Institute for Risk and Reliability

Leibniz Universität Hannover

Treatment of Epistemic and Aleatory Uncertainties with OpenCOSSAN and COSSAN-X

Summer School Graduiertenkolleg 1462

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Institut für Risiko und Zuverlässigkeit
Leibniz Universität Hannover

Outline

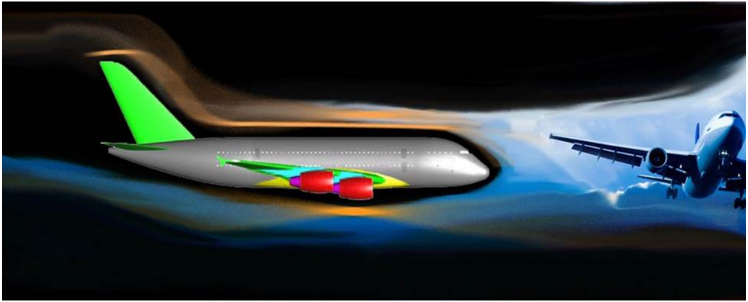
First of all...

A huge acknowledgement!

INSTITUTE FOR RISK AND UNCERTAINTY

Analyses with Uncertainties

- Predictive mathematical and numerical models
- Can contain a fairly large set of parameters
- "True" values are not precisely known, i.e. they are **uncertain**.



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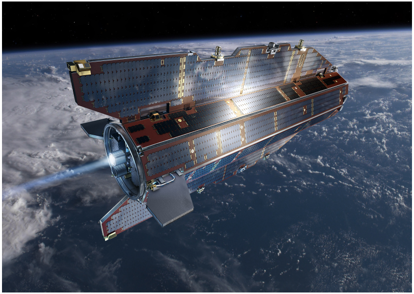

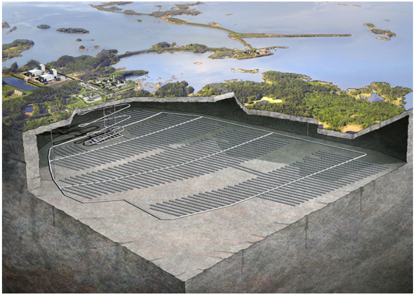

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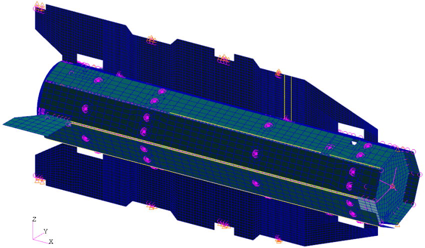
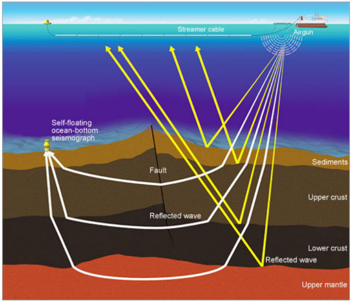




Reliability and Risk Analysis in Engineering
General concepts

Engineers need to assure that engineered systems, components and structures provide acceptable levels of safety.
Requires a proper *Risk analysis and management*

Risk assessment tools

A systematic and comprehensive methodology to evaluate risks associated with a complex engineered technological entity

- Failure modes and effects analysis
- Event tree
- Fault tree analysis
- Reliability and Availability analysis
- ...

Reliability and Risk Analysis in Engineering
Statistical analysis

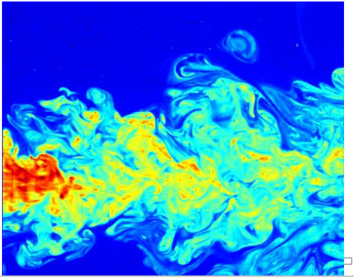
Accidental statistics and historical data are used by industry (essential tools for management)

- monitor risk and safety level
- identify hazards
- analyse accident causes
- evaluate effect of risk reducing measures
- ...

Uncertainties in Engineering systems

Irreducible (aleatory) uncertainties

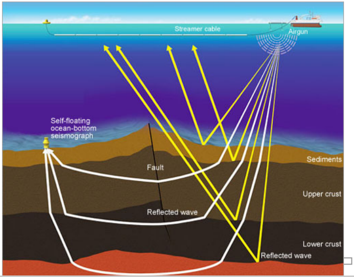
- Parameters intrinsically uncertain
- Value varies at each experiment
- Future environmental conditions, chaotic and stochastic process



Uncertainties in Engineering systems

Reducible (epistemic) uncertainties

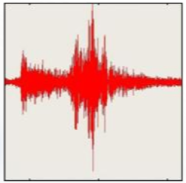
- Quantities that could be in theory fully determined
- Practically they are not available
- Too expensive, only destructive measurement, etc.

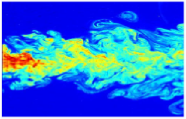


Uncertainties in Engineering systems

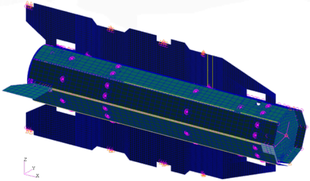
Physical

"Unavoidable" / Aleatory Uncertainties





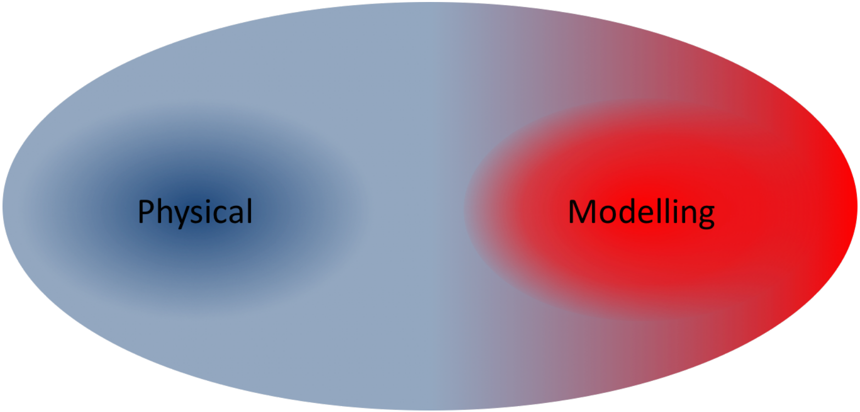
Uncertainties in Engineering systems



Modelling

"Lack-of-knowledge" / Epistemic Uncertainties

Uncertainties in Engineering systems

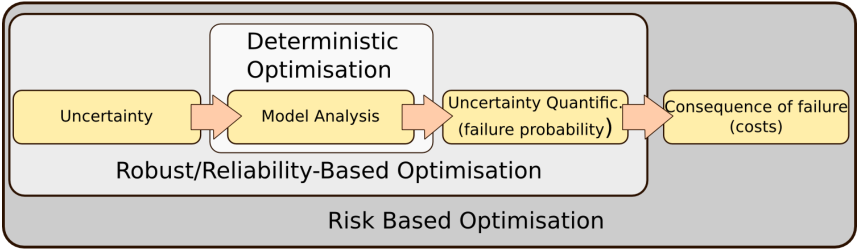


Dealing with Uncertainties
Quantifying Uncertainty

- **Modelling and refinement** of uncertainty based on experimental data, simulations and/or expert opinion
- **Propagation** of mixed aleatory and epistemic **uncertainties** through system models
- Parameter ranking and **sensitivity analysis** in the presence of uncertainty

Dealing with Uncertainties
Managing Uncertainty

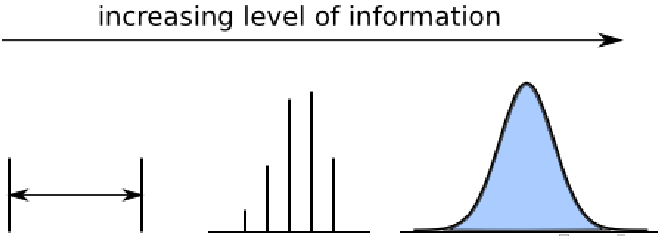
- Identification of the parameters whose uncertainty is the most/least consequential
- Worst-case system **performance assessment**
- **Design** in the presence of uncertainty



Epistemic uncertainties

Different levels of information

- Model the unknowns without introducing unjustified assumptions
- Limited experimental data
- Qualitative and heterogeneous information (expert judgements, constrasting sources)



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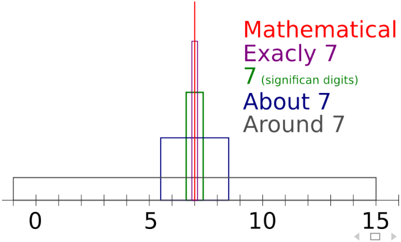

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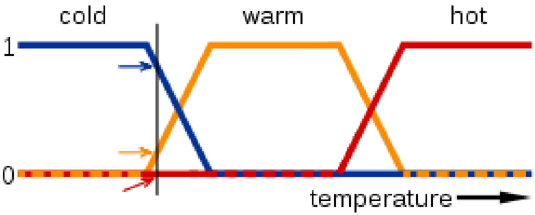
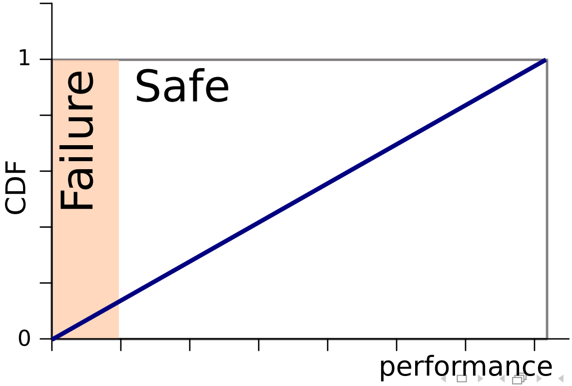


