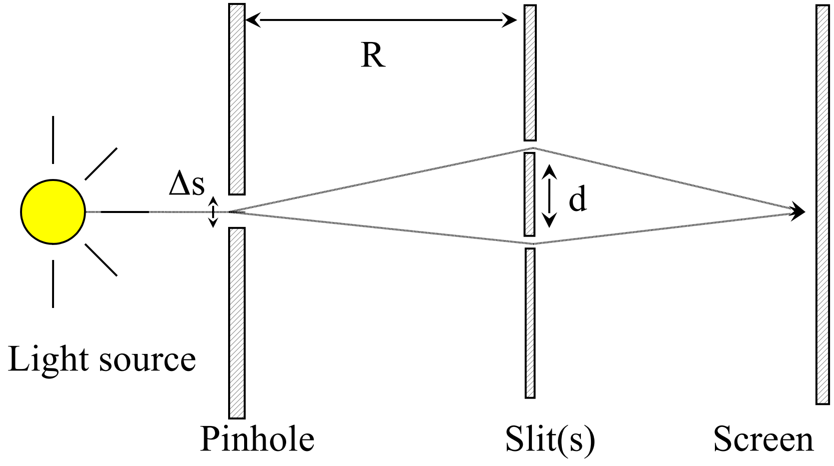
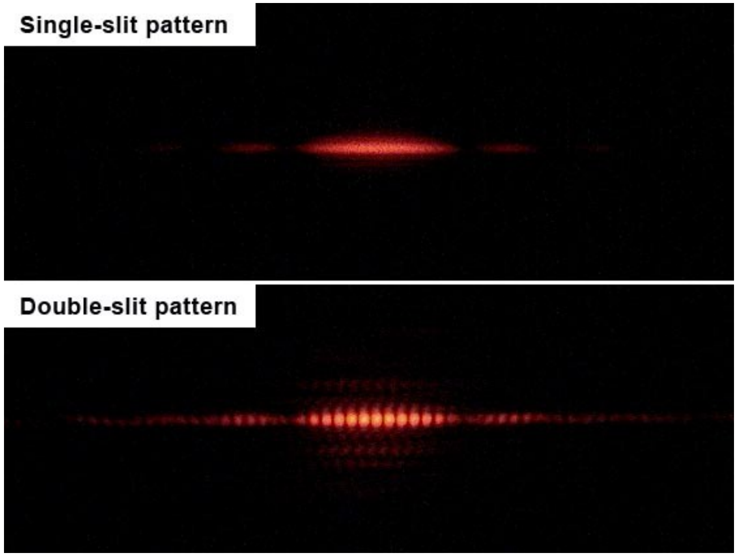


Spatial coherence

- Young's double slit interference experiment
  - A coherent light source S sends light through a pinhole
  - In front of the pinhole there are two slits symmetrically shifted from the perpendicular to the pinhole
  - Because the source is the same, the light travels different distances, and will form interference on the observation screen
  - If one uses only one slit, no interference can be seen
  - For interference to be observable, it must be that $d \leq (R\lambda)/\Delta s$
- Normal light sources, such as bulbs or the sun, can be treated as incoherent





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Maxwell’s equations

- Because light can be seen as electromagnetic waves, one can use for it Maxwell’s equations
- Maxwell equations describe the electric and magnetic fields arising from various distributions of charges and current, and how these fields change in time
- They are four equations:
  - Faraday’s law of induction:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

  - Ampere’s law amended by Maxwell:

$$\nabla \times \mathbf{H} = \mathbf{J} - \frac{\partial \mathbf{D}}{\partial t}$$

