

# Predicting the Drape of Woven Cloth Using Interacting Particles

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## Abstract

We demonstrate a physically-based technique for predicting the drape of a wide variety of woven fabrics. The approach exploits a theoretical model that explicitly represents the microstructure of woven cloth with interacting particles, rather than utilizing a continuum approximation. By testing a cloth sample in a Kawabata fabric testing device, we obtain data that is used to tune the model's energy functions, so that it reproduces the draping behavior of the original material. Photographs, comparing the drape of actual cloth with visualizations of simulation results, show that we are able to reliably model the unique large-scale draping characteristics of distinctly different fabric types.

**Keywords:** cloth, drape, physically-based modeling, particle systems, Kawabata Evaluation System.

## 1 Introduction

The vast number of uses for cloth are mirrored in the extraordinary variety of types of woven fabrics. These range from the most exquisite fine silks, to the coarsest of burlaps, and are woven from such diverse fibers as natural wool and synthetic polyester. Each of these unique fabrics has its own distinguishing characteristics, and is recognizable to the trained eye, perhaps most easily, by the way it drapes. It is not surprising, then, that image makers, designers, and engineers have had a keen interest in characterizing the draping properties of cloth.

In this paper we report on a new technique for reliably reproducing the characteristic drape of particular fabrics. Here, drape means the final configuration of a cloth placed over a solid object. We attempt to answer questions like "What would this shirt look like made from cotton rather than from polyester?" or "Would this dress have a more pleasing drape if made from silk rather than a light wool?". Our work on this problem began several years ago with the development of a theoretical model of woven cloth based on interacting-particle methods [8], that we used to model such complex draping configurations as those in Figure 1. More recently we have been working on a technique for using empirical data from the *Kawabata Evaluation System* [29] fabric measuring equipment

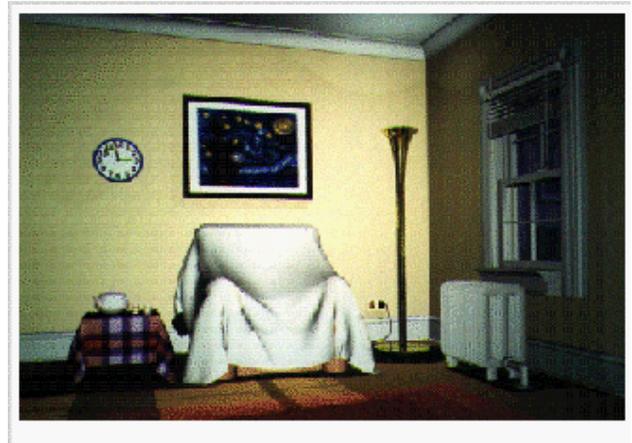


Figure 1: Draping cloth objects

to tune the model. With this technique we can now test a particular cloth sample, derive energy functions based on the sample's non-linear mechanical properties, and then use the model to reproduce the fabric's characteristic large-scale draping behavior.

To date, most of the efforts to create a model of cloth have employed continuum mechanics, with simulations utilizing finite element or finite difference techniques. These models have provided less than satisfactory results when attempting to accurately reproduce the characteristic folds and buckles found in specific types of cloth. In the introduction to a 1978 study on textile mechanics Shanahan, Lloyd and Hearle [39] express the opinion that

"Because of the relative coarse structure of textile materials, . . . it might be more profitable . . . to use non-continuum systems directly in the problems of complex (fabric) deformation."

Despite their reservations, they explored continuum methods for many years, but Hearle finally abandoned this approach, stating that [1]

"In dealing with 3-dimensional buckling of textile fabrics, neither the terminology nor the methodology of established (continuum) theory of bending plates and shells is of much help."

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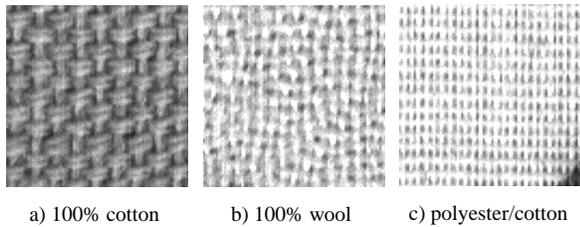


Figure 2: Magnified views of 3 samples of woven cloth

The problem is that cloth is a complex mechanical mechanism whose components are at a scale that is close to the scale of a typical simulation mesh element. Fine fibers are spun into yarns or threads, and these threads are woven into an interlocking network. Significantly, this assemblage is held together not by molecular bonds or welds, but simply by friction. The complexity and variety of the resulting mechanical systems are evident in the magnified views of small pieces of cotton, wool, and polyester/cotton cloth shown in Figure 2. The cotton material is woven with the coarsest thread and has doubled weft threads. The wool material is the most irregular in structure and appears to have the loosest weave. The polyester/cotton is the most regular, is of the finest thread, and has the most open space between parallel threads. The behavior of each of these “mechanisms” depends upon all of these factors, as well as fiber type, and ambient conditions such as humidity and temperature.