Naive approach

Treat epistemic uncertainties as aleatory uncertainties

- Example: Information available as only bounds → assign uniform distribution



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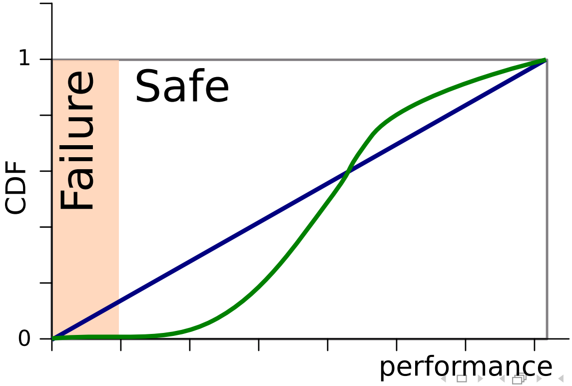

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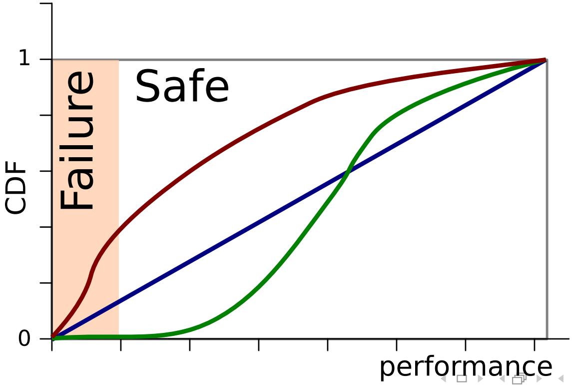
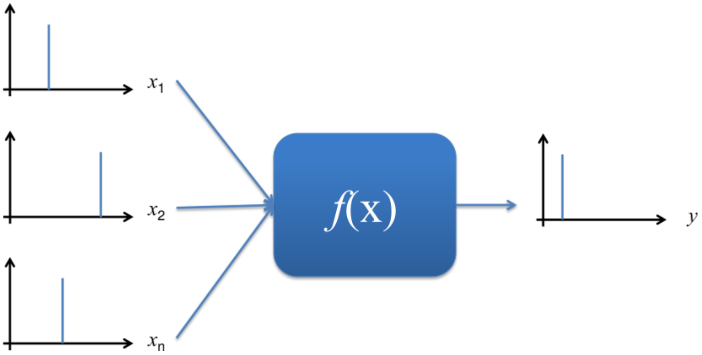


Imprecise parameters



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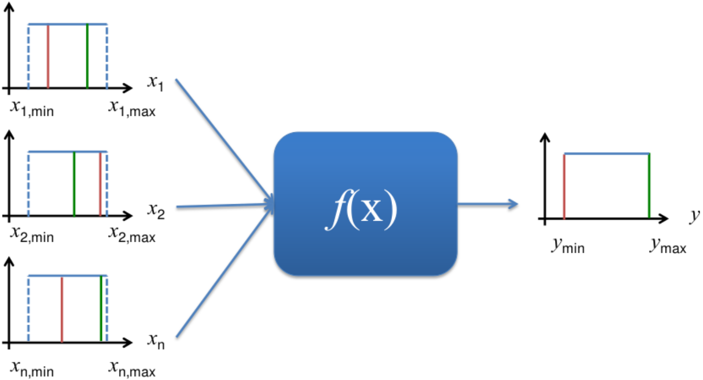
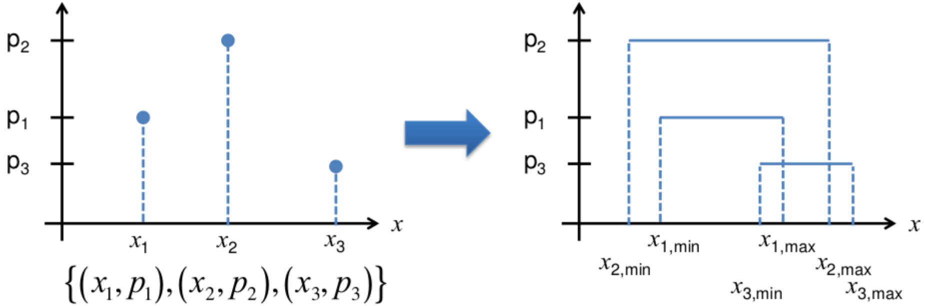


Imprecise random numbers

- Different probability masses associated to distinct intervals (e.g. weighting expert options or assumptions)
- UQ by sampling intervals (solving an optimization problem for each sample)



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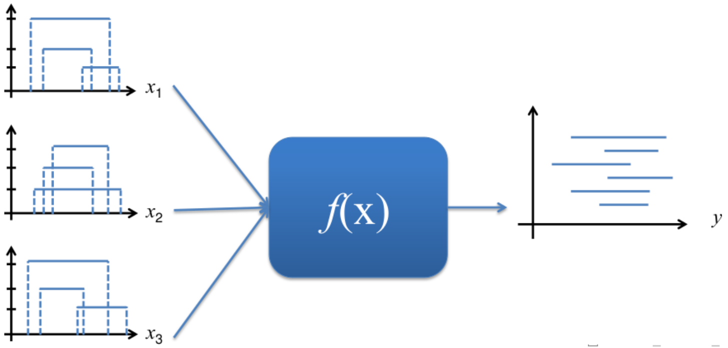
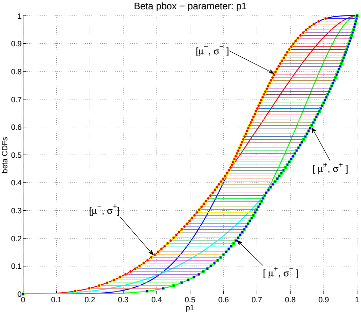
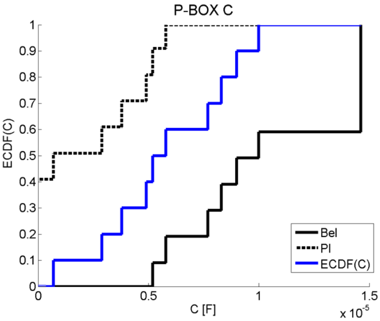


Imprecise random variables
Probability box

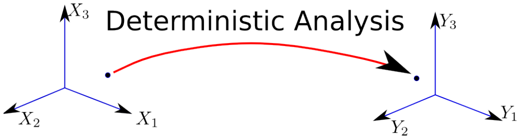


Distributional p-box



Distribution-free p-box

Analyses with Imprecise Probabilities
Computational Challenges



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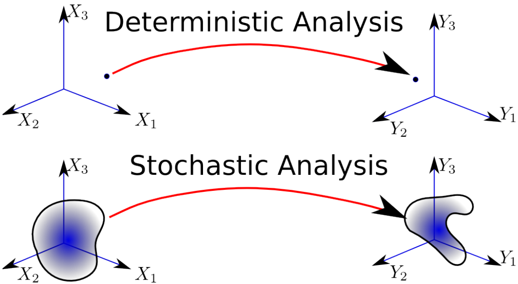

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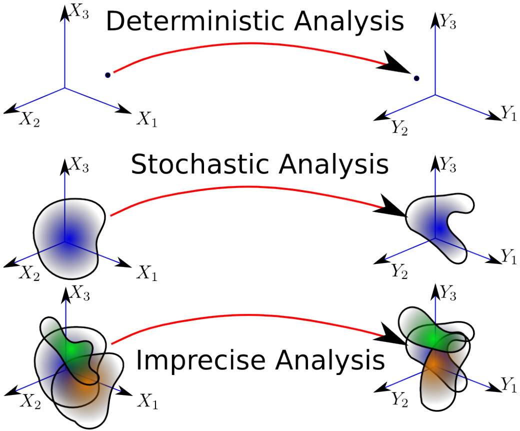


Outline

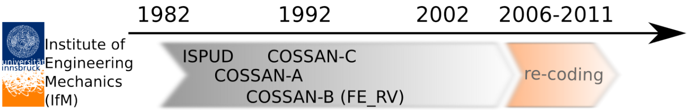
COSSAN
History and developments

Early recognition of the need of:

- Numerically efficient Monte Carlo methods
- Innovative software to solve to realistic, industry-size problems

COSSAN (COmputational Stochastic Stuctural ANalises)

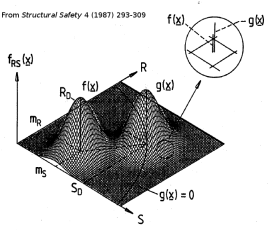
Transfer of stochastic simulation methods into engineering practice



COSSAN software
Past development

- ISPUD (Importance sampling using design points)
- COSSAN-A (Stand alone software)
- COSSAN-B (Interaction with 3rd party solvers)
- 2006 - 2011 development of next generation of COSSAN

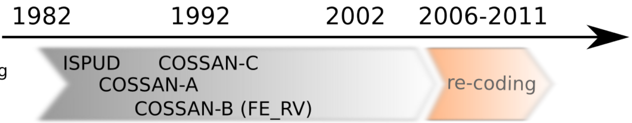
From *Structural Safety* 4 (1987) 293-309



R_D, S_D ..... coordinates of design point

Fig. 3. Schematic sketch for importance sampling procedure using the design point (ISPUD).