Displacement current

  - Gauss laws for the electric and magnetic field:

$$\nabla \cdot \mathbf{D} = \rho$$

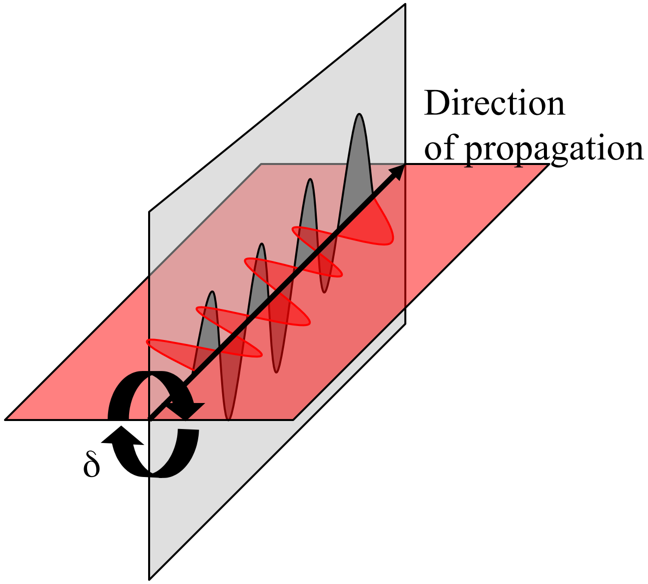
$$\nabla \cdot \mathbf{B} = 0$$

- Here,
  - E and H are electric and magnetic *field intensities*
  - D and B are electric and magnetic *flux densities*
  - ρ and J are *volume charge density* and *electric current density* of any *external charges* (the sources)
  - and ∇ is the nabla operator

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n} \right)$$

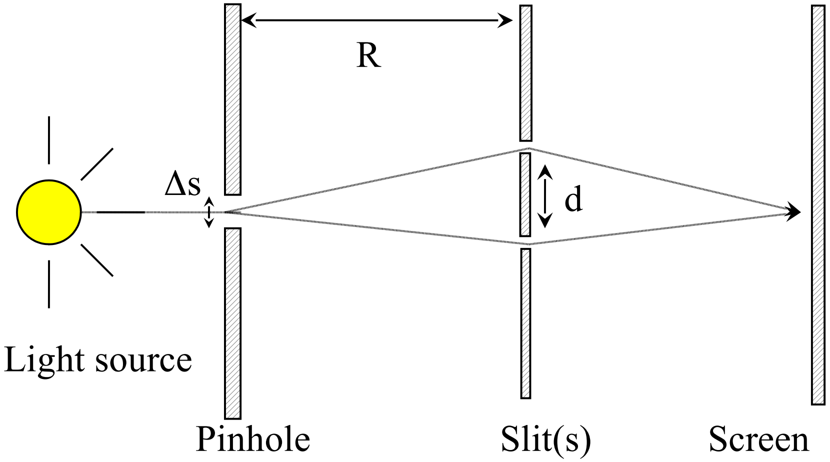
Polarization

- Far from their source, the electric and magnetic fields are
  - perpendicular to each other
  - perpendicular to the direction of propagation.
- This fact leaves one degree of freedom for the pair, the rotation angle δ around the direction of propagation
- When the light is such that this degree of freedom is not there any more, then the light is said to be *polarized*
- In other words, once δ is fixed, the light is polarized



