Fundamentals of Imaging
Colour Spaces

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Today's lesson

- All about colour spaces….
- All flavours, all species, all types

Alternative colour spaces

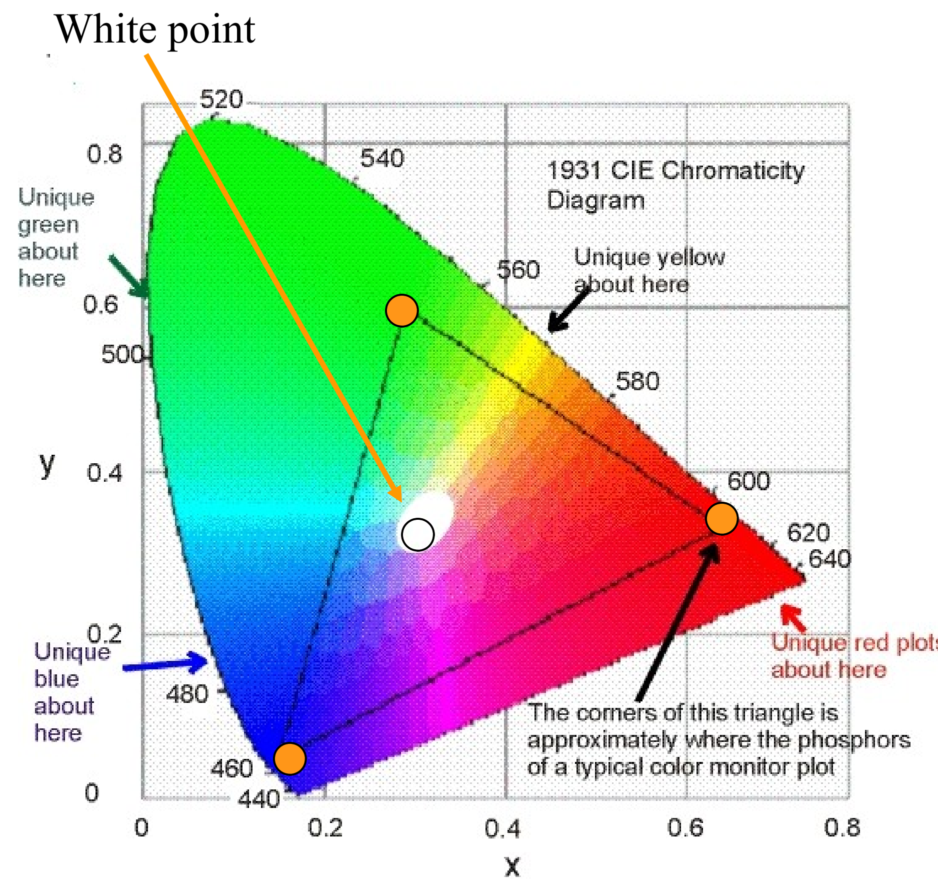
- In general, the XYZ colour space is not suitable for computing colours or encoding colours.
- Many XYZ pairs do not match viewable colours
- In most cases, alternative colour spaces are used
- Reasons for alternate colour spaces:
  - Physical constraints: I/O devices have different characteristics, and one might want to couple these to process colour directly
  - Efficient encoding: some colour spaces were developed for efficient encoding and transmission (NTSC, PAL)
  - Perceptual uniformity: make the colour space so that colour distances are perceptually sound
  - Intuitive usage: RGB is device oriented, and far from being easy to use

Transforming colour spaces

- Typically, the colour spaces are characterized by their bounding volumes in colour space
  - Called gamut of colours the space can represent
  - They usually do not cover all colours: for example, CRT screens can only add the phospor colours.
- *Gamut mapping*: converting one gamut to another
  - For simple, linear, additive trichromatic displays the transformation from XYZ to their colour values can be usually given by a 3x3 matrix
  - This due to Grassman's law
  - Conversion is usually done with
    - a linear transformation to XYZ
    - some non-linear processing to minimize perceptual errors (e.g. gamma correction)

Transforming colour spaces

- To convert, one must know
  - Device primary colours
  - White point of the device
- These are usually described in x,y chromaticity coordinates.
- On the device, the primary colours are usually represented as points between 0 and 1 in 3D space (1,0,0), (0,1,0) (0,0,1).
- Why do I need the white point?
  - Simple: remember the CIE coordinates were projections.
  - So they do not contain luminance!
  - So we have no idea of their mutual strength
  - BUT if one knows the white point on the CIE, one knows the proportions of luminances (and know (1,1,1))
  - Table shows an example: ITU-R BT 709 colour space (HDTV)