Institute of Engineering Mechanics (IfM)



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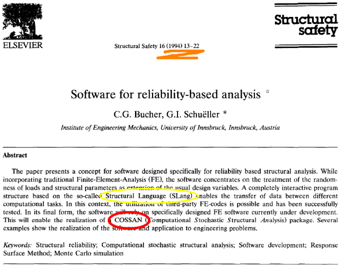


ELSEVIER
Structural Safety 16 (1994) 13–22
Structural safety

Software for reliability-based analysis *

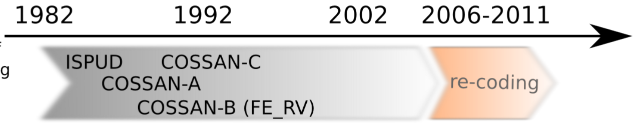
C.G. Bucher, G.I. Schuëller *
Institute of Engineering Mechanics, University of Innsbruck, Innsbruck, Austria

Abstract

The paper presents a concept for software designed specifically for reliability based structural analysis. While incorporating traditional Finite-Element-Analysis (FE), the software concentrates on the treatment of the randomness of loads and structural parameters as extension of the usual design variables. A completely interactive program structure based on the so-called Structural Language (SLang) enables the transfer of data between different computational tasks. In this context, the utilization of third-party FE-codes is possible and has been successfully tested. In its final form, the software will rely on specifically designed FE software currently under development. This will enable the realization of a COSSAN (*Co*mputational *S*tochastic *S*tructural *An*alysis) package. Several examples show the realization of the software and application to engineering problems.

Keywords: Structural reliability; Computational stochastic structural analysis; Software development; Response Surface Method; Monte Carlo simulation

Institute of Engineering Mechanics (IfM)



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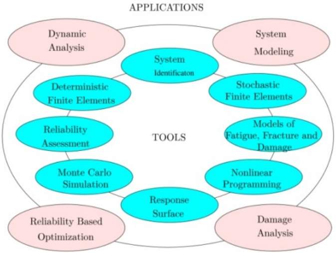

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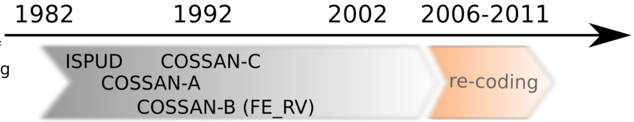

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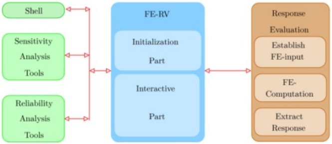

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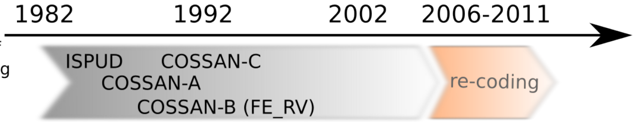

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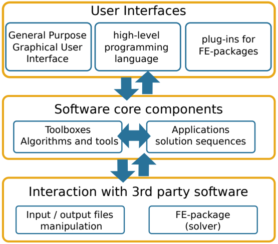

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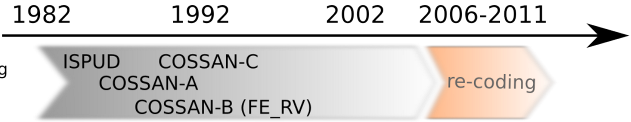
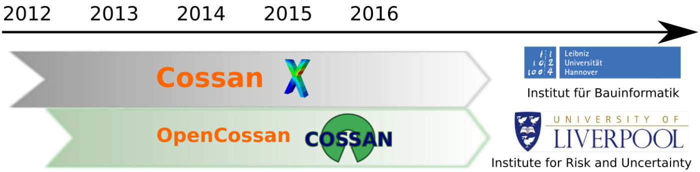


COSSAN Software

COSSAN-X and OpenCossan

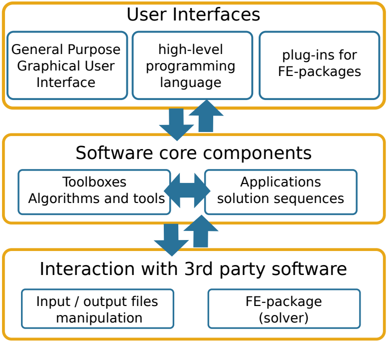
- from 2011 hosted at University of Liverpool
- 2012 First Release of OpenCossan
- 2014 New developments for iepistemic uncertainties
- 2016 Liverpool-Hannover joint development



COSSAN-X

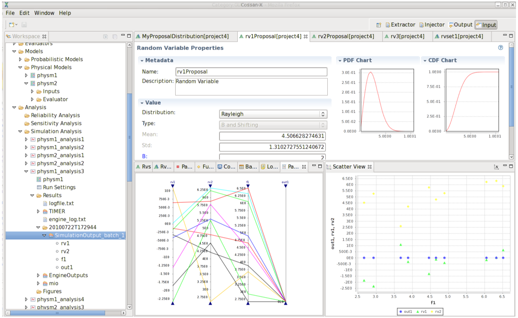
Next generation software for numerical simulation and uncertainty management

- Integrate legacy stand-alone applications
- General Purpose Software
- User friendly interfaces
- Open I/O formats
- Include efficient algorithms/methods
- Better interoperation with 3rd party software
- High-fidelity analysis



General purpose graphical user interfaces

- Coded in Eclipse RCP
- Multiplatform native interfaces
- Help, wizards and graphical tools
- Provide guidance to the users at every step of the analysis



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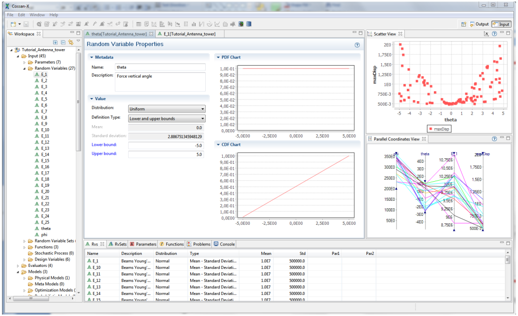
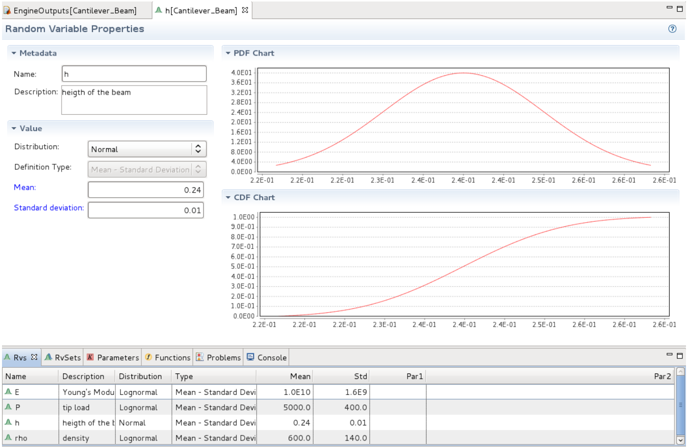


General purpose graphical user interfaces
Problem definition



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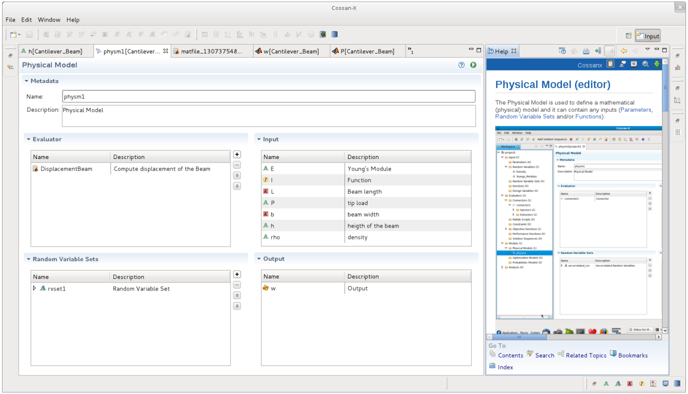
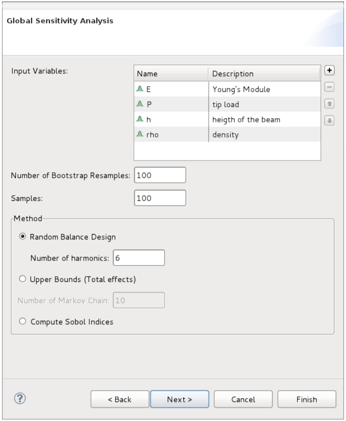
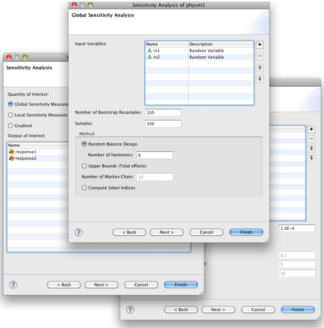


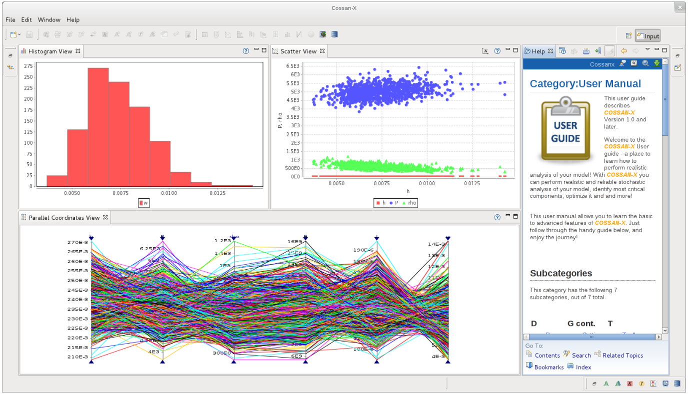
General purpose graphical user interfaces

Wizards (to define analyses)