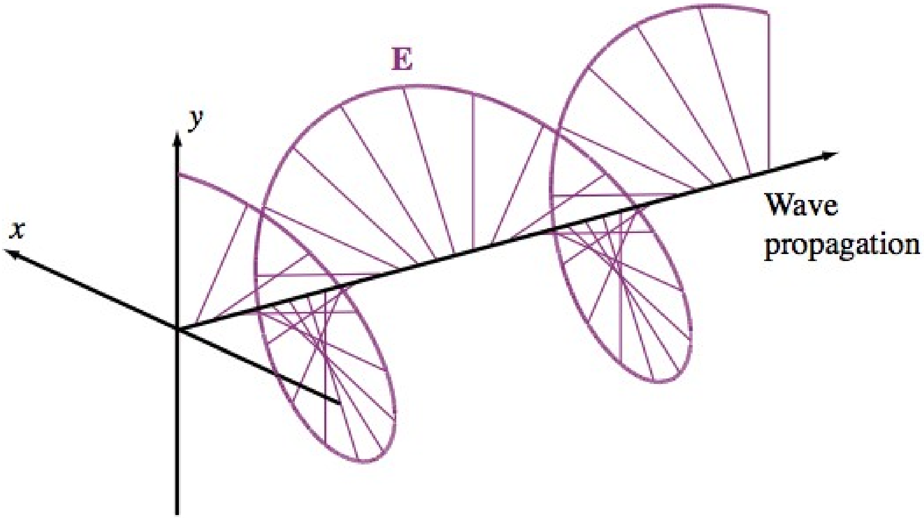
Polarization and interference

- Fresnel and Arago performed again Young's slit experiment, this time with a polarized light source
- Results obtained were:
  - Two beams of light linearly polarized in the same plane can produce interference patterns
  - Two beams of light linearly polarized perpendicularly to each other cannot produce interference patterns.
  - Two beams of linearly polarized light, derived from an unpolarized light beam in perpendicular directions, cannot produce interference patterns even if they are brought back to the same polarization plane by polarizers.
  - Two beams of linearly polarized light, derived from a linearly polarized light beam in perpendicular directions, can produce interference patterns if they are brought back to the same polarization plane by polarizers.



Non-Linear Polarisation

- When we talked about polarization, we said that the angle of the plane of the waves has to be fixed
- In fact, this is not completely true: the degree of freedom has to be eliminated.
- If it is with the fixed angle, we speak of linear polarization
- In the case illustrated at the right, here the polarization is done on a circular pattern
- Note that the drawing shows only the electrical field, and not the magnetic one (which is perpendicular to it)



- We know now the nature of light: Next we will introduce how to measure light

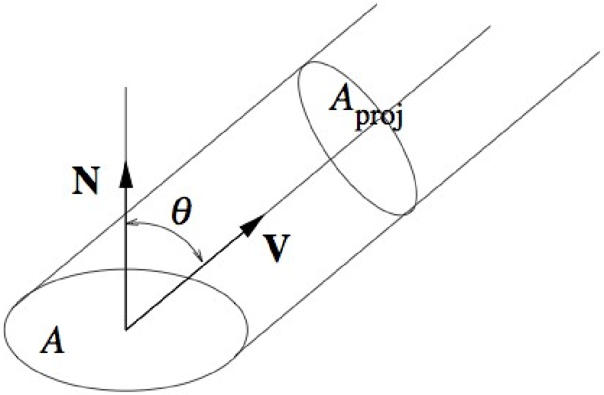
Radiometry

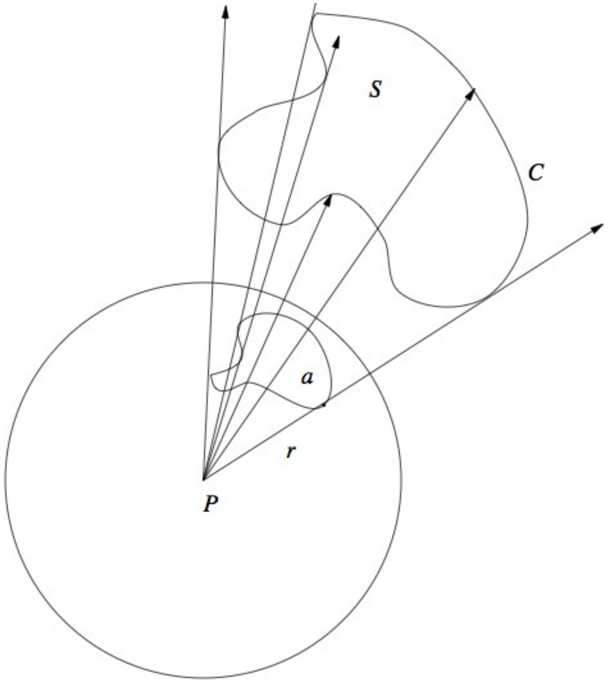
- It is important to know how much light energy reaches a sensor
- Sensors have limited operating range: controlling how much light reaches them is essential.
- The study and measurement of optical energy flow is the subject of *radiometry.*
- Radiometry units for light measurement are standardized by the CIE (Commission Internationale de l'Eclarage)
- The unit of radiant light energy Q is the Joule
- The energy flow per unit time through a point (x, y, z) in space in a direction (θ, φ) is called the *radiant flux*,

