Non-linear Structural Analysis Technique Based on Flexibility Method by Pade Approximants,

Application on Spherical Assemblies

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Outlines

1. Introduction

- Spheres are always considered unique and elegant geometry for structures.
- Due to the geometrical characteristics of spheres, they are used to afford a wide span as a lightweight structure with economical choice.

Fig. 1. Al Wasl plaza spherical structure

1. Introduction

- When they are affected by specific external loads, they face notable deformation.
- Precise computation during the analysis and design process considering their geometrical nonlinear behavior.
- Analysis approaches:

✓Dynamic relaxation, ✓Minimum potential energy, ✓Finite element, ✓Force method.

1. Introduction

- An alternative approach is conducted in analyzing the spherical model which is a nonlinear geometrical structure.
- The proposed technique is based on the nonlinear force method expanded using the Pade approximation method.

2. Geometrical nonlinear force method

- Equilibrium equation

$$Q(d)t = P \quad (1)$$

- Compatibility conditions

$$C(d)d = e\left(d\right) \quad (2)$$

- Constitutive relationship

$$e\left(d\right) - Ft = 0 \quad (3)$$

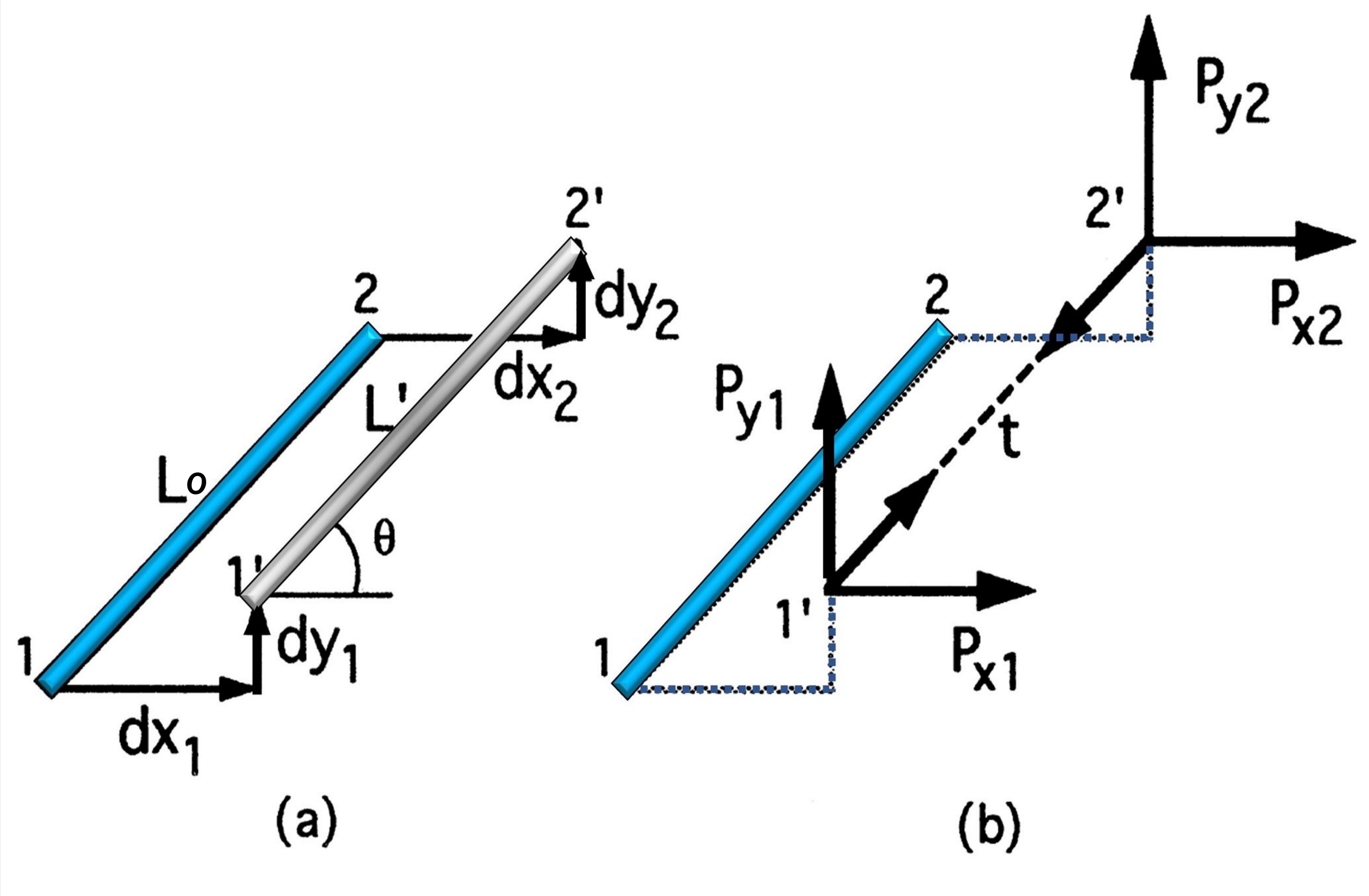


Fig. 2. (a) Original and deformed length of element 1-2.
(b) Original and deformed equilibrium state of element 1-2.

2. Geometrical nonlinear force method

Where:

$Q(d)$: equilibrium matrix at the deformed configuration.

t: internal force.

P: external force.

$C(d)$: the compatibility matrix at the deformed configuration.

d: nonlinear nodal displacement.

$e_o(d)$: nonlinear member actuation.

F: flexibility matrix $F = \dfrac{L_o}{AE}$

2. Geometrical nonlinear force method

$$e = L_c - L_o \quad \text{i}$$

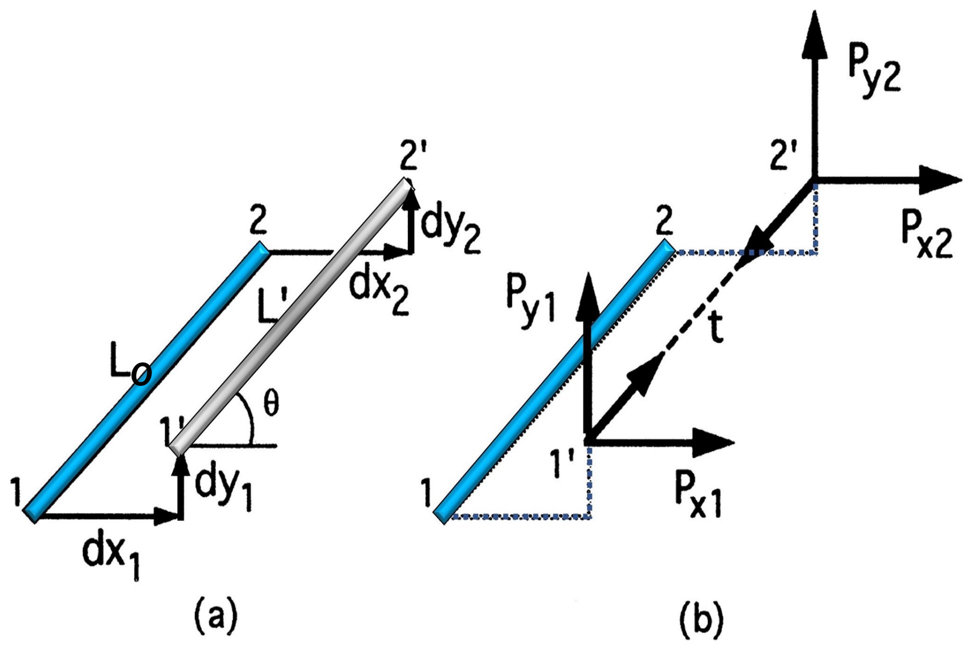
$$L_c = \left\{ \left(x_{ji} + dx_{ji} \right)^2 + \left(y_{ji} + dy_{ji} \right)^2 + \left(z_{ji} + dz_{ji} \right)^2 \right\}^{\frac{1}{2}} \quad \text{ii}$$

$$L_c = \left(L_o^2 + 2x_{ji} dx_{ji} + 2y_{ji} dy_{ji} + 2z_{ji} dz_{ji} + dx_{ji}^2 + dy_{ji}^2 + dz_{ji}^2 \right)^{\frac{1}{2}}$$