	R	G	B	White
x	0.6400	0.3000	0.1500	0.3127
y	0.3300	0.6000	0.0600	0.3290

Transforming colour spaces

- Now, if one on top of this knows the luminance of the white point, one knows it all.
- If we know the xy coordinates of the points RGB we can compute
  $z=1-x-y$
- Thus, we can have the XYZ of RGB: (x_R,y_R,z_R), (x_G,y_G,z_G) , (x_B,y_B,z_B),
- …and of the white: (x_W,y_W,z_W).
- Which gives following equations

$$X_W = x_R S_R + x_G S_G + x_B S_B,$$
$$Y_W = y_R S_R + y_G S_G + y_B S_B,$$
$$Z_W = z_R S_R + z_G S_G + z_B S_B.$$

  where the S_x are scaling factors and unknown.
- This gives: $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_R S_R & x_G S_G & x_B S_B \\ y_R S_R & y_G S_G & y_B S_B \\ z_R S_R & z_G S_G & z_B S_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$

- The problem is that often the luminance of white is not known, but the CIE chromaticity coordinates
- In this case, one adds up the luminances of the single phosphor components (if one has it) to obtain the white light one.

sRGB colour space

- On a typical display, such as CRT, plasma, LED and DLP, the phosphors are RGB.
- Since they may be different, the basic colors are in general different
- Therefore, there is no such thing as an RGB space: they are device dependent!
- You have therefore to convert to a standardized set of primaries.
- The space sRGB is exactly this standardization:
  - It assumes luminance as 80cd/m^2
  - It uses a matrix computation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- …and the following linearization transformation, which minimizes quantization errors in digital applications:

$$R_{\mathrm{sRGB}} = \begin{cases} 1.055\,R^{1/2.4} - 0.055 & R > 0.0031308, \\ 12.92\,R & R \le 0.0031308; \end{cases}$$

$$G_{\mathrm{sRGB}} = \begin{cases} 1.055\,G^{1/2.4} - 0.055 & G > 0.0031308, \\ 12.92\,G & G \le 0.0031308; \end{cases}$$

$$B_{\mathrm{sRGB}} = \begin{cases} 1.055\,B^{1/2.4} - 0.055 & B > 0.0031308, \\ 12.92\,B & B \le 0.0031308. \end{cases}$$

this is called *gamma encoding*

- This is NOT gamma correction! It can be parametrized as:

$$R_{\mathrm{nonlinear}} = \begin{cases} (1+f)\,R^{\gamma} - f & t < R \le 1, \\ s\,R & 0 \le R \le t; \end{cases}$$

$$G_{\mathrm{nonlinear}} = \begin{cases} (1+f)\,G^{\gamma} - f & t < G \le 1, \\ s\,G & 0 \le G \le t; \end{cases}$$

$$B_{\mathrm{nonlinear}} = \begin{cases} (1+f)\,B^{\gamma} - f & t < B \le 1, \\ s\,B & 0 \le B \le t. \end{cases}$$