  $(x, y, z, \theta, \varphi)$.
- Radiant flux is a quantity dependent on position and the direction, which means it is associated with the light beam
- At a far away distance from a light source, the source can be modeled as a point source
- In practice, if the light source size is less then 1/10 of the distance between light source and object, the error in considering a point light source for radiometric calculations is less than 10%

Projected area - Solid angle

- Projected area: area of an object seen from a direction different from the surface normal
- Solid angle of a cone: area cut by the cone onto a unit sphere with center at the apex of the cone
- In other words, the solid angle of a cone is the area of its projection onto a unit sphere
- A sphere has a solid angle of 4π.
- If the sphere has a radius of *r*, then the solid angle is the surface of the projection divided by r^2.
- The unit of a solid angle is the *steradian*.
- The steradian is the solid angle that, having its vertex at the center of a sphere with radius r, cuts off an area of the surface of the sphere equal to r^2





Computing solid angles

- To calculate the solid angle of an area of arbitrary shape, one can use the differential definition of solid angle

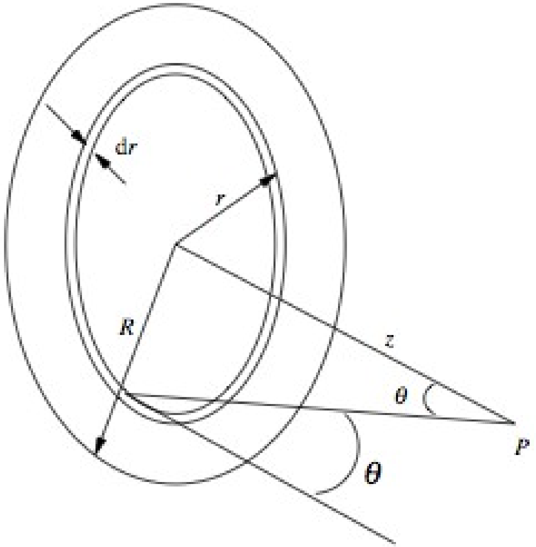
$$d\omega = dA_{proj}/r^2$$

- And integrate it on the whole area

$$\omega = \int_A d\omega = \int_A \frac{dA_{proj}}{r^2}$$

where A_{proj} is the projected area onto the ray connecting A and the point light source

- Exercise for home:
  - Calculate the solid angle subtended by a circular disk of radius *R* to the point source *P* at a distance *z* to the center of the disk.

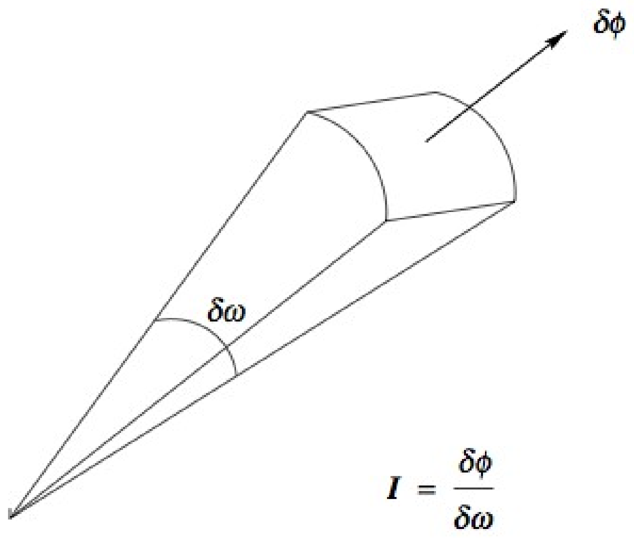


Intensity

- Let $d\Psi$ be the radiant flux leaving a point source through a cone of solid angle $d\omega$.
- Then the quantity that describes the light output from the point source at this cone is called the radiant intensity
  - *I*: radiant flux per unit solid angle