$$L_c = \left(L_o^2 + H \right)^{\frac{1}{2}}$$

$$L_c = L_o \left(1 + H / L_o^2 \right)^{\frac{1}{2}}$$

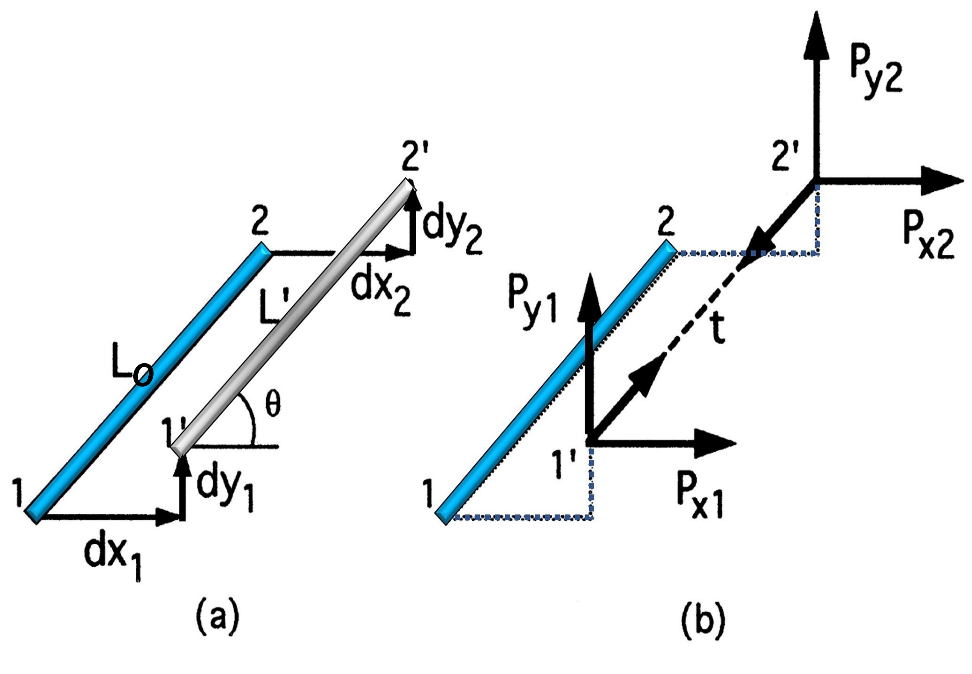
$$\boxed{L_c = L_o \left(\frac{4 + 3H/L_o^2}{4 + H/L_o^2} \right)}$$



2. Geometrical nonlinear force method

$$e = L_o \times \left\{ \left(\frac{4 + \dfrac{3\left(2x_o dx_o + 2y_o dy_o + dx_{ji}^2 + dy_{ji}^2\right)}{L_o^2}}{4 + \dfrac{\left(2x_o dx_o + 2y_o dy_o + dx_o^2 + dy_o^2\right)}{L_o^2}} \right) - 1 \right\} \quad (4)$$

$$P_x = \pm\, t \times \cos\theta$$
$$P_y = \pm\, t \times \sin\theta \quad (5)$$



$$\cos\theta = \frac{x_o + dx_o}{L_o + e} = \frac{4x_o L^2 + 4dx_o L^2 + 2x_o^2 dx_o + 2x_o y_o dy_o}{4L^3 + 6L\left(x_o dx_o + y_o dy_o\right)}; \qquad \sin\theta = \frac{y_o + dy_o}{L_o + e} = \frac{4y_o L^2 + 4dy_o L^2 + 2y_o^2 dy_o + 2y_o x_o dx_o}{4L^3 + 6L\left(x_o dx_o + y_o dy_o\right)}$$