General purpose graphical user interfaces
Output visualization



Tools and algorithms
COSSAN-X and OpenCOSSAN

Toolbox	Main algorithms and procedures
Reliability	Monte Carlo, LatinHyperCube Sampling, Sobol' Sampling, Halton Sampling, Line Sampling, Subset simulation and approximate methods (*FORM*, bounds)
Optimization	*BFGS*, COBYLA, Cross Entropy, Evolution Strategies, Genetic Algorithms, MiniMax, Simplex, Simulated Annealing, SQP, Stochastic Ranking
Meta-modelling	Artificial Neural Networks, Response Surface, Polyharmonic Splines, polynomial-chaos
Sensitivity	Local (finite differences, Monte Carlo), Global (Sobol' and Total indices, upper bounds)

Tools and agorithms
OpenCOSSAN

The following new feature are available and in development

Toolbox	Main algorithms and procedures
Uncertainty quantification and propagation	Imprecise probabilities, Model Updating, Sensitivity with p-boxes
Reliability	Subset-infinite, Advanced line Sampling
System Reliability	Bayesian Networks (crisp and imprecise), Survival Signature (crisp and imprecise)
Meta-Model	Gaussian process, Interval Predictor Model

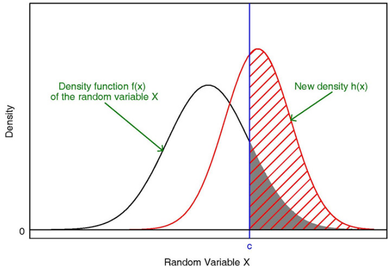
Advanced Monte Carlo Simulation

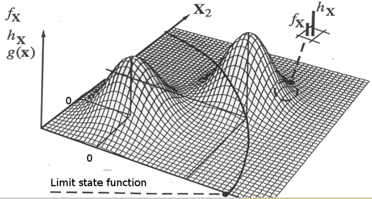
- Importance Sampling
- Line Sampling
- Subset Simulation

$$P_f = \int \mathbb{I}_{\mathcal{F}}(\mathbf{x})\ f_{\mathbf{X}}(\mathbf{x})\ d\mathbf{x}$$

$$P_f = \int \mathbb{I}_{\mathcal{F}}(\mathbf{x})\ g_{\mathbf{X}} \frac{f_{\mathbf{X}}(\mathbf{x})}{g_{\mathbf{X}}(\mathbf{x})}\ d\mathbf{x}$$

$$\hat{P}_f = \frac{1}{N} \sum_{k=1}^{N} \mathbb{I}_{\mathcal{F}}(\mathbf{X}^{(k)}) w(\mathbf{X}^{(k)})$$



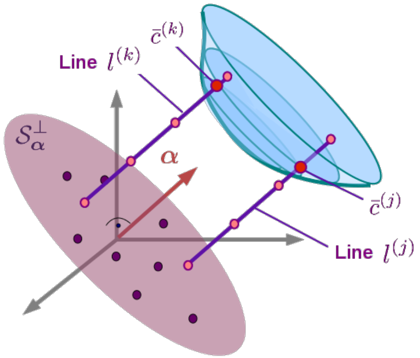


Advanced Monte Carlo Simulation

- Importance Sampling
- Line Sampling
- Subset Simulation

$$\hat{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} p_f^{(i)}$$

$$\hat{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} \Phi(-\bar{c}^{(i)})$$



Advanced Monte Carlo Simulation

- Importance Sampling
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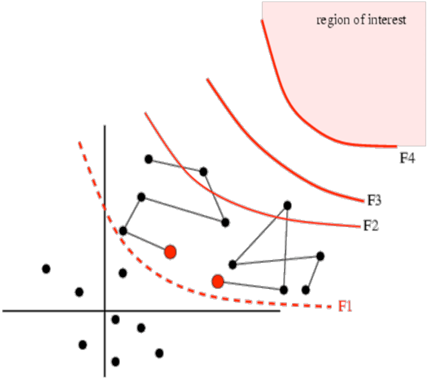
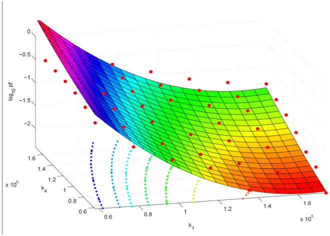
$$\hat{P}_f = P\left(\bigcap_{i=1}^{m} F_i\right) P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i)$$



Surrogate models

Approximation of input/output relations with simple mathematical expression

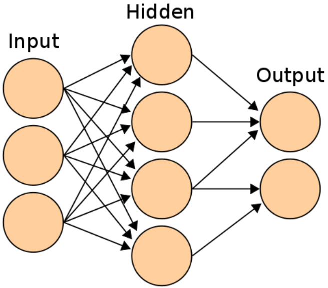
- Response surface
- Kriging / Gaussian Process
- Artificial Neural Networks
- Polyharmonic Splines
- Interval Predictor Model



Surrogate models

Approximation of input/output relations with simple mathematical expression

- Response surface
- Kriging / Gaussian Process
- Artificial Neural Networks
- Polyharmonic Splines
- Interval Predictor Model



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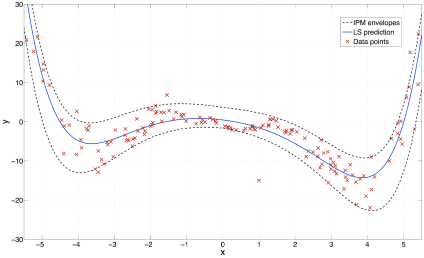
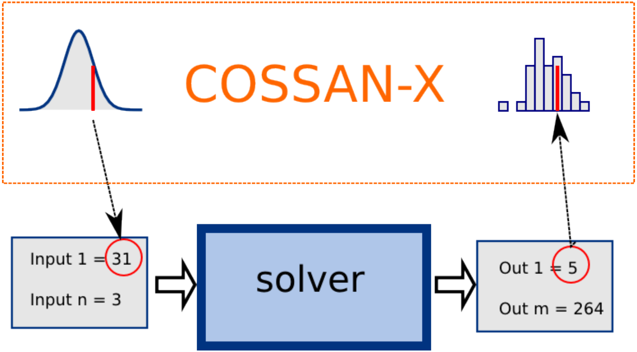


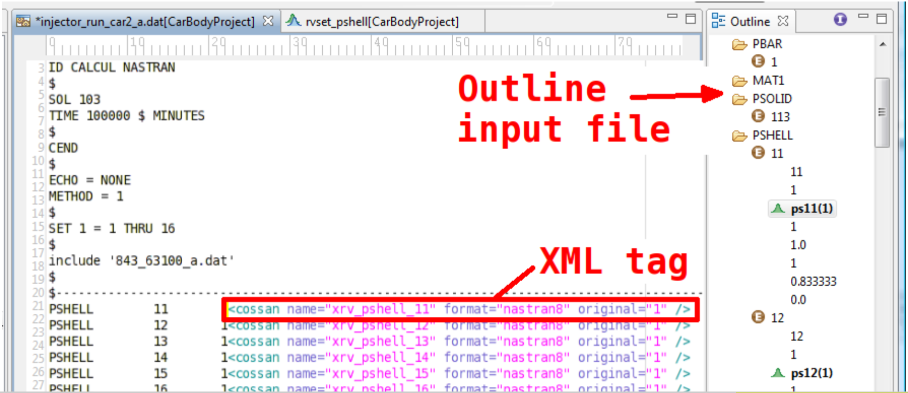
Interaction with 3rd party software

- Third party software are considered as black-boxes
- ASCII input/output files manipulation



Interaction with 3rd party software

- User friendly editors (Human readable XML tags)
- Injector (Input files) / Extractor (output file)



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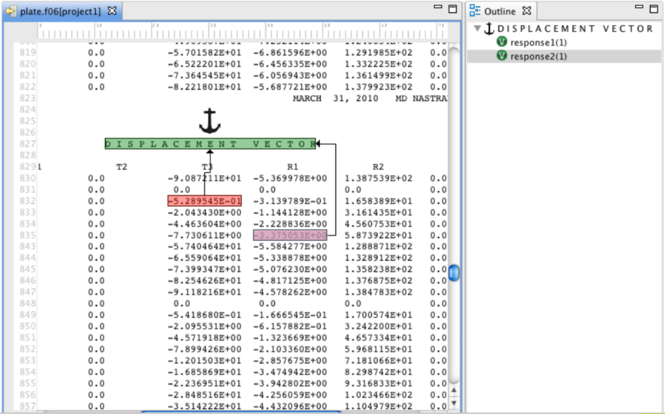
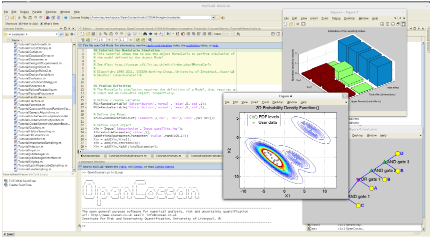


OpenCossan

Open Matlab toolkit for risk and uncertainty quantification

- Programmed **object oriented** fashion in MATLAB
- Open Source
- Allows to control, modify, and implement new algorithms/tools



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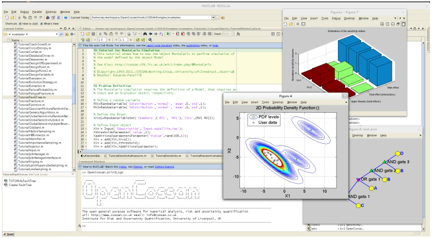
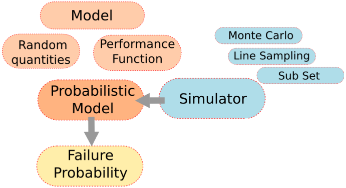