$$I(x,y,z,\theta,\phi) = \frac{d\Phi(x,y.x,\theta,\phi)}{d\omega} W sr^{-1}$$

- Intensity is an idealization not taking into account diffraction and other parameters
- For example: stars are point sources, and exibit diffraction on telescope images

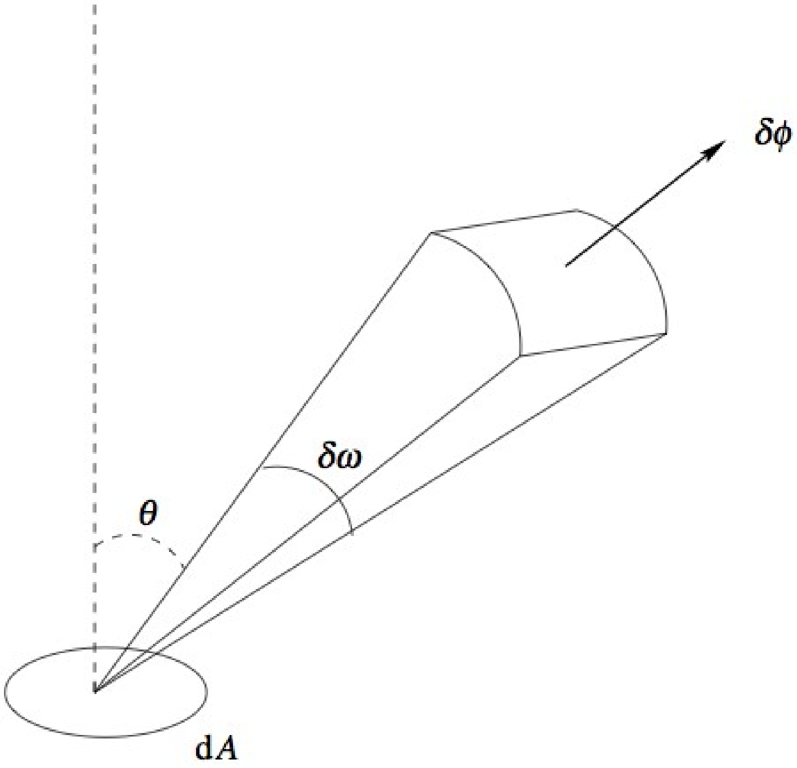


Radiance

- If the light source is close, it is not possible any more to treat it as a point light source
- Dividing the area source in small elements *dA* one can treat each one as a point source, but the intensity is proportional to its area *dA*.
- And if the surface is not perpendicular to the rays, then its projected area *dAcosθ* has to be used
- The quantity describing the amount of light coming from a surface is called radiance *L*:
  it is the light flux per solid angle per projected surface area

$$L(x,y,z,\theta,\phi) = \frac{d^2\Phi(x,y,z,\theta,\phi)}{dA\cos\theta d\omega} W sr^{-1} m^{-2}$$

- Radiance is used to measure how much light is reflected by walls, or how much light comes from the sun



Radiant flux, Irradiance, Radiant Exitance

- To describe how much light passes through a surface area, the term *radiant flux* (surface) *density* is used

  $W(x,y,z,\theta,\varphi)$

  measured in $[Wm^{-2}]$
- The radiant flux irradiating onto a surface instead is called irradiance

  $E(x, y, z, \theta, \varphi)$

  measured in $[W\,m^{-2}]$
- For the radiant flux leaving a surface one uses radiant exitance

  $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \theta, \varphi)$

  measured in $[W\,m^{-2}]$
- W, E and M are dependent on the direction and on the position
- Many times, one is interested in the total power received by a sensor surface
- In this case, the flux is integrated over all angles and is no longer dependent on the direction