3. Numerical example

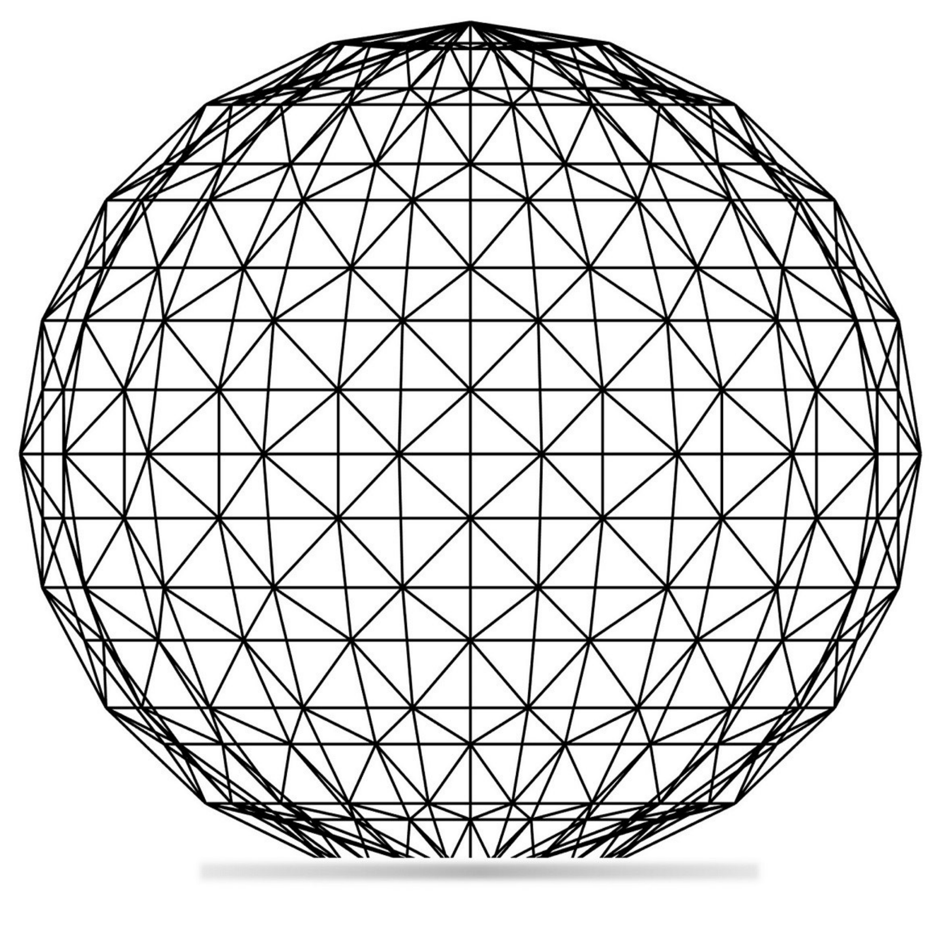


Fig. 3. Double-layer spherical model

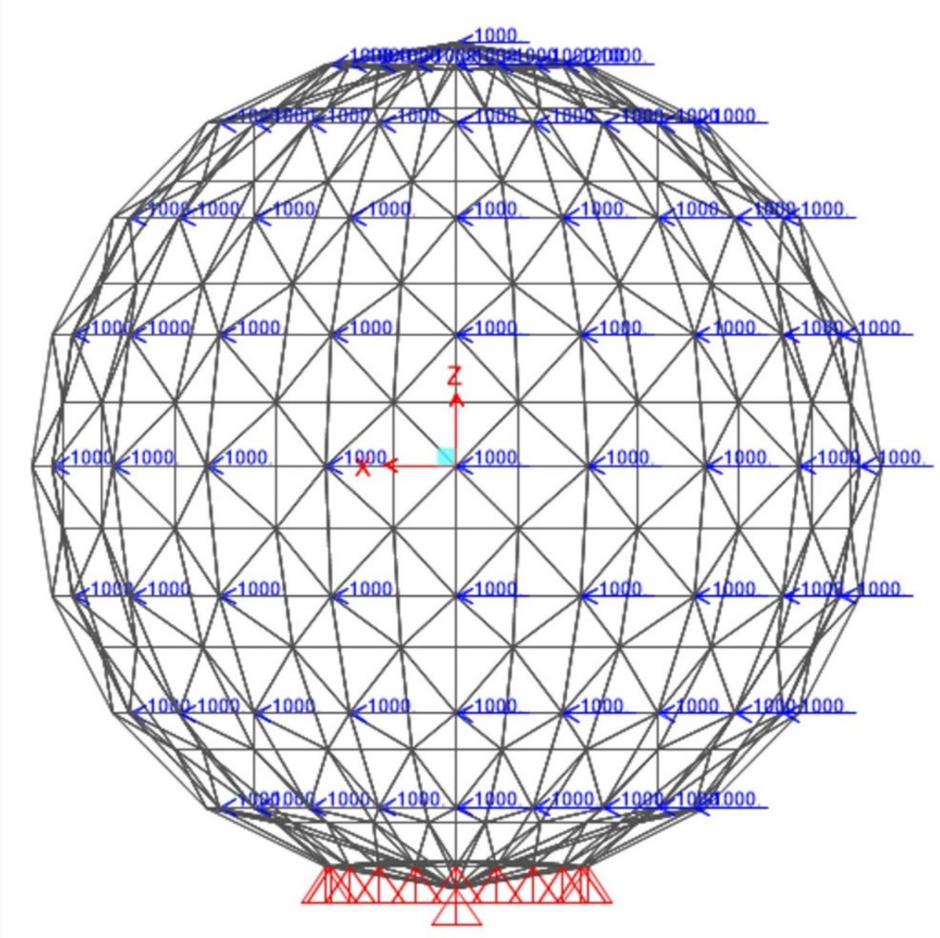


Fig. 4. Laterally loaded in (N) spherical model