Our experience is that it is now both possible and practical to construct a model that captures key elements of the small-scale structure of woven cloth. In contrast to continuum techniques, our model utilizes interacting particles. This approach is founded on the premise that by modeling the low-level structures of a material and computationally aggregating their small-scale interactions, correct macroscopic behavior will emerge.

## 2 Background

Our approach to cloth modeling builds upon and merges concepts developed for particle systems and previous cloth models. The following summary of particle system and cloth modeling work is necessarily brief. For further information, we refer the reader to the extensive particle system bibliography found in [9], and to the detailed review of cloth modeling work found in [11].

### 2.1 Particle systems

Particle Systems were first used in computer graphics by Reeves in 1983 [36]. He defined a particle system model as “a cloud of primitive particles,” where each particle is generated into the system, moves, ages, and then dies. This work was later extended by Reeves and Blau [37], Fournier and Reeves [20], Sims [42], and many others to model such diverse phenomena as trees and grass, ocean spray, fireworks, waterfalls, fire, snowstorms and explosions. Reynolds [38], in his work on flocking behavior, greatly enhanced the power of the particle system as a modeling tool. He proposed the idea of coupling the particles so that they interact with each other as well as with their environment, and demonstrated that it is possible to exploit simple local rules of interaction between large numbers of simple primitives to produce complex aggregate behaviors. Miller and Pearce [32], Terzopoulos, Platt and Fleischer [46], and Tonnesen [48] all explored coupled particle systems as a way to model liquid-like and melting materials. Miller et al. [33], Szeliski and Tonnesen [44], and van Wijk [49] proposed particle interactions that are a function of direction, producing deformable

sheets and surfaces of particles. Our own interest in coupled particle systems has led to a variety of explorations into specialized modeling and visualization tools [6, 24, 25, 47], into computational issues [26, 27, 34], and CAD technologies [3, 4].

## 2.2 Cloth modeling

### 2.2.1 computer animation models

The first computer animation model of cloth was by Weil [50], who used a two-step geometric process to model a rectangular cloth hanging from several constraint points. Dhande et al. [17] present a hybrid drape model, which relates the parameters of a swept surface to fabric mechanical properties. Feynman [19] developed the first true physically-based cloth model. His model utilizes a set of energy equations based on the theory of elastic shells, distributed over a grid of points. Haumann and Parent [23] produced several cloth animations, including a flag waving and curtains blowing in a breeze. Terzopoulos and Fleischer [45] developed a wide range of models for computer graphics based on elasticity theory. Their finite difference and finite element simulations demonstrated 3-D cloth-like structures that bend, fold, wrinkle, interact with solid geometry, and tear. Others have extended their model to simulate complete sets of clothing [13], and how cloth responds to air flow [30]. Aono [2] also used elasticity theory to simulate ripples in cloth-like structures.

### 2.2.2 engineering and design models

The first cloth draping work published in the engineering community was by Shanahan et al. [39], who used the theory of sheets, shells and plates to characterize a matrix of elastic parameters for a sheet of material. Lloyd [31] later provided non-linear extensions to the matrix and used finite element methods to simulate a 3-D circular cloth being deformed by a projectile. Eischen et al. [18] modeled cloth structures using a large deformation beam and shell theory recently proposed by Simo et al. [41]. Collier et al. [15] present a finite element approach to modeling draping behavior. They also tested fabric using a drape measuring device called the Drapemeter [14], and showed that coefficients produced by their simulations compared favorably with measured values. As part of an apparel CAD system [35], Imaoka et al. [28] developed a continuum mechanics model of cloth based on the large deformation shell theory of Green and Zerna [22]. They also attempted to incorporate data from the Kawabata mechanical tester, but were unable to find a clear mapping of test data into their model.

