Computer Graphics:
11 - Aliasing and Antialiasing

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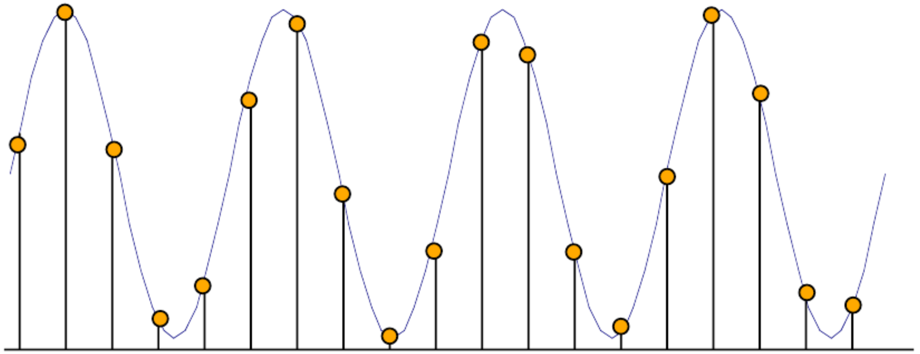
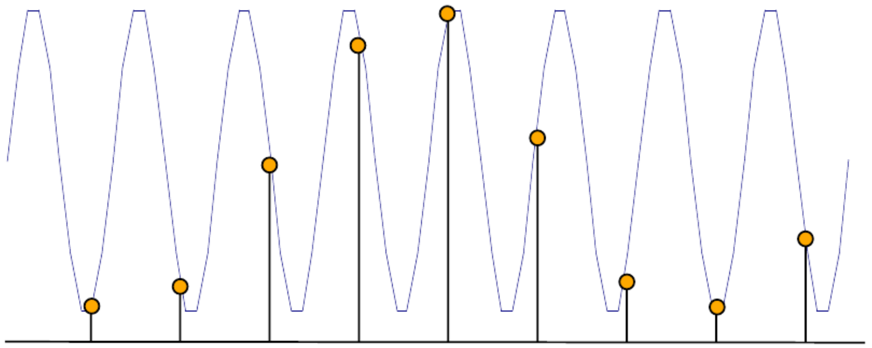
Introduction

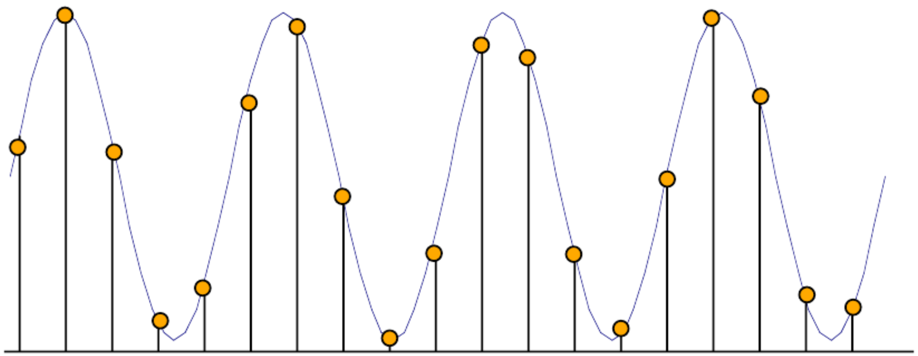
- Suppose you have a signal you want to sample at regular intervals: sampled points are marked in orange
- In the top sinus wave, the sample is fast enough that the reconstructed signals will have the same frequency than the original signal
- In the second wave, instead, the reconstructed wave will be appearing to have a much lower frequency than the original
- This is called an aliased signal