4. Result

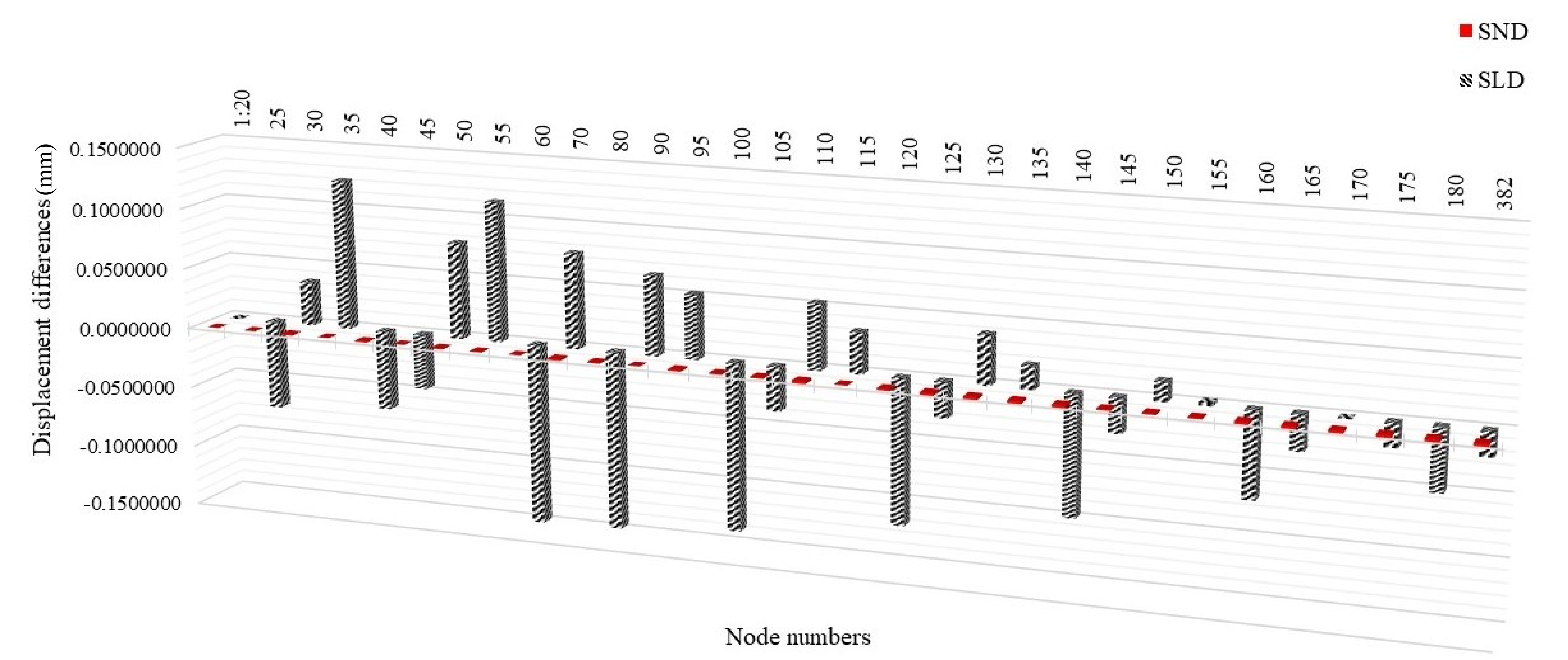


Fig. 5. Differences of resultant displacement obtained through nonlinear and linear force methods with respect to SAP2000