## 3 A Particle-Based Model of Cloth

The fundamental principles of our particle-based model of cloth have been fully described elsewhere [8, 27], but they are briefly reviewed here, since a basic understanding is essential to the theme of this paper.

We model cloth as a collection of particles that conceptually represents the crossing points of warp and weft threads in a plain weave. Important mechanical interactions that determine the behavior of woven fabric occur at these points. Most significantly, the tension is typically so great at crossings that the threads are clamped together, providing an axis around which bending can occur in the plane of the cloth. Other more distributed interactions, such as stretching of threads and out of plane bending, can be conveniently discretized and lumped at the crossing points.

In the model, we represent the various thread-level structural constraints with energy functions that capture simple geometric relationships between particles within a local neighborhood. These

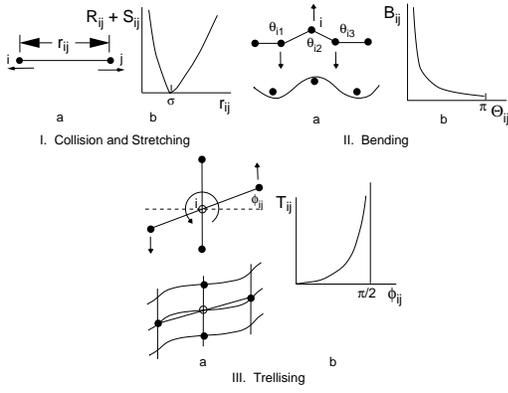


Figure 3: Cloth model energy functions

energy functions are meant to encapsulate four basic mechanical interactions: thread collision, thread stretching, out-of-plane bending, and trellising. These are shown graphically in Figure 3, and are captured in the energy equation for particle  $i$ ,

$$U_i = U_{repel_i} + U_{stretch_i} + U_{bend_i} + U_{trellis_i} + U_{grav_i}. \quad (1)$$

In this equation,  $U_{repel_i}$  is an artificial energy of repulsion, that effectively keeps every other particle at a minimum distance, providing some measure of thread collision detection, helping prevent self intersection of the cloth.  $U_{stretch_i}$  captures energy of tensile strain between each particle and its four-connected neighbors.  $U_{bend_i}$  is the energy due to threads bending out of the local plane of the cloth, and  $U_{trellis_i}$  is the energy due to bending around a thread crossing in the plane.  $U_{grav_i}$  is the potential energy due to gravity. Repelling and stretching are functions only of interparticle distance  $r_{ij}$  (Figure 3-Ia), whereas bending and trellising are functions of various angular relationships between segments joining particles (Figure 3-IIa and 3-IIIa).  $U_{grav_i}$  is a function of the height of the particle. Trellising occurs when threads are held fast at a crossing and bend to create an ‘‘S-curve’’ in the local plane of the cloth, and is related to shearing in a continuous sheet of material, but since our model treats cloth as an interwoven grid of threads, trellising is a more descriptive term.

We assume that the threads in the fabric do not stretch significantly when a cloth is simply draping under its own weight. Therefore, the combined stretching and repelling energy function  $R + S$  shown in Figure 3-Ib is not empirical, and is meant only to provide collision prevention and a steep energy well that acts to tightly constrain each particle to a nominal distance  $\sigma$  from each of its 4-connected neighbors. We have had good success with the functions

$$R(r_{ij}) = \begin{cases} C_0[(\sigma - r_{ij})^5/r_{ij}] & r_{ij} \leq \sigma \\ 0 & r_{ij} > \sigma, \end{cases} \quad (2)$$

and

$$S(r_{ij}) = \begin{cases} 0 & r_{ij} \leq \sigma \\ C_0[((r_{ij} - \sigma)/\sigma)^5] & r_{ij} > \sigma, \end{cases} \quad (3)$$

where  $C_0$  is a scale parameter.

The function  $U_{repel_i}$  prevents collision and self intersection, so it is calculated by summing over all particles, as given by

$$U_{repel_i} = \sum_{j \neq i} R(r_{ij}). \quad (4)$$

In practice, our simulation algorithm maintains a spatial enumeration, so that the summation need only be done over near neighbors.

An energy well is produced by directly coupling each particle with the stretching function  $S$  only to its 4-connected neighbors, as given by

$$U_{stretch_i} = \sum_{j \in N_i} S(r_{ij}), \quad (5)$$

where  $N_i$  is the set of particle  $i$ 's four-connected neighbors.