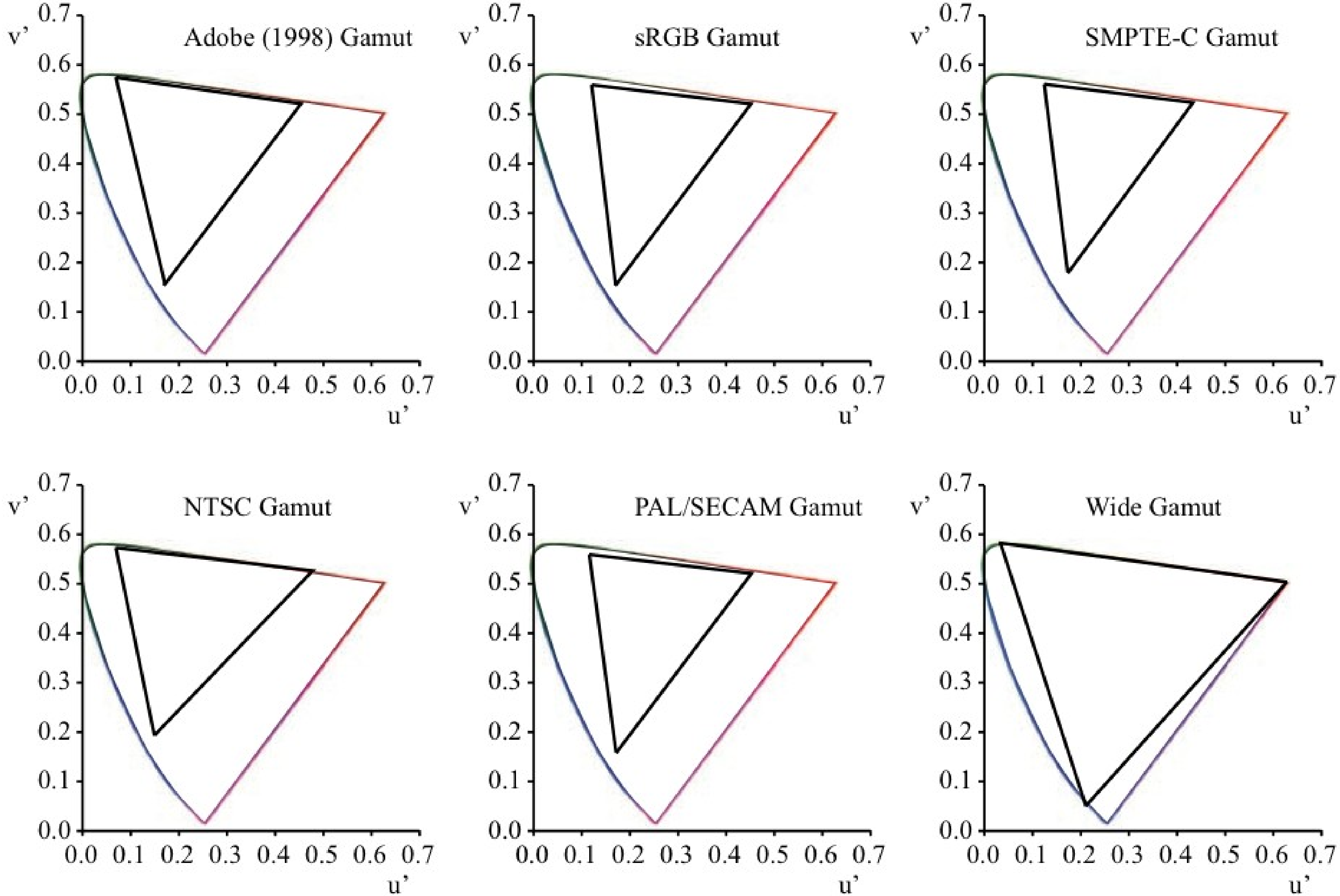
RGB Colour spaces

- In the course of time many standards have been developed, depending on their application:
  - Print
  - Screen
  - Draw
  - Net

Color space	XYZ to RGB matrix	RGB to XYZ matrix	Non-linear transform
sRGB	$\begin{bmatrix} 3.2405 & -1.5371 & -0.4985 \\ -0.9693 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0572 \end{bmatrix}$	$\begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$	$\gamma = 1/2.4 \approx 0.42$ $f = 0.055$ $s = 12.92$ $t = 0.0031308$
Adobe RGB (1998)	$\begin{bmatrix} 2.0414 & -0.5649 & -0.3447 \\ -0.9693 & 1.8760 & 0.0416 \\ 0.0134 & -0.1184 & 1.0154 \end{bmatrix}$	$\begin{bmatrix} 0.5767 & 0.1856 & 0.1882 \\ 0.2974 & 0.6273 & 0.0753 \\ 0.0270 & 0.0707 & 0.9911 \end{bmatrix}$	$\gamma = \frac{1}{2\frac{51}{256}} \approx \frac{1}{2.2}$ $f =$ N.A. $s =$ N.A. $t =$ N.A.
HDTV (HD-CIF)	$\begin{bmatrix} 3.2405 & -1.5371 & -0.4985 \\ -0.9693 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0572 \end{bmatrix}$	$\begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$	$\gamma = 0.45$ $f = 0.099$ $s = 4.5$ $t = 0.018$
NTSC (1953)/ ITU-R BT.601-4	$\begin{bmatrix} 1.9100 & -0.5325 & -0.2882 \\ -0.9847 & 1.9992 & -0.0283 \\ 0.0583 & -0.1184 & 0.8976 \end{bmatrix}$	$\begin{bmatrix} 0.6069 & 0.1735 & 0.2003 \\ 0.2989 & 0.5866 & 0.1145 \\ 0.0000 & 0.0661 & 1.1162 \end{bmatrix}$	$\gamma = 0.45$ $f = 0.099$ $s = 4.5$ $t = 0.018$
PAL/SECAM	$\begin{bmatrix} 3.0629 & -1.3932 & -0.4758 \\ -0.9693 & 1.8760 & 0.0416 \\ 0.0679 & -0.2289 & 1.0694 \end{bmatrix}$	$\begin{bmatrix} 0.4306 & 0.3415 & 0.1783 \\ 0.2220 & 0.7066 & 0.0713 \\ 0.0202 & 0.1296 & 0.9391 \end{bmatrix}$	$\gamma = 0.45$ $f = 0.099$ $s = 4.5$ $t = 0.018$
SMPTE-C	$\begin{bmatrix} 3.5054 & -1.7395 & -0.5440 \\ -1.0691 & 1.9778 & 0.0352 \\ 0.0563 & -0.1970 & 1.0502 \end{bmatrix}$	$\begin{bmatrix} 0.3936 & 0.3652 & 0.1916 \\ 0.2124 & 0.7010 & 0.0865 \\ 0.0187 & 0.1119 & 0.9582 \end{bmatrix}$	$\gamma = 0.45$ $f = 0.099$ $s = 4.5$ $t = 0.018$
Wide Gamut	$\begin{bmatrix} 1.4625 & -0.1845 & -0.2734 \\ -0.5228 & 1.4479 & 0.0681 \\ 0.0346 & -0.0958 & 1.2875 \end{bmatrix}$	$\begin{bmatrix} 0.7164 & 0.1010 & 0.1468 \\ 0.2587 & 0.7247 & 0.0166 \\ 0.0000 & 0.0512 & 0.7740 \end{bmatrix}$	$\gamma =$ N.A. $f =$ N.A. $s =$ N.A. $t =$ N.A.