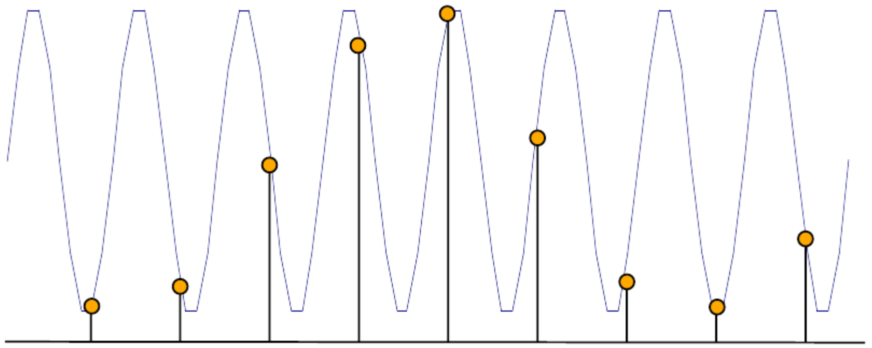




The Nyquist theorem

- In point sampling theory, there is the Nyquist theorem that states that to reconstruct accurately a signal, the sampling rate must be >= 2 times the highest frequency in a signal
- The consequence of this is the fact that music is sampled at 44khz to reproduce the audible spectrum up to 22khz
- Any frequences over 22khz are removed froom the system so as not to have low frequencies due to aliasing





An easy intro to Fourier Analysis

- Let F(x) be a continuous function of a variable
- The same function can be written as the integral

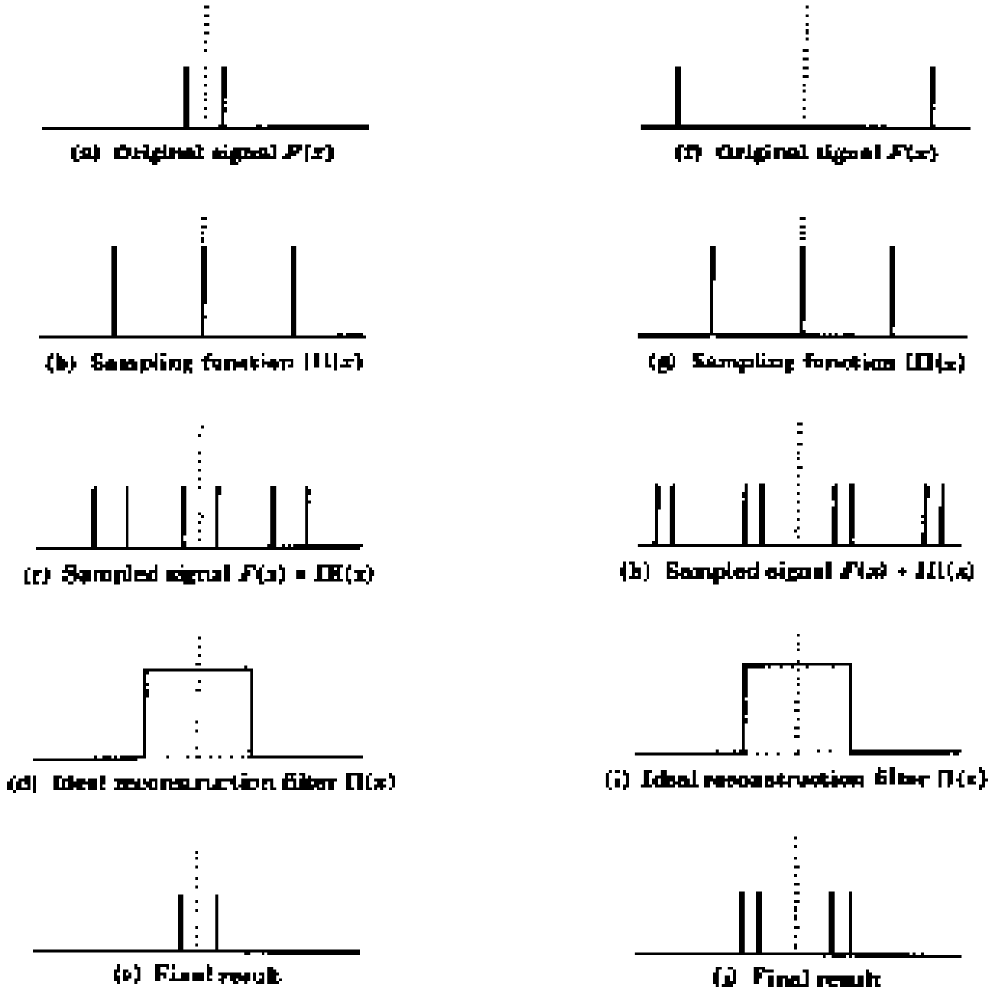
$$F(\omega)=\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi\omega t}\, dt$$