The particle energy due to gravity is simply defined as

$$U_{grav_i} = m_i g h_i, \quad (6)$$

where  $m_i$  and  $h_i$  are the mass and height of particle  $i$ , and  $g$  is gravitational acceleration. The mass is of the small patch of cloth represented by the particle.

In contrast to stretching, we assume that bending and trellising are the significant contributors to the overall drape of cloth, when it is simply draping under its own weight.

We define a unit of the bending energy  $B$  shown in Figure 3-IIb as a function of the angle formed by three particles along a weft or warp ‘‘thread line’’, as shown in Figure 3-IIa. The complete bending energy is

$$U_{bend_i} = \sum_{j \in M_i} B(\theta_{ij}), \quad (7)$$

where  $M_i$  is the set of six angles  $\theta_{ij}$  formed by the segments connecting particle  $i$  and its eight nearest horizontal and vertical neighbors. This definition is used so that the derivative of bending energy reflects the total change in bending energy due to change in position of particle  $i$ . The redundancy in this formulation is taken care of later by proper scaling.

The phenomenon of trellising is diagramed in Figure 3-IIIa and a corresponding unit of the trellising energy  $T$  is shown in Figure 3-IIIb. Two segments are formed by connecting the two pairs of neighboring particles surrounding a central particle. An equilibrium crossing angle of  $90^\circ$  is assumed, but this angle could easily change over the course of a simulation to model slippage. The trellis angle  $\phi$  is then defined as the angle formed as one of the line segments moves away from this equilibrium. The complete function for our energy of trellising is

$$U_{trellis_i} = \sum_{j \in K_i} T(\phi_{ij}), \quad (8)$$

where  $K_i$  is the set of four trellising angles  $\phi_{ij}$  formed around the four-connected neighbors of particle  $i$ . As with bending, this redundant formulation was chosen so that change in total energy with change in the particle's position is completely accounted for locally.

The simulation of the model is implemented as a three-phase process operating over a series of small discrete time steps [27]. The first phase for a single time step calculates the dynamics of each particle as if it were falling freely under gravity in a viscous medium, and accounts for collisions between particles and surrounding geometry. The second phase performs an energy-minimization to enforce interparticle constraints. A stochastic element of the energy minimization algorithm serves to both avoid local minima and to perturb the particle grid, producing a more natural asymmetric final configuration. The third phase corrects the velocity of each particle to account for particle motion during the second phase.

The energy functions indicated in the curves in Figure 3 are similar in shape to those that we first used to verify the theoretical model. These initial functions were simply convenient ones that we knew would smoothly interpolate reasonable boundary conditions. Even with these ‘‘sketched-in’’ energy functions, the simple interactions governing the particles aggregate to produce a macroscopic draping behavior that is convincingly close to that of cloth. We were able to produce visually satisfying results, such as the cloths draped over both the easy-chair and end-table in Figure 1, after just a few runs to tune constants.

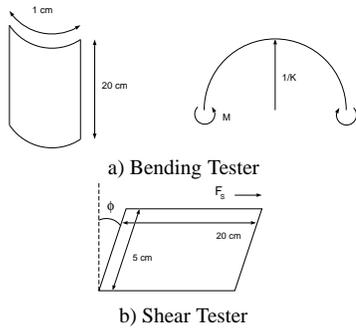


Figure 4: Kawabata Evaluation System measurements

## 4 The Kawabata Evaluation System

Even though early experiments confirmed that we could generate reasonable looking draping behavior, there were many things about our simulated cloth that we did not know. The model was not based on physical units, so we did not know the actual size of our simulated cloth sample, and we could not query the model for any kind of mechanical information. Most importantly, we did not have a methodical means of tuning the model to simulate particular kinds of cloth.

In order to tie the model directly to the draping behavior of actual cloth, we have developed a method for deriving the model's energy equations from empirical mechanical data produced by the Kawabata Evaluation System [29]. This system is a standard set of fabric measuring equipment that can measure the bending, shearing and tensile properties of cloth, as well as its surface roughness and compressibility. For bending, shearing and tensile properties, the equipment measures what force or moment is required to deform a fabric sample of standard size and shape, and produces plots of force or moment as a function of measured geometric deformation. Since we assume that threads do not stretch significantly when a cloth is simply draping under its own weight, we make use only of the Kawabata bending and shear plots. There is, however, no reason why Kawabata tensile data could not be used if one wished to model fabric under tensile load.