Modular Framework: easy to use/reuse components

No need to “reinvent the wheel”

OpenCossan
Intuitive and Flexible toolbox



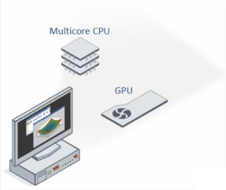

```
% ====================================================================
% This file is part of openCOSSAN.  The open general purpose matlab
% toolbox for numerical analysis, risk and uncertainty quantification.
% Author: Edoardo Patelli
% http://cossan.co.uk
% ====================================================================

% Definition of the Model
MyModel= Model('Xevaluator',ModelSteelRoofTruss,'Xinput',MyInput);

% Definition of the Performance Function
MyPerformanceFunction = PerformanceFunction('Sdemand','maxDisp', ...
                    'Scapacity','displacementCapacity','Soutputname','Vg');

% Definition of the Probabilistic Model
MyProbabilistiModel=ProbabilisticModel('Xmodel',MyModel, ...
                              'XperformanceFunction',MyPerformanceFunction);

                        % Definition of the Reliability solver
MySimulator=LineSampling('NmaxLines',20);
```

OpenCossan
Parallelization Solutions

1. Loop Parallelism
   - shared memory multicore
   - use Matlab parfor command
2. Job parallelism
   - Distributed memory cluster
   - independent jobs sent through GridEngine
3. Hybrid Job+Loop parallelism
   - Each job start a pool of Matlab workers

Local parallelism

Matlab parfor



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Job parallelism

Job manager

OpenCossan
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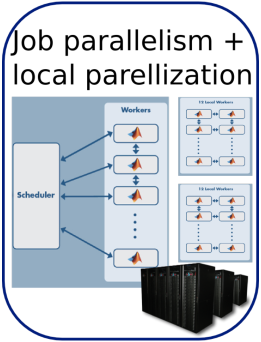
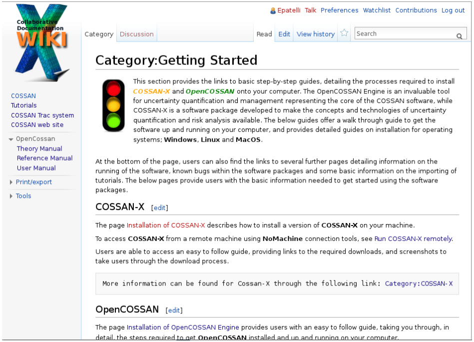


Collaborative development
Documentation

Open Documentation based on MediaWiki

- Theory manual
- Learn-by examples
- Tutorials and examples
- Reference Manual



Collaborative development

Code development

Based on Apache Subversion (SVN)

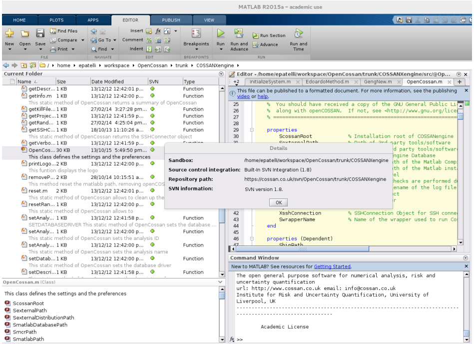
- Software versioning and revision control system
- Maintain versions of the files such as source code, web pages, and documentation.

JENKINS

- Integration tool
- Automatically validate every new proposed code change

TRAC system

- Effective way to track issues and software bugs



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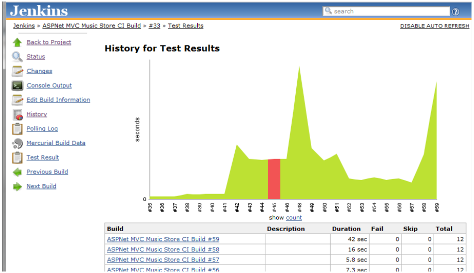

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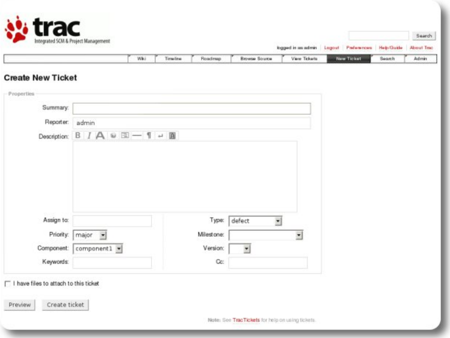


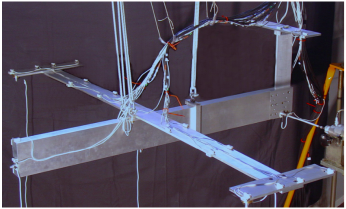
Outline

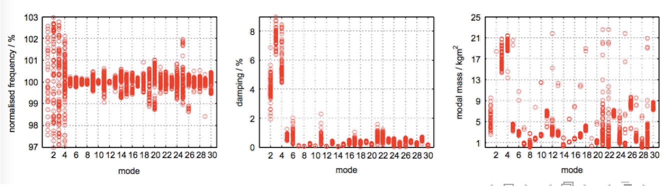
Switch to MATLAB for a quick demo of OpenCossan

Outline

Model updating of the DLR Airmod

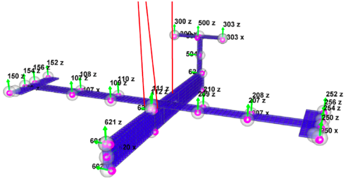
- Replica of GARTEUR SM-AG19 benchmark structure
  - 2m wingspan, 1.5m length, 0.46m height, 44kg weight
  - Disassembled and reassembled 130 times (86 tests usable)
- Excited with random signal in the frequency 0-400 Hz





Numerical model

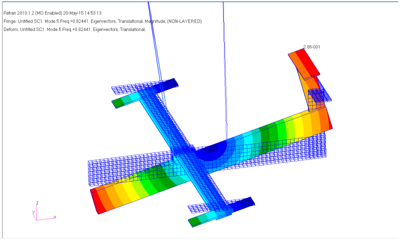
- Modeled in MSC.NASTRAN
- 3136 grid points
- 1446 solid elements (CHEXA, CPENTA) for the main aluminium structure
- 561 CELAS1 for joints modelling
- 73 concentrated mass for cables, instrumentation

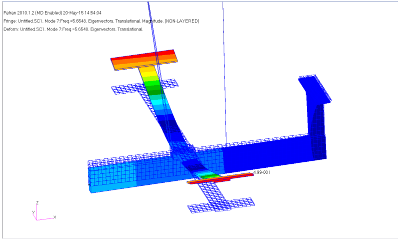


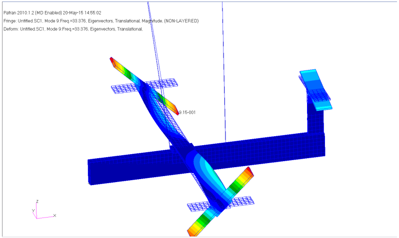
Scope

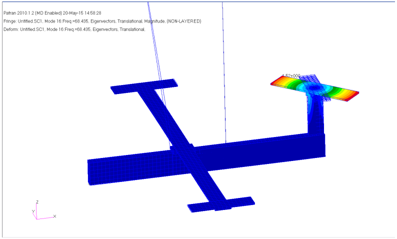
Comparison between Covariance Model Updating (Frequentist updating, optimization based) and Transitional Markov Chain Monte-Carlo (Bayesian updating)

Numerical model









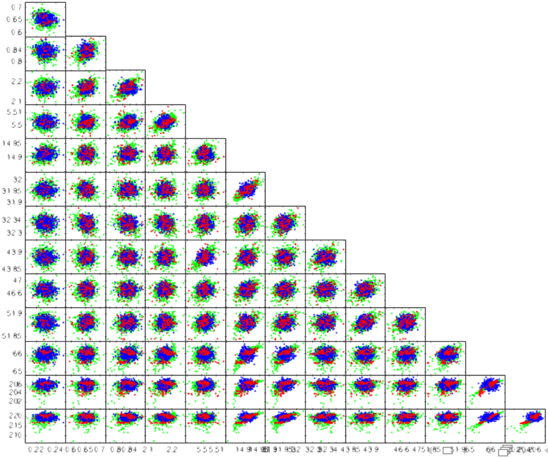
Uncertain Parameters

	Type	Location	Description	Init. val.	Unit
θ_{01}	Stiffness	Front bungee cord	Support stiffness	1.80E+03	N/m²
θ_{02}	Stiffness	Rear bungee cord	Support stiffness	7.50E+03	N/m²
θ_{03}	Stiffness	VTP/HTP joint	Sensor cable – *y* dirⁿ	1.30E+02	N/m
θ_{04}	Stiffness	Wing/fuselage joint	Sensor cable – *y* dirⁿ (top)	7.00E+01	N/m
θ_{05}	Stiffness	Wing/fuselage joint	Sensor cable – *y* dirⁿ (bottᵐ)	7.00E+01	N/m
θ_{06}	Stiffness	VTP/HTP joint	Joint stiffness – *x*, *y* dirⁿˢ	1.00E+07	N/m
θ_{07}	Stiffness	VTP/HTP joint	Joint stiffness – *z* dirⁿ	1.00E+09	N/m
θ_{08}	Mass	VTP/HTP joint	Sensor cables	2.00E-01	kg
θ_{09}	Mass	Wingtip right wing	Screws and glue	1.86E-01	kg
θ_{10}	Mass	Wingtip left wing	Screws and glue	1.86E-01	kg
θ_{11}	Mass	Wingtip left/right	Sensor cables on wings	1.50E-02	kg
θ_{12}	Mass	Outʳ wing left/right	Sensor cables on wings	1.50E-02	kg
θ_{13}	Mass	Innʳ wing left/right	Sensor cables on wings	1.50E-02	kg
θ_{14}	Stiffness	Wing/fuselage joint	Joint stiffness – *x* dirⁿ	2.00E+07	N/m
θ_{15}	Stiffness	Wing/fuselage joint	Joint stiffness – *y* dirⁿ	2.00E+07	N/m
θ_{16}	Stiffness	Wing/fuselage joint	Joint stiffness – *z* dirⁿ	7.00E+06	N/m
θ_{17}	Stiffness	VTP/fuselage joint	Joint stiffness – *x* dirⁿ	5.00E+07	N/m
θ_{18}	Stiffness	VTP/fuselage joint	Joint stiffness – *y* dirⁿ	1.00E+07	N/m