  here we basically changed the function space base functions, from the cartesian functions to the Fourier basic functions
- The Fourier representation has the advantage of expressing explicitly the frequencies present in the function, since each coordinate function coefficient tells me how much that frequency is present in F
- The Fourier transform can be discretized for N samples

$$F(u)=\frac{1}{N}\sum_{i=0}^{N-1} x(i)\, e^{-j2\pi u\frac{i}{N}}$$

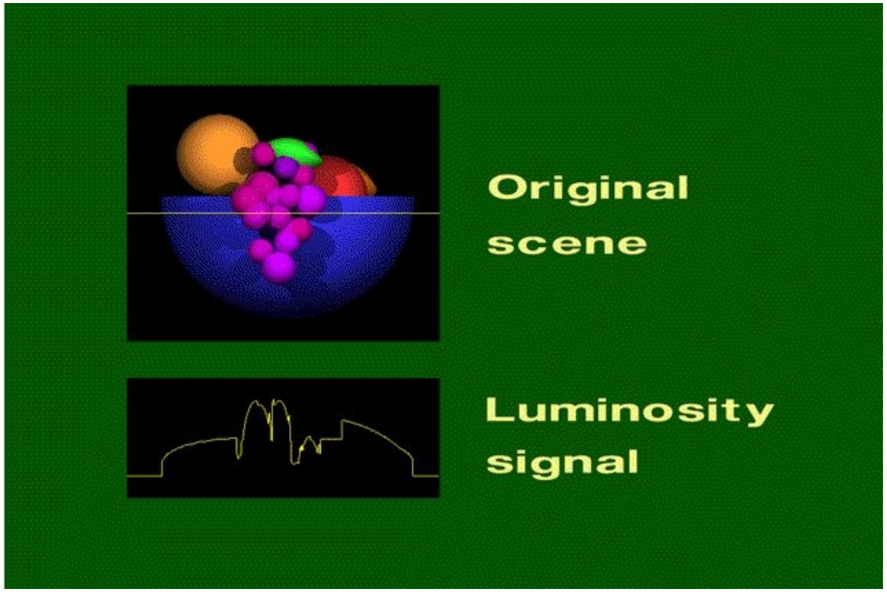
  This allows to compute the DFT of samples at uniform intervals
- In Fourier space, functions can be filtered by the operation of convolution
- Convolution „multiplies“ two functions in Fourier space and allows to filter out too high frequencies
- Filtered functions can be the retransformed back in original space to obtain a „better“ image