The Kawabata bending measurement is done by clamping a  $20\text{ cm} \times 1\text{ cm}$  sample of cloth along both its long edges. The sample is then bent between the clamps, as diagramed in Figure 4a, and the moment necessary to accomplish the bending is recorded. A plot of bending moment  $M$  versus curvature  $K$  is produced by assuming that the  $1\text{ cm}$  cross-section bends with constant curvature. The shearing measurement is done by applying a shearing force along one of the long edges of a  $20\text{ cm} \times 5\text{ cm}$  cloth sample, as diagramed in Figure 4b. A plot of the force  $F_s$  versus shear angle  $\phi$  is produced. By cutting samples out of the original cloth in two orthogonal directions, it is possible to measure bending and shear in both the warp and weft directions.

Kawabata bending and shear plots for the 100% cotton, 100% wool, and cotton/polyester samples of Figure 2 are shown in Figure 5. Each curve plots a full deformation cycle for fabric oriented in both the warp (solid curve) and weft (dashed curve) directions. The curves are produced by applying a force (or moment) in one direction, releasing the force, reversing the direction of the force, and releasing the force once again. The plots clearly show the hysteric behavior of cloth – the path of deformation when the cloth is stressed is different from the path when the stress is released, producing a loop in the plot.

The plots for each type of cloth are obviously quite different. Although the shear plots differ little between warp and weft, there are dramatic differences between warp and weft bending for both the 100% cotton and the polyester/cotton materials. In the 100% cotton

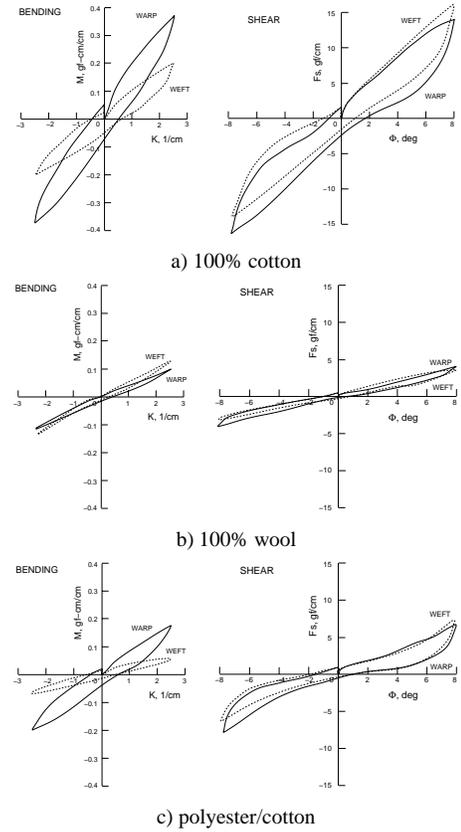


Figure 5: Kawabata Bending and Shear Plots

this is due to the doubling of weft threads, and in the polyester/cotton this is due to the differing warp and weft materials. The shallower shear curve for the 100% wool indicates that it will be the most limp and easily shaped.

We have found it useful to think of the bending and shearing as being divided into three loosely defined regions, a region of initial resistance to deformation, a region of low deformation, and a region of high deformation. The mechanical behavior of cloth throughout its range of deformation is non-linear, especially during initial deformation. The mechanical properties of cloth in the low-deformation region are usually well-behaved. This is why the assumption of linear elasticity made in continuum cloth models yields a reasonable “cloth-like” behavior. The mechanical properties in the initial-resistance and high-deformation regions are not as well-behaved, and defy a simple, general mathematical description. The Kawabata plots provide information only in the initial-resistance, and low-deformation regions.

## 5 Derivation of the Energy Equations

The process of generating particle energy functions for woven cloth from Kawabata data has three steps. First, we determine functions that approximate the Kawabata plots. Next, we relate these approximating functions to the model's energy functions. This is crucial, since in the case of bending, the approximating functions relate bending moment to curvature, and in the case of trellising, they relate force to shear angle, but what is needed is energy as a function of the bending and trellising angles shown in Figures 3-II and 3-III. Finally, we scale the resulting equations so that they will produce energy values in standard physical units.

## 5.1 Approximating the Kawabata curves

For purposes of calculating drape, we assume that the hysteresis of cloth does not play an important role. Thus, in approximating the Kawabata curves, we look only at the first of the four stages of the deformation cycles shown in Figure 5.