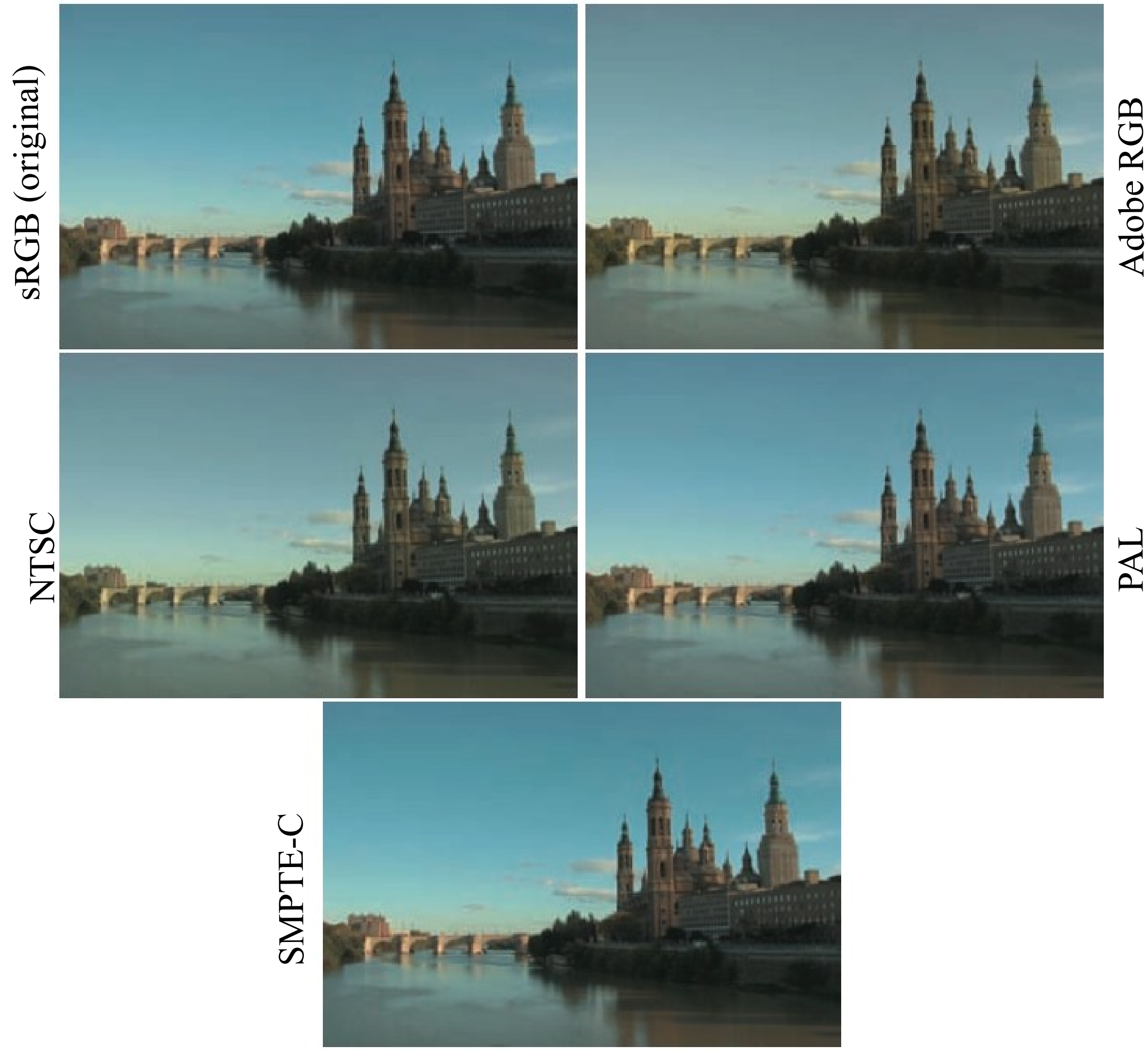
RGB Colour spaces

- Here the corresponding colour space gamuts:



RGB Colour spaces

- Converting modifies colours:



CMY and CMYK

- CMY and CMYK are used for printers:
  - Cyan
  - Magenta
  - Yellow
- These are *subtractive* colours: printers deposit pigments subctracting light.
- Inherently device dependent: ink has pigments
- Usually, K (=black) is added, because C+Y+M is never real black
- Modern printers use many more primary colours, but we focus on 4 colour ones