Aliasing in the frequency domain

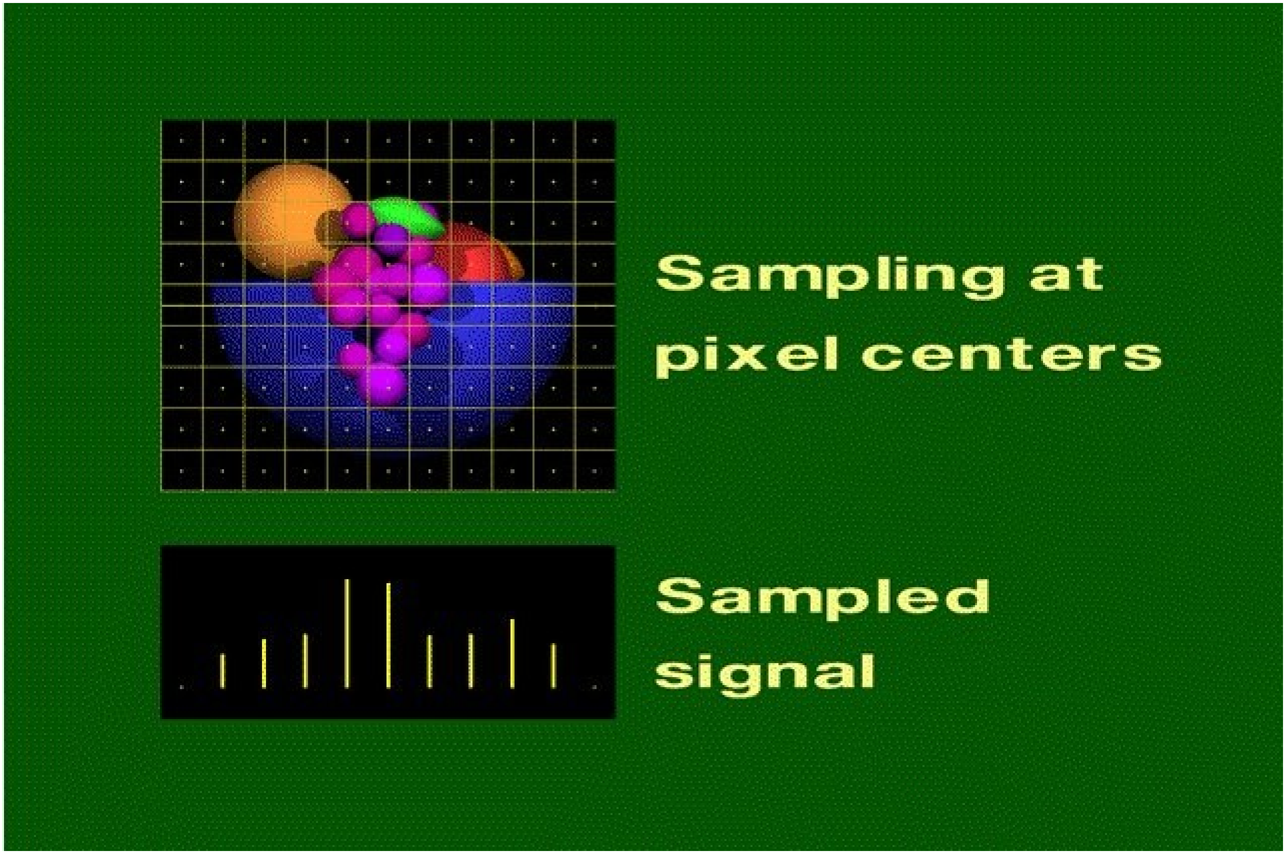


Images as functions

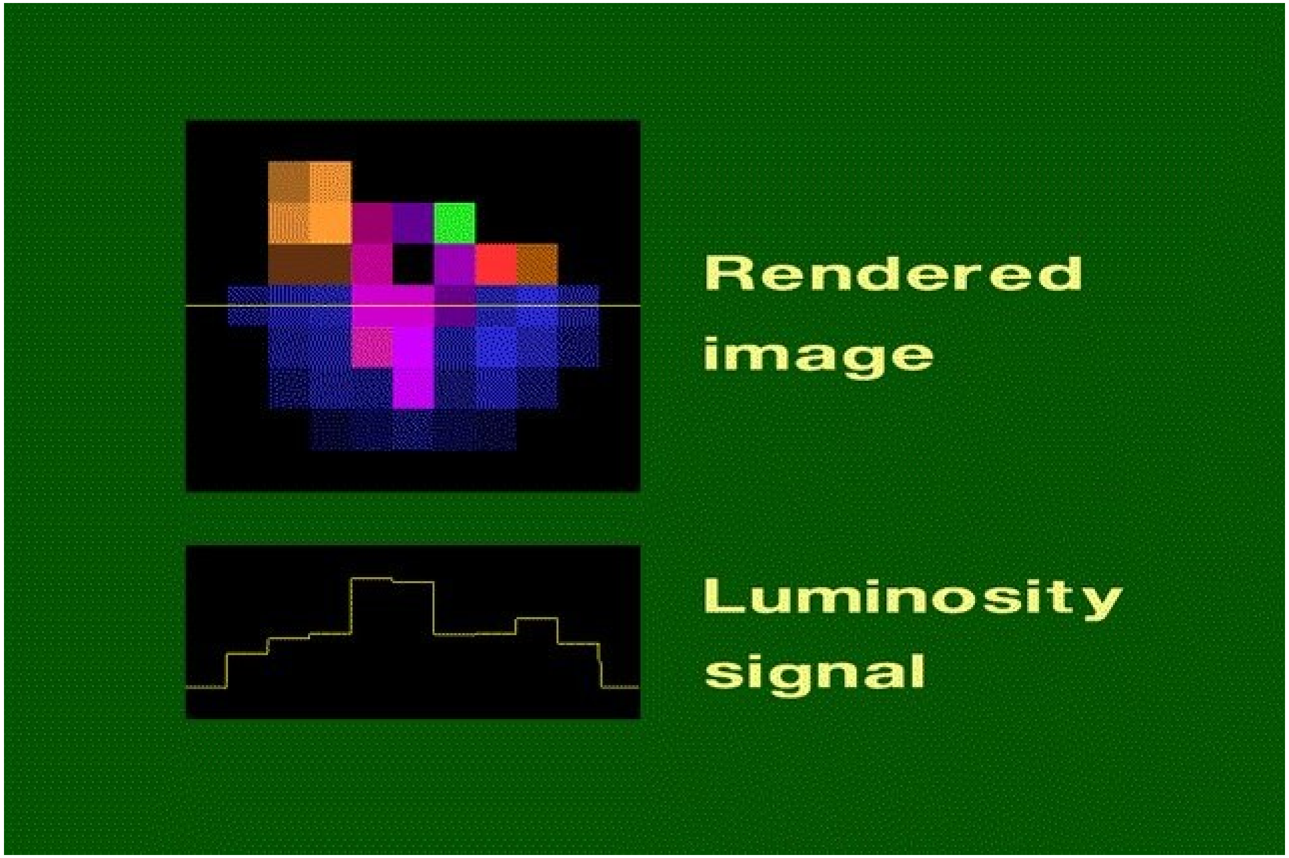
- What does this have to do with graphics?
- An Image can always be seen as a luminosity function F(x,y) of values defined at the pixel centres
- As such it can be seen as the point sampling of a continuous function
- A row of pixels can be therefore seen as a function of the variable x
- Writing pixel values is exactly like sampling the function at the pixel centres