The most convenient way to approximate these curves is with piecewise polynomial functions. This can be done in any number of ways, but the important thing is that we obtain reasonable approximating functions  $M(K)$  for bending moment, and  $F(\phi)$  for shear force. We decided to interpolate the inflection points of the curves using the lowest order polynomials that were practical, and the Kawabata plots are sufficiently simple that we were able to do this with quadratic and linear segments. In each case, we first fit a function to the outer, more stable segment of the Kawabata curve, then fit additional segments to the initial segment maintaining position and slope continuity at the segment boundaries, using standard interpolation techniques [12]. In general, the slope of the Kawabata plots is difficult to determine at the origin. Therefore, although the first segment must pass through the origin, we did not enforce any slope constraint there.

## 5.2 The bending energy equations

Within the low deformation region within a single thread, we assume that the theory of elastic bending beams [40] is applicable, and can be used to calculate the energy of bending. The strain energy  $dU$  due to bending stored in a segment  $dS$  of an elastic beam is given by

$$dU = \frac{M dS}{2\rho}, \quad (9)$$

where  $M$  is the bending moment acting on the segment and  $\rho$  is its radius of curvature, which is related to curvature  $K$  by  $K = 1/\rho$ . Within our model, each particle is separated from its 4-connected neighboring particles by the equilibrium distance  $\sigma$ , which is a function of the grid dimensions and particle density. Therefore each particle represents a  $\sigma \times \sigma$  square of cloth. This square of cloth can be thought of as a series of elastic beams (threads) lined parallel to each other. The energy of bending in one of these threads is defined by the integral

$$U = \int_0^\sigma \frac{MK}{2} dS. \quad (10)$$

We assume that within one  $\sigma \times \sigma$  patch that the moment and the curvature are constant, simplifying the energy equation for a single thread to

$$U = \frac{MK}{2} \sigma. \quad (11)$$

Since  $M$  is given in units of moment per unit width of sample, we can simply multiply Equation 11 by the width  $\sigma$  of each patch, in order to sum up the contributions of each beam (thread) within the  $\sigma \times \sigma$  patch. The energy of bending in just one direction then becomes

$$B = \frac{MK}{2} \sigma^2 \quad (12)$$

for each particle. This calculation is performed twice for each particle, once for bending in the warp direction, and once for bending in the weft direction. Recall that our approximation of the Kawabata bending plot provides bending moment  $M$  as a function of curvature  $K$ , so that equation 12 yields bending energy  $B$  as a function only of curvature. Thus, we need only relate curvature to the bending angles of the model.

Curvature, along a thread, at the position of a single particle can be approximated by assuming that the curvature is constant from the particle to its two neighbors. Given this assumption, a

circle can be fit to the three points and the circle's curvature can be calculated [5]. Unfortunately, this assumption becomes poor as the bending angle  $\theta$  becomes small (i.e. as the threads bend in on one another). We would like the curvature  $K$  to become arbitrarily large for small bending angle, in order to give reasonable high-deformation behavior. As an approximation, we fit the curve  $a/\theta + b$  to the small-angle portion of the curvature equation to yield the complete curvature equation

$$K(\theta) = \begin{cases} \frac{2}{\sigma} \cos(\theta/2), & \pi/4 < \theta \leq \pi \\ -\left(\frac{\pi}{4}\right)^2 \beta/\theta + \alpha + \frac{\pi}{4} \beta, & 0 \leq \theta \leq \pi/4, \end{cases} \quad (13)$$

where the constants  $\alpha = \frac{2}{\sigma} \cos(\pi/8)$  and  $\beta = \frac{1}{\sigma} \sin(\pi/8)$  were chosen to maintain  $C_0$  and  $C_1$  continuity at  $\theta = \pi/4$ .

## 5.3 The trellising energy equations

We can calculate the energy stored in the 20 cm  $\times$  5 cm cloth sample that is sheared in a Kawabata Shear Tester from the work  $W$  produced by a force  $F$  acting over a displacement  $dS$ ,

$$W = \int F dS. \quad (14)$$

If we assume that the width  $l$  of the sample remains constant during shearing, then the path traveled by the point at which the shearing force is applied is a circular arc whose length is defined by  $S = l\phi$ , where  $\phi$  is the shearing angle. If the force point is moving along a circular arc, the component of the applied shearing force in the direction of motion is  $F \cos(\phi)$ . Applying these results to Equation 14 yields the equation for shearing energy as a function of shear angle first derived by Cusick [16],

$$T = \int F \cos(\phi) l d\phi. \quad (15)$$

Recall that the Kawabata shearing plots provide the shearing force  $F$  as a function of shearing angle  $\phi$ . Therefore, substituting the approximating equation for  $F$  into Equation 15 and integrating yields the required energy of shearing strictly as a function of angle.