- Here,

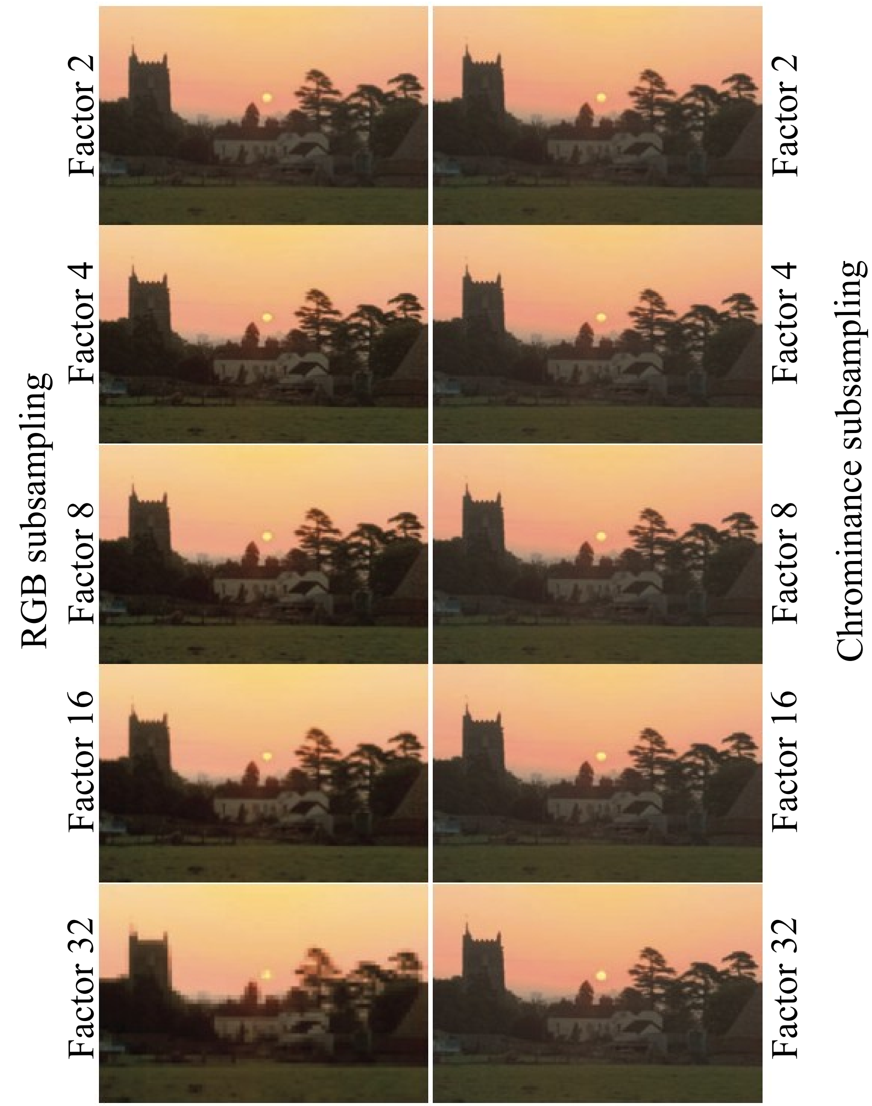
$$C=1-R$$
$$M=1-G$$
$$Y=1-B.$$

- If one has K, then

$$K=Min(C,M,Y)$$
$$C=1-R$$
$$M=1-G$$
$$Y=1-B.$$

Luminance-Chrominance

- RGB is useful especially for hardware devices and transmission efficiency
- For intuition, it is easier to work with
  - one luminance channel
  - Two chrominance channels
- Luminance represents how light or dark (similar to Y channel)
  - Note that this can be used at the same tome for B/W transmission
- Chrominance determine chromatic content
- Similar to visual system:
  - Rods sense luminance
  - Cones sample colour, in two colour-opposing axes:
    - Reddish-greenish
    - Blueish-yellowish
  - Because of this, colour spatial resolution (pixel) is only on the area of 4 cones
  - It is therefore possible to transmit colour at a lower resolution without a major quality decrease



CIE Yxy and YUV

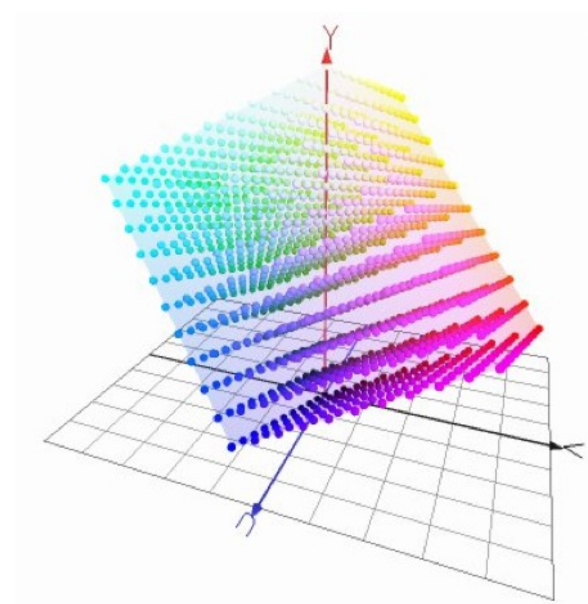
- The CIE Yxy can be seen as a colour space itself
  - Y carries luminance
  - Although xy do not match the eye sensors (color wise), nor do they map perceptual distances well
- As an alternative, the Cie proposed the space YUV:
  - Keep the Y of XYZ and Yxy
  - Map the other two coordinates so they are uniform:

$$u = \frac{2x}{6y - x + 1.5},$$
$$v = \frac{3y}{6y - x + 1.5}.$$

- CIE corrected later to improve uniformity into the CIE Yu'v' space:

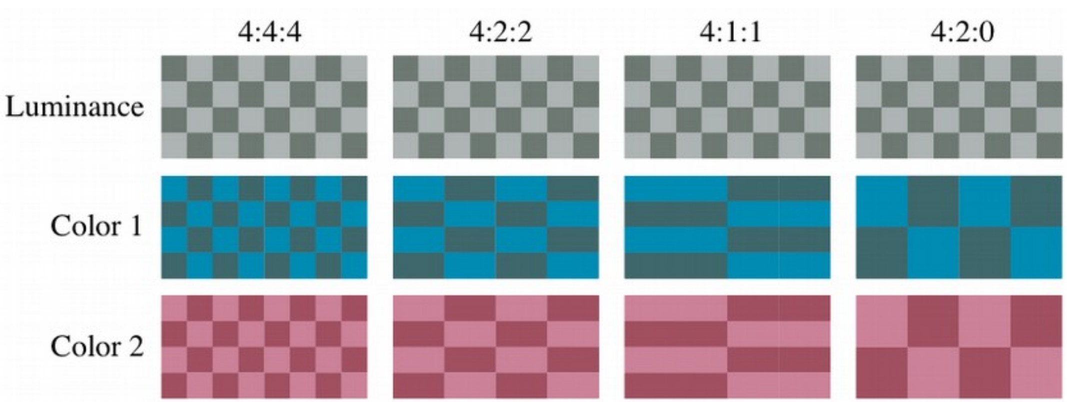
$$u' = \frac{2x}{6y - x + 1.5},$$
$$v' = \frac{4.5y}{6y - x + 1.5}.$$

nice about these spaces: since they are defined from Yxy, they are device independent



Broadcasting

- For broadcasting, luminance-chrominance systems are well suited
- Chrominance can be subsampled, as seen a few slides ago
- Typical subsamplings are done for both chroma components:



- Here, a:b:c means:
  - a: luminance sampling WRT a sampling rate of 3.375 MHz
  - b: chrominance horizontal factor with respect to a
  - c: either the same as b, or 0 when the vertical resolution is subsampled at a factor of 2
    caveat! CONSTANT 2!
- So 4:2:2 means
  - luminance sampled at 13.5 MHz,
  - horizontal chrominance subsampled at factor 2,
  - vertical not subsampled

Broadcast colour spaces: PAL and NTSC

- We saw that broadcasting converts the RGB signal into a composite one for analog broadcasting
- In PAL, we saw before how R'G'B' is obtained from XYZ.
- From this, the luminance Y' is subtracted from R' and B' to obtain the chroma components

$$U' = 0.492111(B' - Y'),$$
$$V' = 0.877283(R' - Y').$$

this leads to the following transforms for the transmission variables Y'U'V':

$$\begin{bmatrix} Y' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147141 & -0.288869 & 0.436010 \\ 0.614975 & -0.514965 & -0.100010 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}$$

- NTSC uses instead the Y'I'Q' coordinates, defined as

$$I' = -0.27(B' - Y') + 0.74(R' - Y'),$$
$$Q' = 0.41(B' - Y') + 0.48(R' - Y').$$

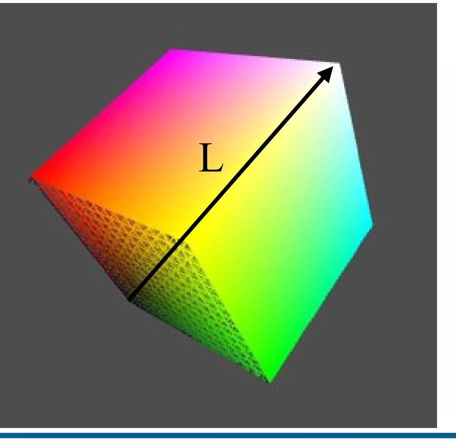
  - I' is orange-blue
  - Q': purple-green
- They lead to the transformation

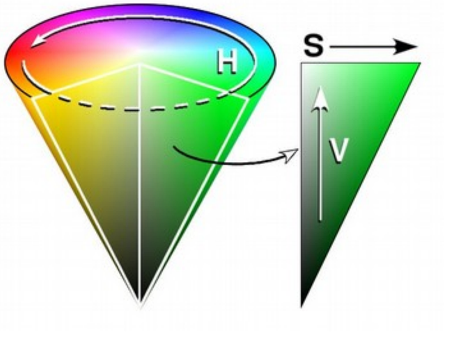
$$\begin{bmatrix} Y' \\ I' \\ Q' \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}$$

- Quite recently, these standards have been replaced by SMPTE-C and the HDTV standards
  - similar transformation matrices
  - not yet stable (see HD+)

HSL space

- Until now we introduced colour spaces defined for the hardware or transmission.
- Their main flaw is usability, since it is difficult to understand how their variables work
- A space that overcomes this is the Hue, Saturation and Lightness space (HSL)
- HSL comes in a few variants:
  - HIS (intensity)
  - HSV (value)
- These spaces are defined on the basis of an abstract RGB space which is supposed to be normalized
- Rotate RGB so that the diagonal forms the lightness axis
- Perpendicular to the lightness axis is a circular plane which defines color
  - Saturation: distance of the point on the circular plane to the lightness axis
  - Hue: angle between predetermined reference colour and colour we are interested in.