Sampling images



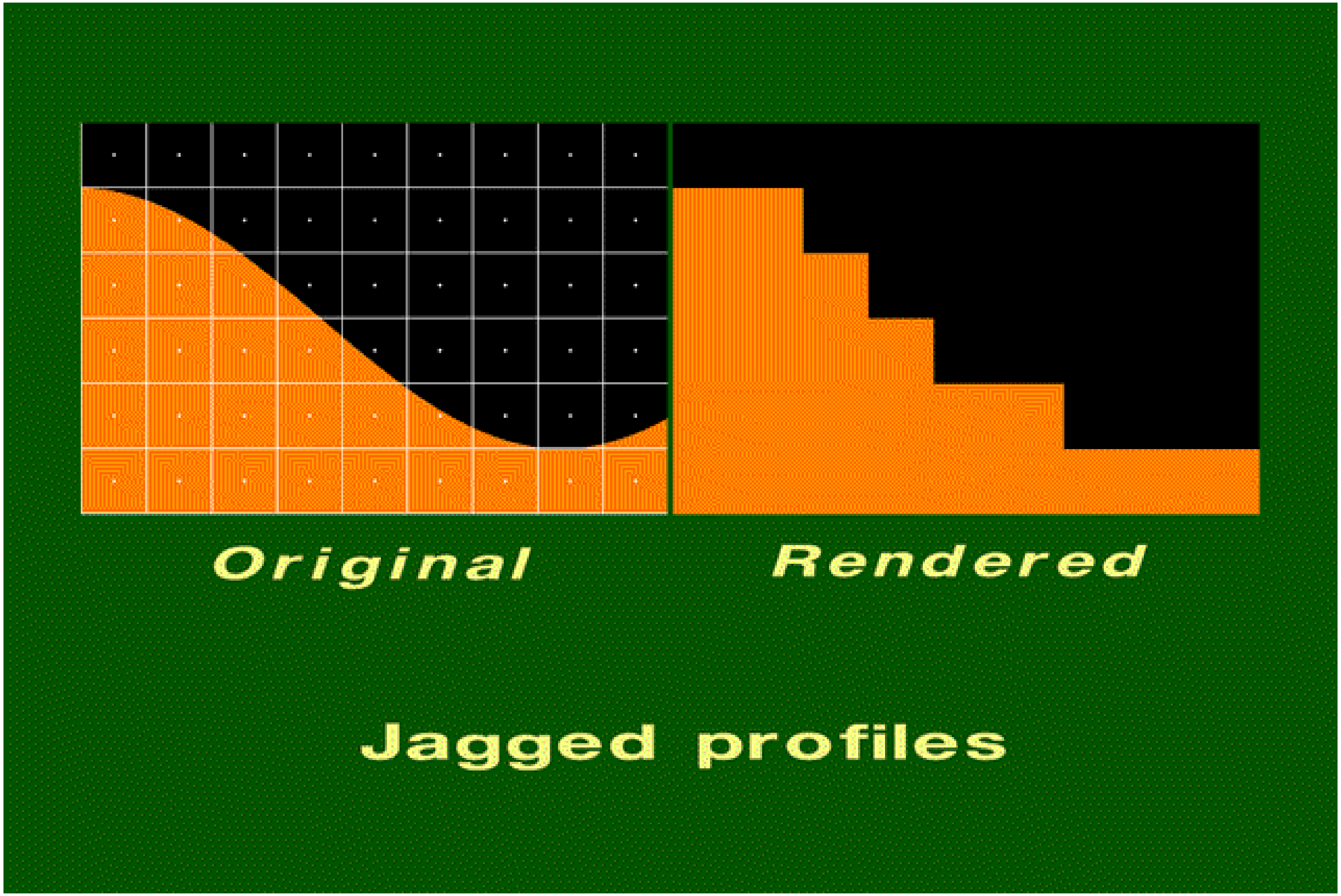
Rendering images



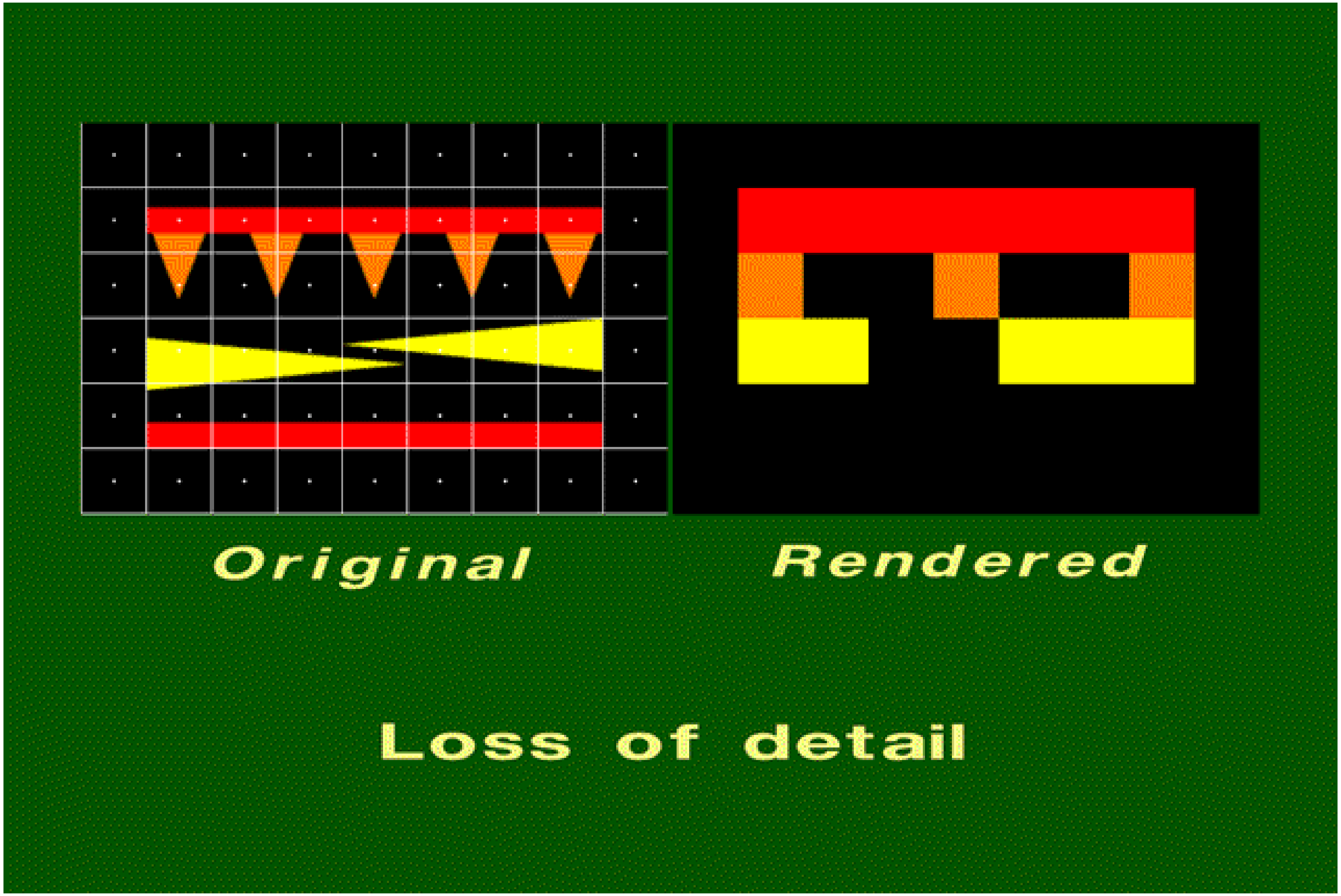
Sampling theory and graphics

- When the pixel distance is higher than the Nyquist limit of the sampled signal frequencies, one becomes jaggies