Lambertian surfaces

- Many reflecting surfaces appear to be approximately equally bright, independently of the viewing angle.
- There are two reasons for this: one Physical and one phsychophysical
  - except for highlights, most surfaces reflect light with approximately the same radiance in all directions, and the retinal image irradiance is proportional to the radiance of the scene viewed

- A surface whose radiance is independent from the viewing angle is said to be a *Lambertian surface*
(it can reflect or transmit)
- A source whose radiance is independent from the viewing angle is called a Lambertian source
(it emits light)
- A Lambertian surface that reflects (transmits) 100% of the incident light is called a *perfect reflecting (transmitting) diffuser*.
- This idealized reflector is used in the definition of the radiance factor, which describes how bright a surface is in an imaging system

- On the book: several example of how to compute these quantities.

Radiant exposure

- Some sensors allow reading of the light signal continously, for example the eye, others no.
- For these a shutter has to be used to control the time interval during which the sensors are exposed to the image signals
- Such sensors respond to the energy per unit area, and not to the power per unit area
- All photographic films, and most CCDs work this way

- Here, one has to use the integral of the product of image irradiance and time, which gives energy per unit area
- This is called *radiant exposure H*, which for an exposure time *Δt* is defined as

$$H = \frac{dQ}{dA} = \int_{\Delta t} E dt$$

and is measured in *[Jm^{-2}]*

Reflectance

- Light reaching an object surface is
  - Reflected
  - Absorbed
  - Transmitted
- The ratio between the flux of reflected light and the flux of incident light is called the *reflectance* ρ.
- In a passive material, it is always ≤1.
- A lambertian surface having a reflectance of 1 is called a *perfect diffuser*.
- For calculating the reflectance, the whole light reflected is used: surfaces reflect differently in different directions
- To take this into account, one defines the *reflectance factor*:
  - ratio of radiant flux reflected in the direction delimited by the measurement cone to that reflected in the same direction by a perfect reflecting diffuser identically irradiated
- When the measure cone is very small, and one is measuring radiances, it is called the *radiance factor*.
- A similar definition is given for the *transmittence factor*.
- The radiance factor correlates better with what we see than the reflectance

Responsivity

- In most cases, the terms *responsitivity* and *sensitivity* are used interchangeably.
- In vision science, instead, sensitivity is the inverse of the input power required to produce a threshold response.
- If the system response is a nonlinear function of the input power, measured sensitivity is a function of the threshold response that is chosen for the measurement.
- Radiometric measurements are difficult to do: they depend from wavelength, variation of the radiation in time, direction of incident radiation, position of the detector, temperature, coherence and polarization of the radiation….
- Suppose you have a detector (*sensor*):
  the ratio between the input power (light flux) and its output signal is called *responsitivity s*.
- In case of an electrical sensor, this ratio depends on the sensor itself and its characteristics.
- There are detectors, such as *film*, that respond to the total light energy, I.e. the exposure:
  here, responsivity is defined as the ratio of the output signal to the input light energy.
- Sometimes, responsitivity is measured with respect to a single wavelength λ, and is called s_λ.

Spectral radiometry

- Until now, we have not mentioned the spectral composition of light being measured.
- If we want to measure light energy flow with respect to certain light frequency intervals we have to measure in small intervals.
- In this case, we speak of spectral quantities, and their units become per unit wavelength interval (e.g., nanometer) or per unit frequency interval.

The radiance theorem

- In geometrical optics, light is treated as rays
- Tracing rays from one point in space to another is an operation carried out in diverse applications, such as lens design, thermal radiative transfer, scanner engineering, and computer graphics.
- An implicit assumption in ray tracing is that each ray carries with it some measure of energy flux through space and time.
- The radiometric quantity associated with each ray is the so-called basic radiance:
  - it is L/n^2, the radiance divided by the index of refraction squared.
- It can be proven that the basic radiance is conserved when light is propagated through non-absorbing and non-scattering media.
- This conservation property of the basic radiance is called the *radiance theorem*.