HSI and HSV spaces

- For HSV, the minimum and maximum of the RGB triplets are computed: $v_{\min} = \min(R,G,B)$

$$h = \frac{1}{360}\cos^{-1}\left(\frac{\frac{R-G}{2}+(R-B)}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right)$$

$$I = \frac{R+G+B}{3},$$

$$S = 1-\frac{v_{\min}}{I},$$

$$H = \begin{cases} 1-h & \text{if } \frac{B}{I} > \frac{G}{I} \wedge S > 0, \\ h & \text{if } \frac{B}{I} \leq \frac{G}{I} \wedge S > 0, \\ \text{undefined} & \text{if } S = 0. \end{cases}$$

- Notice that H is in angle degrees, and that when R=G=B we have a gray value, and H is undefined.

- For HSI: by using the intermediate variables

$$v_{\min} = \min(R,G,B),$$
$$v_{\max} = \max(R,G,B).$$

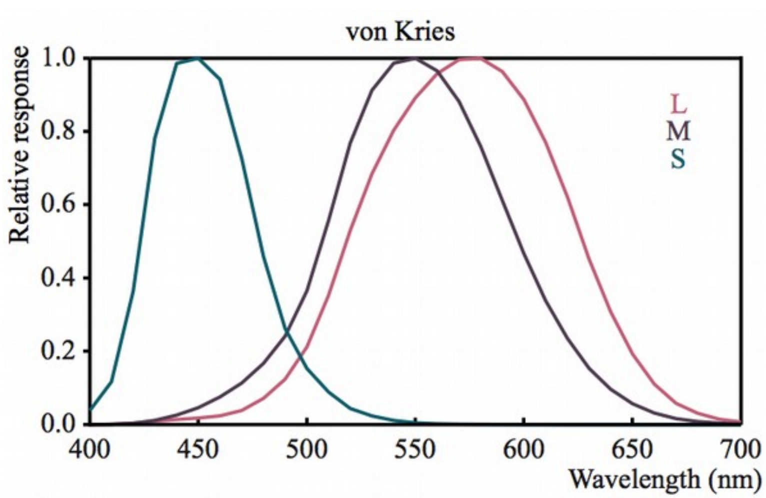
we obtain

$$S = \frac{v_{\max}-v_{\min}}{v_{\max}},$$
$$V = v_{\max}.$$

$$H = \begin{cases} 60\,\frac{G-B}{v_{\max}-v_{\min}} & \text{if } R = v_{\max}, \\ 60\left(2+\frac{B-R}{v_{\max}-v_{\min}}\right) & \text{if } G = v_{\max}, \\ 60\left(4+\frac{R-G}{v_{\max}-v_{\min}}\right) & \text{if } B = v_{\max}. \end{cases}$$

LMS cone excitation space

- Although the CIE XYZ functions are closely related to a linear transform of the LMS cone signals, they are not exact, and an approximation is needed
- A widely used approximation is the Hunt-Pointer-Estevez cone fundamentals, which are used in colour appearance models



- Remember:
  - L=long wavelength,
  - M=middle wavelength
  - S=short wavelength
- Conversion from XYZ to LMS is simply given through the transforms:

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

the inverse of which is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9102 & -1.1121 & 0.2019 \\ 0.3710 & 0.6291 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

Colour opponent spaces

- Characterized through
  - One achromatic variable
  - Two channels representing colour opponency
  - Usually red-green opponency+blue-yellow opponency
- We will examine two of these spaces:
  - CIE L*a*b* (CIELAB)
  - CIE L*u*v* (CIELUV)
  - Both standardized in 1976
- These spaces are interesting because they use a non-linear compression to achieve perceptual uniformity
- This perceptual uniformity is useful because it allows to compute colour differences by simply computing Euclidean distance in the space
- Finally, we will introduce the $L\alpha\beta$ space
- This last space was designed by applying principal component analysis to a set of pictures, encoded first in the LMS space

CIE 1976 L*a*b*

- Input to CIE L*a*b* are
  - (X,Y,Z), the stimulus tristimulus values
  - Tristimulus values of a diffuse white reflecting surface lit by a known illuminant (X_n,Y_n,Z_n)
- Equations calculate
  - the lightness L* of a colour
  - Two opponent chromatic channels, a* and b*

$$\begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \begin{bmatrix} 0 & 116 & 0 & -16 \\ 500 & -500 & 0 & 0 \\ 0 & 200 & -200 & 0 \end{bmatrix} \begin{bmatrix} f(X/X_n) \\ f(Y/Y_n) \\ f(Z/Z_n) \\ 1 \end{bmatrix}$$

where the function f is defined as

$$f(r) = \begin{cases} \sqrt[3]{r} & \text{for } r > 0.008856, \\ 7.787\,r + \dfrac{16}{116} & \text{for } r \le 0.008856. \end{cases}$$