Once again we are faced with the problem of defining energy curves in the high deformation region not covered by the Kawabata data. Skelton [43] states that most woven materials cannot shear more than  $45^\circ$ . We approximate this constraint by introducing a singularity in the trellising energy curve at about  $60^\circ$ . The extra  $15^\circ$  permits the material to shear all the way to  $45^\circ$  if necessary. This singularity is introduced by fitting the function  $a/(1.05 - \phi) + b$  to the slope and position at the endpoint of the Kawabata-derived energy curve (the magic number 1.05, is simply a rough approximation to  $\pi/3 = 60^\circ$ ).

## 5.4 Scaling the energy equations

Up to this point no attention has been paid to the physical units of the energy equations, although we would like them to be in CGS units.

The Kawabata bending plots (see Figure 5) give moment  $M$  in units of gf-cm<sup>1</sup>/cm and curvature  $K$  in cm<sup>-1</sup>. If we take interparticle distance  $\sigma$  to be in cm, then from equation 12 we see that the units for bending energy are gf-cm. Thus, scaling equation 12 by 978.80 will yield energy in ergs.

The Kawabata shearing plots (see Figure 5) give shearing force in units of gf/cm. Scaling this force by 978.80 yields dyne/cm, and multiplying the measured force by the 20 cm length of the sample

<sup>1</sup>The unit gf, or gram-force, is the Earth weight of one gram. It is equivalent to 978.80 dynes, and one gf-cm is equivalent to 978.80 ergs.

gives force in dynes. Since the trellising energy is produced by a moment, the energy unit is dyne-cm or ergs. Since one particle represents a  $\sigma \times \sigma$  patch of cloth, we want the trellising energy per unit area. This can be computed by dividing the total energy stored in the 20 cm  $\times$  5 cm sample by the area of the sample. Therefore the scale factor that converts energy  $T$  of equation 15 into trellising energy in ergs is  $195.76\sigma^2$ .

## 6 Experimental Results

We derived energy equations for the 100% cotton, 100% wool, and polyester/cotton cloth samples shown in Figure 2 using the Kawabata data shown in Figure 5. A full derivation may be found in [10].

Given the energy equations for all three samples, we performed two sets of experiments to verify that the model was able to capture the characteristic draping behavior of each type of cloth. The first experiment was to drape real cloth over a cube and then perform the same drape in simulation, using computer visualizations of the simulation results to make visual comparisons with the actual cloth. The second experiment was to recreate the Kawabata Bending and Shear Testers in simulation, to show that the model can accurately reproduce physical measurements. Results of the first set of experiments are of most interest to the computer graphics community and are detailed below. The second set of experiments produced excellent results that are of most interest to the engineering and design community, and are detailed in [10].

In the actual draping experiments, 1 m  $\times$  1 m sections of our three cloth samples were draped over a 0.5 m  $\times$  0.5 m  $\times$  0.5 m cube. The results of these drapings were photographed and are presented in the left column of Figure 8. The same scenario was recreated in simulation. Models of 1 m  $\times$  1 m samples of 100% cotton, 100% wool and polyester/cotton, represented by a 51  $\times$  51 particle grid, were draped over a 0.5 m  $\times$  0.5 m  $\times$  0.5 m geometric model of a cube. Each simulation started with a flat cloth positioned just above and centered over the cube. The simulation was allowed to run until the cloth had draped over the cube and had settled into an equilibrium position. Equilibrium was judged manually, by examining the maximum particle movement between successive time steps. The final drapings produced by the simulations are shown in the computer graphic visualizations in the right column of Figure 8. Camera positions were chosen to accentuate the unique draping characteristics of each type of cloth.