Bayesian Updating

- Due to excessive time required by TMCMC a Neural Network has been employed
- *Step 1*
  - Uniform prior for TMCMC, Neural Network calibration data
  - Calibration: 10000 LHS samples
  - TMCMC: 500 Markov chain (convergence in 29 steps)
  - Parallelization of LHS and of Markov chains
  - Validation of posterior results running the full FE model
- *Step 2*
  - New prior distribution of the input parameters
  - Gaussian mixture distribution based on posterior of step 1
  - TMCMC with full FE model, 200 Markov chains
  - Convergence in 5 steps

Numerical Results - Covariance and Bayesian Updating



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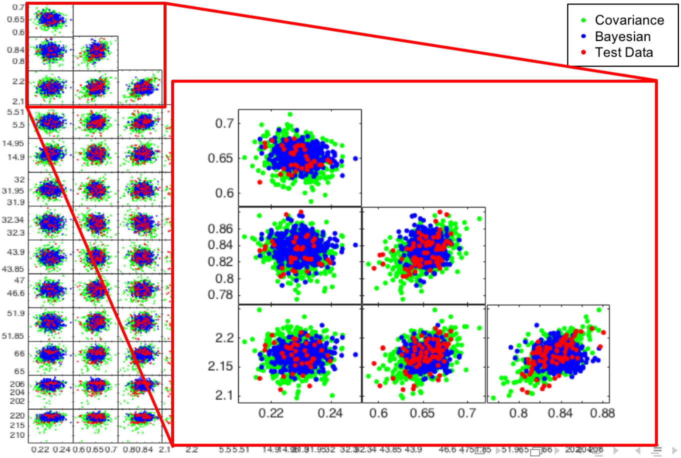

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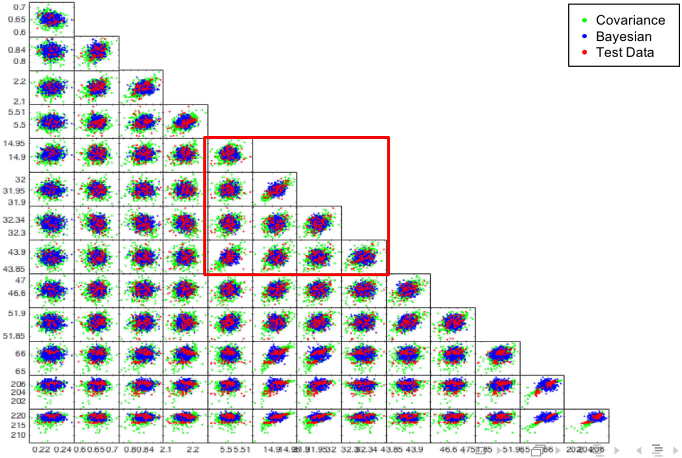

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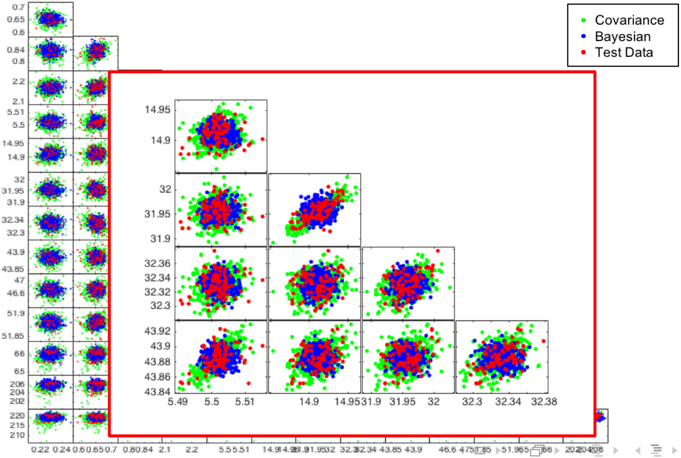

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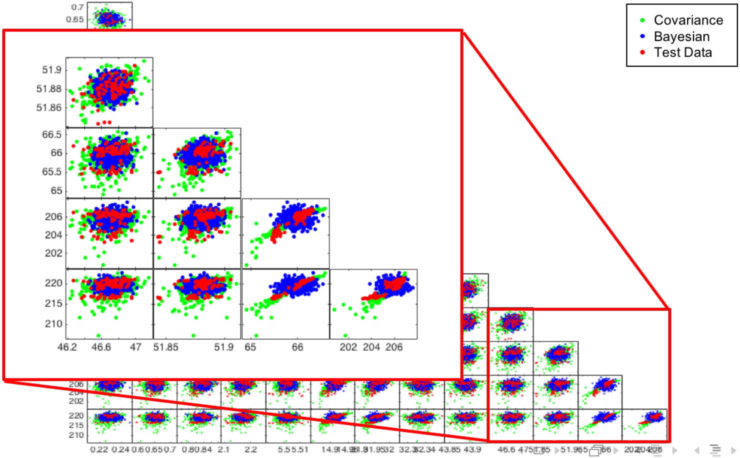
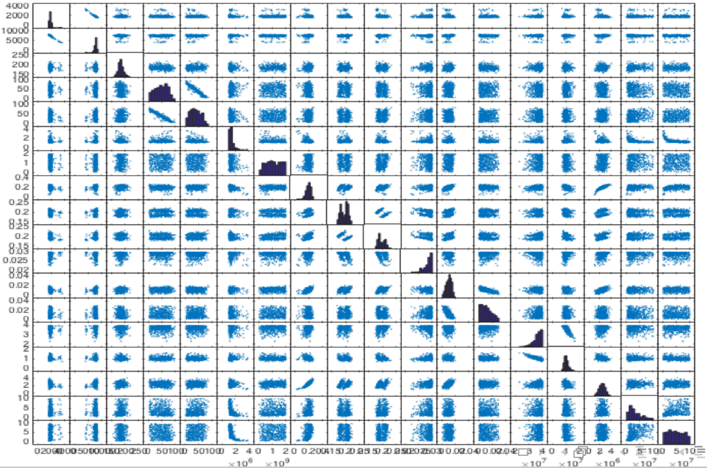


Numerical Results - Posterior distribution of inputs



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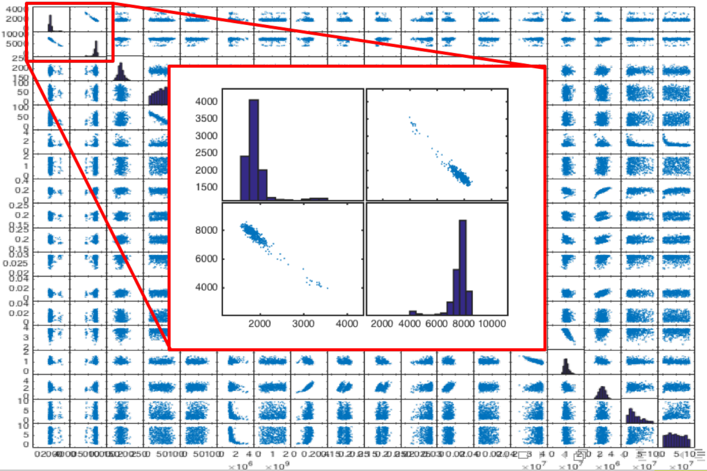

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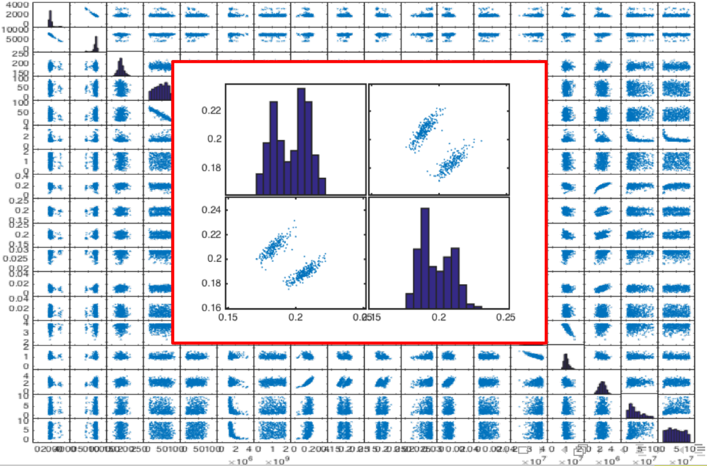
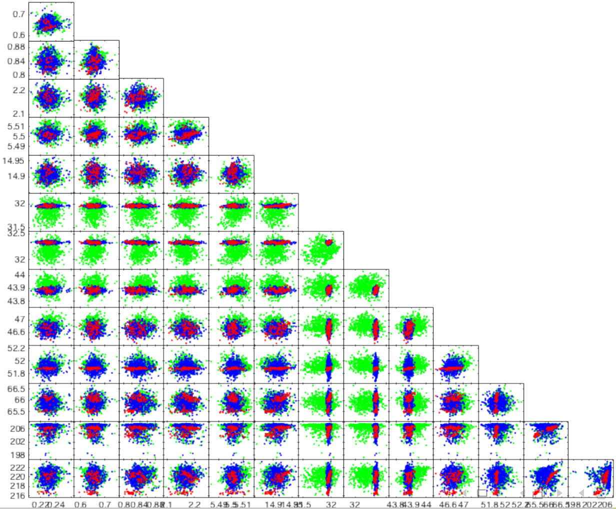
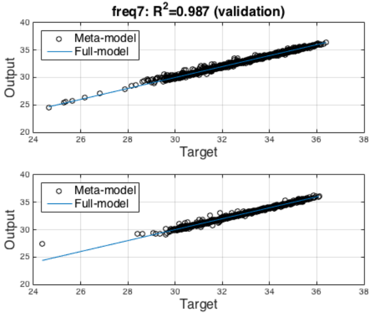