4. Result

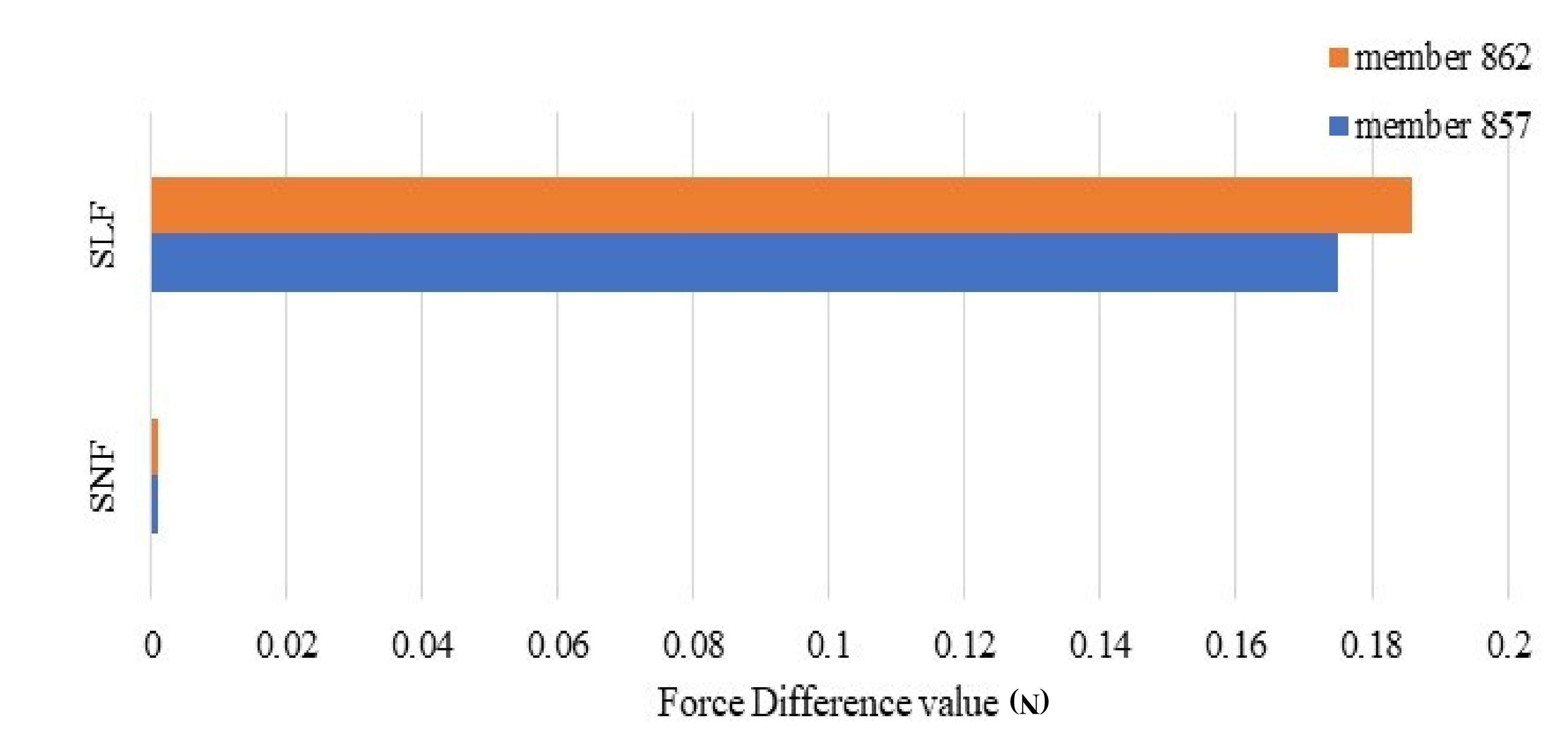


Fig. 6. Difference of maximum compressive and tensile member forces of SNF and SLF

5. Conclusion

1) The newly derived nonlinear geometrical analysis force method is proposed and applied to the double-layer spherical structure.

2) The analysis result was validated using the nonlinear finite element software SAP2000.

3) The maximum discrepancy for both the displacement and member force between nonlinear analysis results were 0.2% and 0.1% respectively.

4) Discrepancy came out as 14.6% for displacement differences, and 17.5% and 18.6% for tensile and compressive force differences in comparison to the linear approach.

5. Conclusion

5) The results showed the applicability of the technique in analyzing such a complex structure by concerning the nonlinear behavior of the structures.

6) The employment of the Pade approximation method in expanding the nonlinear member variation and internal force components provided a very convergeable function in solving the nonlinear equations.

6. References

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