Integrating Cavities

- An integrating cavity is a physical cavity enclosed by reflecting surfaces
- They allow to integrate non-uniform radiant fluxes to produce a source of uniform radiant flux.
- Most common shape: spherical
- The study of the radiant flux distribution in an integrating cavity involves radiometric calculation of the mutual reflection of surfaces.
- This is done
  - Either with a finite element method
  - or through Monte-Carlo raytracing
- An integrating sphere can be used to measure reflectances

- A sample is placed, and illuminated by a light beam.
- The light is reflected to and integrated by the sphere
- This allows to measure irradiance, which is proportional to the amount of reflected light

Blackbody radiation

- A blackbody is an idealized object that absorbs all incident radiant energy
  - No reflection, no transmission
- If it absorbs all incoming energy, its temperature must rise
- This until it reaches thermal equilibrium, which is ruled through the heat transfer equations
- Planck derived the energy spectrum of blackbody radiation

$$L_\lambda(\lambda, T) = \frac{2C_1}{n^2\lambda^5(e^{C_2/n} - 1)}$$

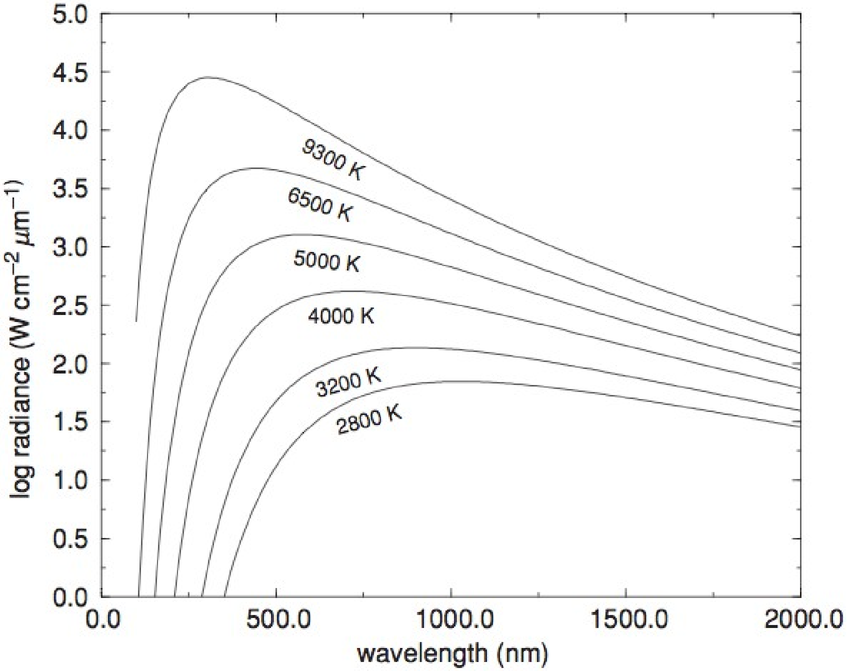
$C_1=hc^2=0.59552\times10^{-16}$ Wm^2

$C_2=hc/k=21.438769\times10^{-2}$mK

c: speed of light

λ: wavelength in the medium

- h Planck constant
  k Boltzmann constant
  K: absolute temperature
- A blackbody is a Lambertian radiator
- Its color variation is similar to that of daylight



Recap

- In this lesson we have touched the following themes:
  - Light, its definition
  - Characteristics of light: coherence and polarization
- How to measure light: radiometry
  - Spectrometry
  - Perfect diffusors
  - The radiant theorem
  - How light reflection is measured through a blackbody
- Granted, this sounds a little bit dry, but it is the basis of both imaging and colour, ultimately of Computer Graphics

Thank you!

- Thank you for your attention!
- Web pages
  http://www.uni-weimar.de/medien/cg

- First exercitation: Next week Friday 11:00, HK7