- Jaggies are high frequencies appearing as low frequencies which produce regular patterns easy to see

Effects of aliasing



Effects of aliasing



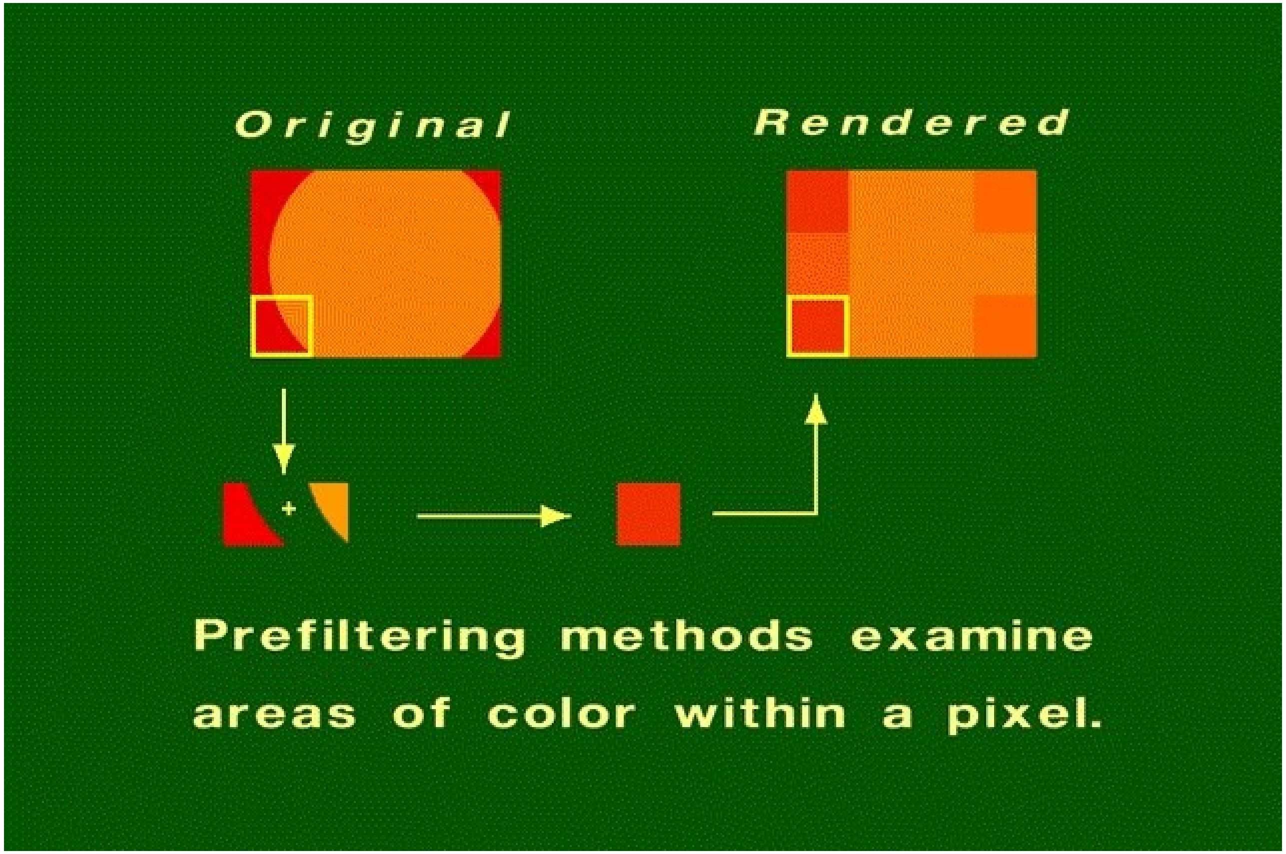
Effects of aliasing

Antialiasing

- Aim of anti-aliasing is to try to avoid the effects of aliasing as much as possible
- There are two main categories of algorithms for doing anti-aliasing
  - prefiltering: treats pixels as an area, and compute pixel color based on the overlap of the scene's objects with a pixel's area.
  - postfiltering: render the scene at higher resolution, and compute the pixel value by (weighted) average of the subpixels (supersampling)