The similarities between the actual and simulated drapings are quite evident in Figure 8. Each kind of material has a characteristic drape that is captured by the simulation. Both the actual and simulated 100% cotton develop a single large billow that comes out from the corner of the cube at a 45° angle. By contrast, the bending stiffness of the wool sample is significantly weaker than the cotton's. Therefore, it does not have the bending strength needed to support a single large billow and the corner structure collapses into two smaller folds, as seen in both the actual and simulated views. In the polyester/cotton material, the bending stiffness is significantly stronger in the warp than in the weft direction. The effect on the draping of the cloth can be clearly seen. Since bending is so much stronger in one direction than the other, the billow is literally pushed around the corner by the warp threads. This produces an asymmetric structure that wraps around the corner of the cube, as can be seen by comparing the front and side views.

## 7 Discussion

The draping experiments show that the model can be used to reproduce the large-scale draping behavior of specific types of cloth, but the cloth simulations do not exactly produce the drape of the actual

cloth in Figure 8. This, in a sense, would be impossible, since cloth will never drape twice in exactly the same way. Instead, what we found when working with real materials is that each material does have its own "preferred" draping tendency. For example, at the corner of the cube, the cotton sample usually produced a single draping structure, the wool would form either a single fold or would collapse into more than one fold, and the polyester/cotton always produced an asymmetric fold. Of course, each material could be forced into many kinds of draping configurations, but when allowed to drape naturally, they generally produced their own characteristic structures.

Another difference evident in Figure 8 between the simulated and real drapings is in the sharpness of edges and corners. The simulated samples appear to have "soft" folds, as if being draped over a rounded cube. These differences are related to the fineness of the particle grid. A 51  $\times$  51 grid is capable of reproducing large-scale draping structures, but it is not sufficient for capturing the sharp bends over the edges or at the corners of a cube. We believe that utilizing a finer particle grid will remove these differences, and are currently working on an adaptive scheme that will sample a region of the cloth more or less finely based on the region's total energy.

The computational speed of our implementation is currently its major drawback. Each simulation, starting with a flat cloth placed above the cube and falling to its final draped configuration, required about 1 CPU-week on an IBM RS/6000 workstation. The issue of speed is one that we chose to ignore for a period while developing and proving the model. There are several ways to improve speed, the most obvious being to write custom simulation code. Currently we work in an object-oriented, message-passing environment that has been excellent for rapid prototyping, but entails a heavy overhead [7, 21]. A more fundamental speed improvement could be had by improving the simulation technique. We currently use a stochastic method that follows a numerically-determined approximation to the energy gradient at each particle [27]. We have begun work custom coding an efficient implementation of our model, that uses pre-calculated tables to more exactly and efficiently determine energy gradients, and are experimenting with a pure gradient-descent approach to energy minimization. Preliminary results indicate speed-ups well beyond an order of magnitude. Another approach would be to use an approximate, purely-geometric pre-drape, followed by the physical simulation to perfect the drape. This has already been tried with success by others [35, 51]. Finally, parallelism has been shown to be an especially efficient way of computing uncoupled particle systems [42]. The highly distributed form of our particle model, with its simple local computations and well defined neighbor interactions, should also be especially amenable to this approach [26].

To designers, one of cloth's most important characteristics is its ability to be shaped and creased. Since we ignore hysteresis, our model, as it stands, is conservative – no energy is lost during a deformation. One consequence of this is that we cannot yet mimic shaping and creasing. This has not been an important issue in the kinds of free-draping studies that we have conducted, but would be of very great importance when looking at fabric under the high stresses that occur in manufacturing. It should, however, be relatively easy to extend our model in a natural way to simulate non-conservative deformation. The stretching, bending, and trellising energy functions all either explicitly or implicitly represent a "rest" value for their independent variables. It would be straightforward to represent all of these rest values explicitly, and then vary them as a function of local strain, thus mimicking the effects of slippage within the weave. This could be put on a firm physical basis, at least for the low deformation region, by adjusting "slippage" so that the full hysteresis curves from the Kawabata tester are matched.

## 8 Conclusion

We have presented a particle-based model capable of being tuned to reproduce the static draping behavior of specific kinds of woven cloth. There are several significant aspects to the work. It has demonstrated that a microstructural model may be used to reproduce the macroscopic mechanical behavior of real flexible materials. It has shown that the use of such an approach can allow for the straightforward incorporation of non-linear empirical test data. The model has been verified by experiments. One generates the low-level mechanical properties of real fabrics, and the other recreates the distinctive macroscopic geometric structures of draping cloth and compares them to actual cloth drapings. This kind of evidence has not been presented in previous cloth modeling studies. With this approach, real materials may now be measured, and the measured data used to derive energy equations, allowing the draping behavior of specific materials to be confidently simulated on a computer.

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