Numerical Results - Validation of results



Numerical Results - Validation of results

- Good performance when inputs are on a wide range
- Not enough calibration data in area of interest for posterior
  - Poor R^2 values (< 0.5)
  - Need new prior PDF with more data in this area
- Step 1 posterior used as new prior
- Full FE model used in next updating step to remove meta-model error



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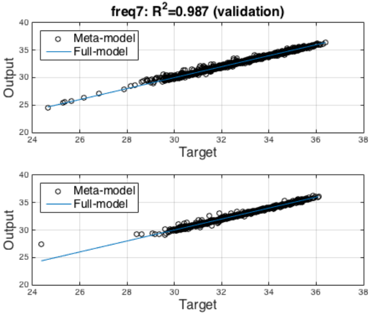

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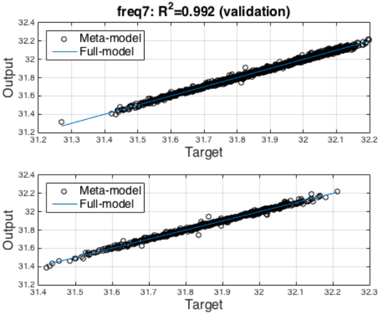
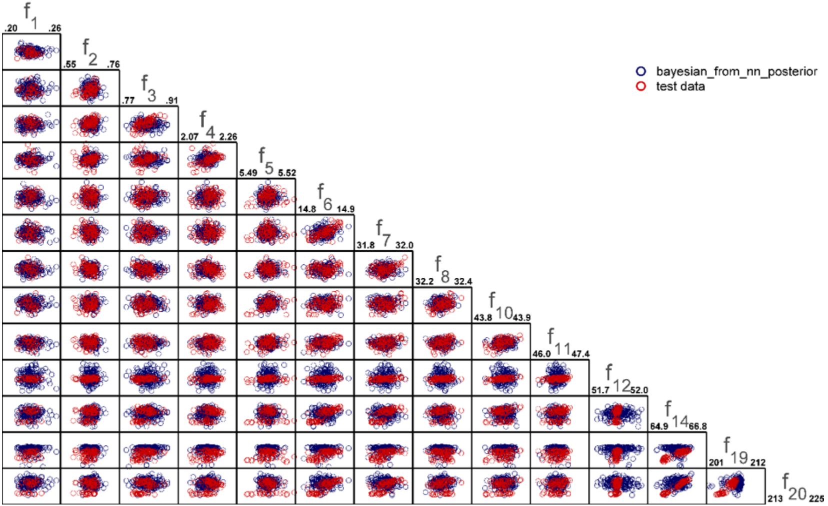


Numerical Results – Step 2



NASA Langley UQ challenge problem

Motivations and Timeline

Aim

- Determine limitations and ranges of applicability of existing UQ methodologies
- Develop **new discipline-independent** UQ methods

Timeline

- January 2013: 100+ UQ experts were invited to participate
- January 2014: 11 groups presented at Scitech 2014
- January 2015: Special edition AIAA Journal of Aerospace Information Systems

Solving each problem with at least **two** different approaches

- Cross validate results
- Increase confidence
- Test different hypotheses
- Different numerical implementation

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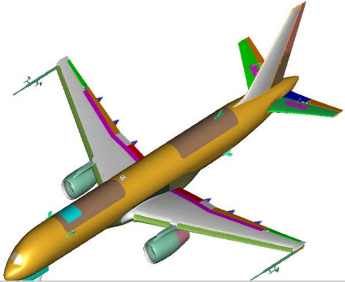
Physical System
NASA Langley Generic Transport Model (GTM)

- The GTM is a 5.5% dynamically scaled, remotely piloted, twin-turbine, research aircraft used to conduct experiments for the NASA Aviation Safety Program
- Aircraft motion that extends outside the normal flight envelope
- the dynamics are driven by **nonlinearities** and **coupling**, having **oscillatory** and **divergent** behaviour

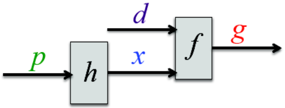


Generic Transport Model (GTM)

Dynamically scaled, highly instrumented, flight test article

The mathematical Model
Black-box model

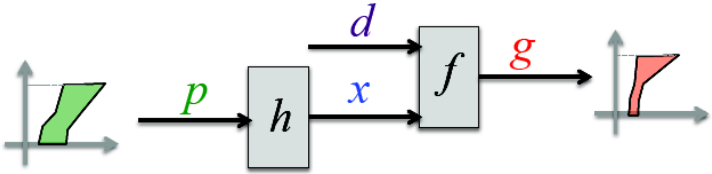
- **Specialized aircraft knowledge is not required**
- 5 tasks: Uncertainty Characterization, Sensitivity analysis, Uncertainty Quantification, Extreme Case analysis and Robust Design



$$\boldsymbol{x} = h(\boldsymbol{p}) \qquad g = f(\boldsymbol{p}, \boldsymbol{x})$$

- 21 Uncertain Parameters $\boldsymbol{p}$: loss in control effectiveness, actuator failure, icing, deadzone, and desired range in operating conditions
- 5 Intermediate variables $\boldsymbol{x}$ (e.g. control effectiveness of elevator, time delay due telemetry and communications)
- 14 Design variables $\boldsymbol{d}$: controller gains
- 8 Performance metrics $\boldsymbol{g}$ (e.g. Lon stability, lat/dir stability, elevator actuation)

Uncertain Parameters (p)
Challenges



Uncertainty models for p are given

Cat. I Random Variables (aleatory uncertainty)

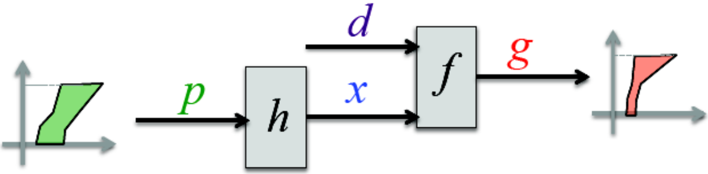
Cat. II Intervals (epistemic uncertainty)

Cat. III Probability boxes (aleatory + epistemic uncertainty)

Some of these Uncertainty Models can be reduced/improved

The propagation of p for a fixed d makes g a p-box

Uncertain Parameters (p)
Challenges



Uncertainty models for $\boldsymbol{p}$ are given

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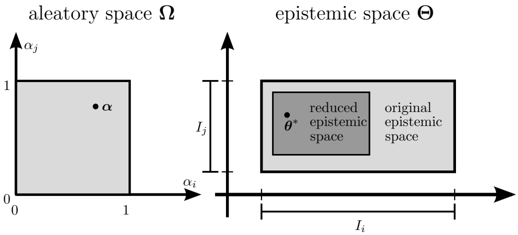
Cat. III Probability boxes (aleatory + epistemic uncertainty)

Some of these Uncertainty Models can be reduced/improved

The propagation of $\boldsymbol{p}$ for a fixed $\boldsymbol{d}$ makes $\boldsymbol{g}$ a p-box

Uncertainty Characterization
Aim and strategy

- Refine the uncertainty model of $\boldsymbol{p}$ given n observations
- What is the effect of the number of observations n?



Strategies

- Bayesian updating on the epistemic space
- Non-parametric statistical methods

Bayesian updating on the epistemic space
Transitional Markov Chain Monte Carlo

Adaptive MCMC algorithms used to explore the posterior distribution via sampling

- Do not sample directly from the posterior distribution
- Use intermediate distributions (weighted likelihood)
- Converge slowly to the posterior distribution

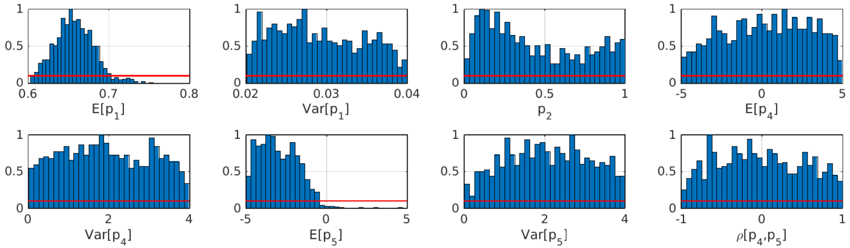
Advantages

- Allows to deal with multi-modal or very peaked distributions

Transitional Markov Chain Monte Carlo (TMCMC)

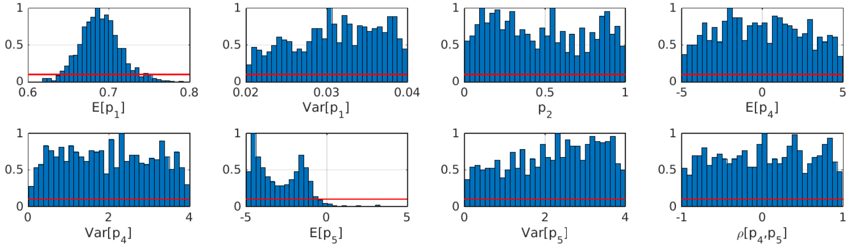
J. Ching and Y.-C. Chen. Transitional Markov Chain Monte Carlo method for Bayesian updating, model class selection, and model averaging. Journal of Engineering Mechanics, 133:816-832, 2007