Pre-filtering

- Pixel color is determined by how much percentage of subarea is which colour



Post-filtering

- Pixel color is determined by subsamples:
  - For each pixel, several samples are taken: usually N=4, 9, 16 or 25 subsamples
  - Resulting pixel “subcolors” I_i (i=1,…,N) of the subsamples are then averaged to lead to a pixel color value I

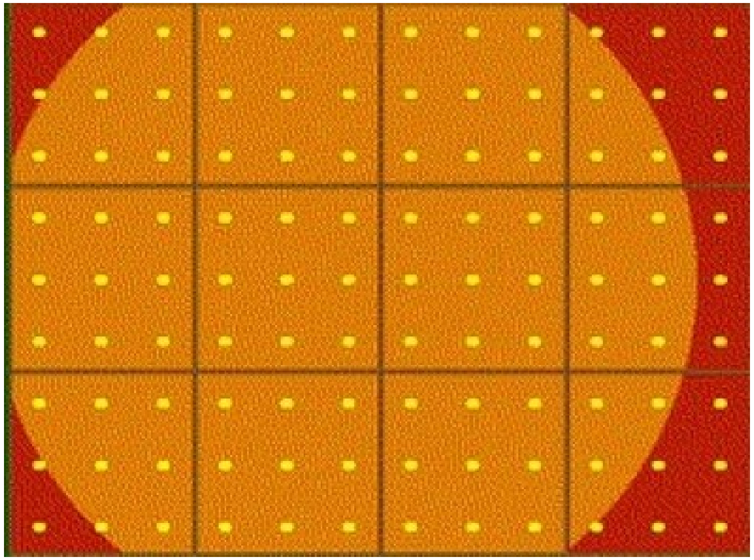
$$I=\Sigma_{i=1,\dots,N} I_i/N$$

  - Sometimes weights w_i are used

$$I=\Sigma_{i=1,\dots,N} w_i I_i/N$$

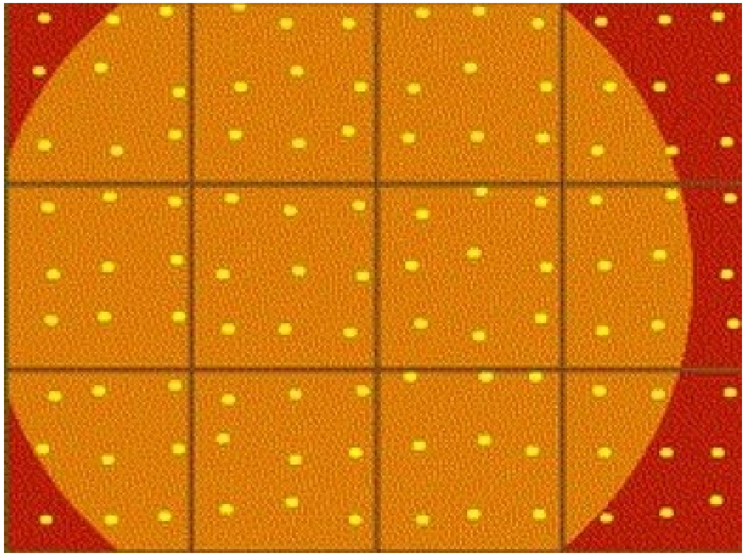
Post-filtering

- There are different ways to determine where to take the subsamples too:
  - Uniform sampling: the samples are taken on a grid (here 9 subsamples)



Post-filtering

- There are different ways to determine where to take the subsamples too:
  - Jittered sampling: the samples are centered on a grid, but random values are added to avoid aliasing



Examples

Examples

Examples

Examples

Comparison


No antialiasing


3x3 supersampling
5x5 weighted filter


3x3 supersampling
3x3 unweighted filter


3x3 jittered supersampling
5x5 weighted filter

Credits

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