Reduced epistemic space
Bayesian updating (25 observations)



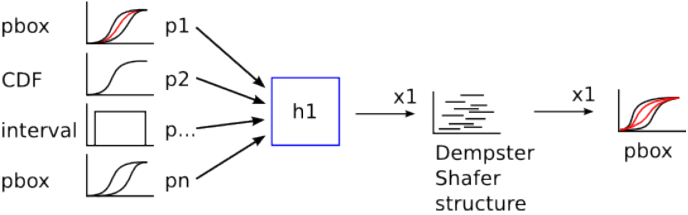
Reduced epistemic space

Bayesian updating (50 observations)



Sensitivity Analysis

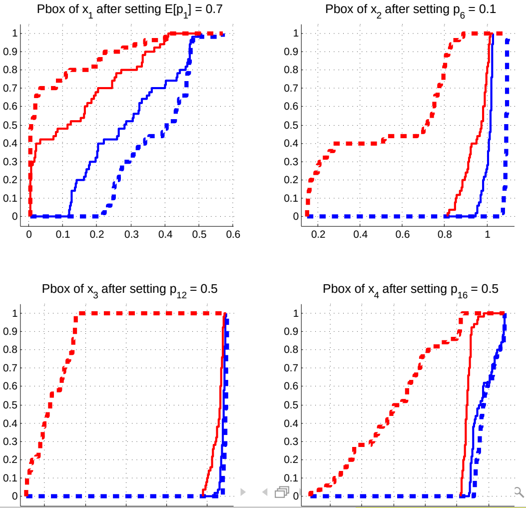
Pinching: reduce the width of an input interval / p-box, and evaluate reduction in output p-box



- Klir, G. J. and Wierman, M. J., Uncertainty-Based Information: Elements of Generalized Information Theory, Vol. 15 of Studies in Fuzziness and Soft Computing), Physica-Verlag, Heidelberg, Germany, 1998.
- Klir, G. J., Uncertainty and Information : Foundations of Generalized Information Theory, John Wiley and Sons, New Jersey, 2006.
- Alvarez, D. A., Reduction of uncertainty using sensitivity analysis methods for infinite random sets of indexable type, International Journal of Approximate Reasoning, Vol. 50, No. 5, 2009, pp. 750 – 762.

Sensitivity Analysis

Probability boxes corresponding to the original (thick lines) and pinched (thin lines) output probability boxes.



Uncertainty Propagation
Aim and strategy

Aim

- Find the range of the metrics with *reduced* and *improved* models

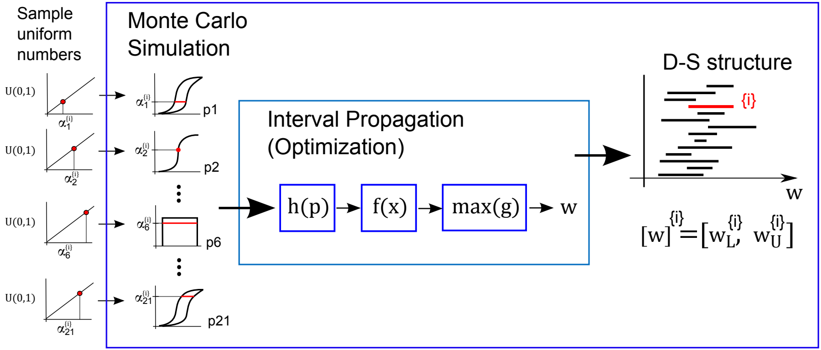
$$J_1 = E[w(\boldsymbol{p}, \boldsymbol{d}_{\text{baseline}})] \quad \text{and} \quad J_2 = 1 - P[w(\boldsymbol{p}, \boldsymbol{d}_{\text{baseline}}) < 0]$$

- 21 input parameters (random variables, intervals, p-boxes)
- 8 performance functions

Strategies:

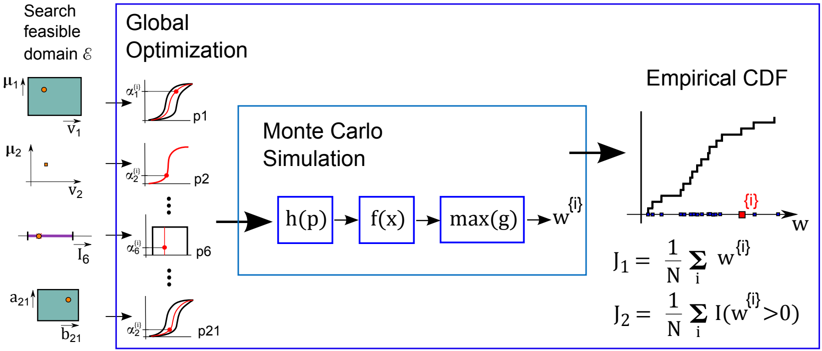
- Approach **A**: propagate intervals obtained from given distribution-free p-boxes and construct Dempster-Shafer structure
- Approach **B**: global optimization in the epistemic space (search domain). Monte Carlo Simulation to estimate J_1 and J_2

Approach A
Solution Strategy



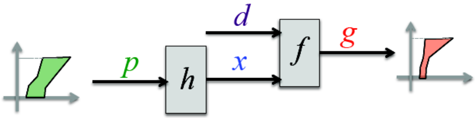
$$[J_1] = \frac{1}{N}\sum_{i}^{N}[w]^{\{i\}} \qquad [J_2] = \frac{1}{N}\sum_{i}^{N}\left[\mathcal{I}(w_L^{\{i\}} \geq 0),\ \mathcal{I}(w_U^{\{i\}} \geq 0)\right]$$

Approach B
Solution Strategy



$$[J_1] = \left[\min_{\mathcal{E}} J_1,\ \max_{\mathcal{E}} J_1\right], \qquad [J_2] = \left[\min_{\mathcal{E}} J_2,\ \max_{\mathcal{E}} J_2\right]$$

Robust Design
Aim and strategy



- **Design Variable:** 14 control parameters ($\boldsymbol{d}$)
- **Objective Function:** minimize $max(J_1)$ (expected value)
- **Objective Function:** minimize $max(J_2)$ (failure probability)

Computational challenges

- Each candidate solution: $\approx$ 3 days (i.e. $max(J_1)$, $max(J_2)$)

Strategies:

- Surrogate model (Artificial Neural Networks)
- Optimizer: Genetic Algorithms (and BOBYQA)

Robust Design

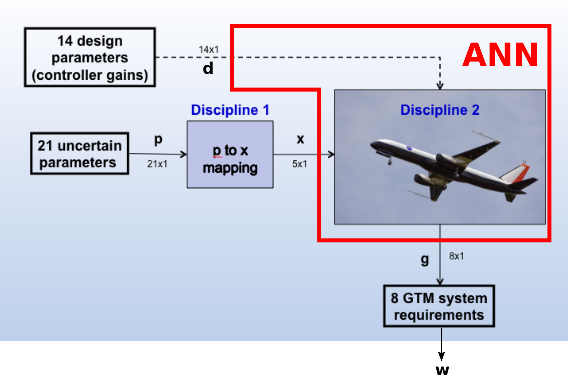
Surrogate model (Artificial Neural Networks)

Approximation of the most computationally expensive part

- Inputs: $\mathbf{x}$ and $\mathbf{d}$
- Outputs: $\mathbf{g}$

Results:

- $max(J_1) = 0.0044$ (baseline 3.05)
- $max(J_2) = 0.34$ (baseline 0.41)



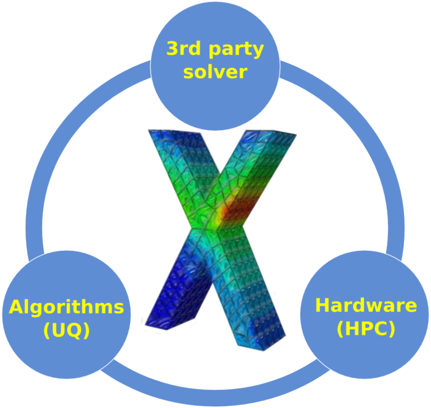
Outline

Summary

- Epistemic uncertainties allow UQ analysis with limited information
- Updating improves uncertainty model when experiments are available
- Sensitivity ranks uncertainty parameters and allow to ignore non important inputs helping simplifying model and analysis
- Advanced algorithms and meta-models make very computational intensive computations feasible

Summary
COSSAN Software

- General purpose software with interaction with 3rd party solvers
- Advanced simulation methods for treatment of uncertainties
- High Performance Computing capabilities
- Different interfaces for advanced and inexperienced users



Modular Framework: easy to use/reuse components

Summary
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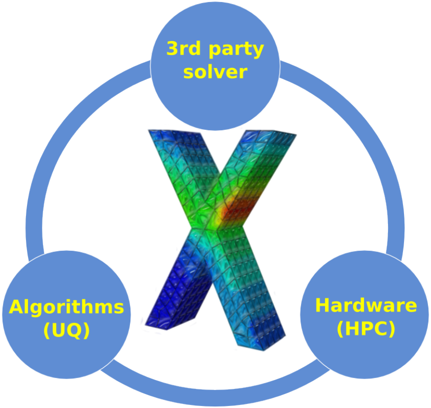

Modular Framework: easy to use/reuse components