- The inverse is

$$X = X_n \begin{cases} \left(\dfrac{L^*}{116} + \dfrac{a^*}{500} + \dfrac{16}{116}\right)^3 & \text{if } L^* > 7.9996, \\ \dfrac{1}{7.787}\left(\dfrac{L^*}{116} + \dfrac{a^*}{500}\right) & \text{if } L^* \le 7.9996, \end{cases}$$

$$Y = Y_n \begin{cases} \left(\dfrac{L^*}{116} + \dfrac{16}{116}\right)^3 & \text{if } L^* > 7.9996, \\ \dfrac{1}{7.787}\,\dfrac{L^*}{116} & \text{if } L^* \le 7.9996, \end{cases}$$

$$Z = Z_n \begin{cases} \left(\dfrac{L^*}{116} - \dfrac{b^*}{200} + \dfrac{16}{116}\right)^3 & \text{if } L^* > 7.9996, \\ \dfrac{1}{7.787}\left(\dfrac{L^*}{116} - \dfrac{b^*}{200}\right) & \text{if } L^* \le 7.9996. \end{cases}$$

- What is interesting of CIELAB is that it is almost perceptually linear, so one can compute Euclidean distances between colours: E=Empfindung

$$\Delta E^*_{ab} = \left[(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2\right]^{1/2}$$

CIE 1976 L*u*v*

- Input to CIE L*u*v* are like CIELAB:
  - (X,Y,Z), the stimulus tristimulus values
  - Tristimulus values of a diffuse white reflecting surface lit by a known illuminant (X_n,Y_n,Z_n)
- Here

$$L^* = \begin{cases} 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16 & \frac{Y}{Y_n} > 0.008856 \\ 903.3\,\frac{Y}{Y_n} & \frac{Y}{Y_n} \le 0.008856 \end{cases}$$

$$u^* = 13L^*(u' - u'_n),$$

$$v^* = 13L^*(v' - v'_n).$$

where

$$u' = \frac{4X}{X + 15Y + 3Z}, \qquad u'_n = \frac{4X_n}{X_n + 15Y_n + 3Z_n}$$

$$v' = \frac{9Y}{X + 15Y + 3Z}, \qquad v'_n = \frac{9Y_n}{X_n + 15Y_n + 3Z_n}$$

- Also CIELUV is perceptually more or less linear, which means that one can calculate distances

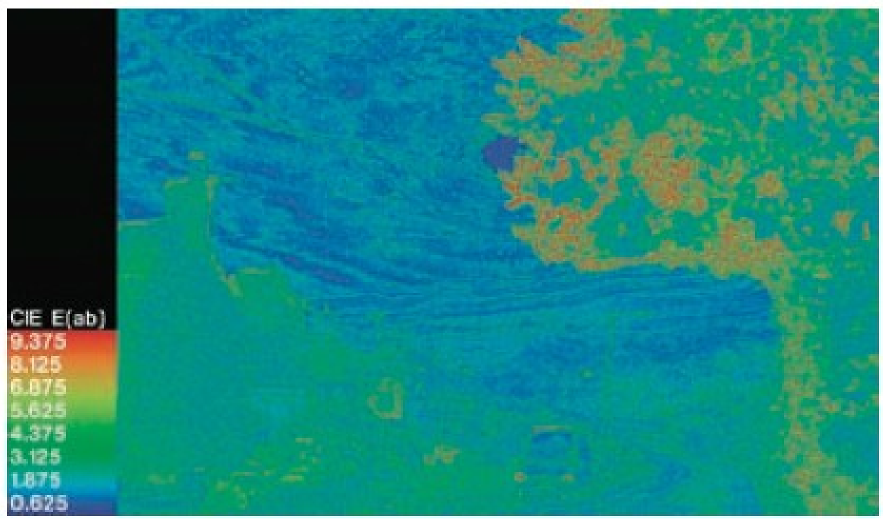
$$\Delta E^*_{uv} = \left[(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2\right]^{1/2}$$

where the Δ represent differences in the respective components

Colour metrics example: E^*_{ab}

Lossless: PPM

Compressed: JPG



Difference E_{ab}: CIELAB works well on small differences

Lαβ space

- For Lαβ, a set of natural images was used
- Then converted into LMS space
- And finally principal component analysis was done: rotating the data so that the first component captures most of the variance
- Then axes are rotated to coincide with first principal component
- Same is done for
  - 2nd principal component
  - 3rd principal component
- What results is a 3x3 transformation matrix
- In the first axis, one has luminance
- Second and third axes are yellow-blue and red-green components

$$\begin{bmatrix} L \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \log L \\ \log M \\ \log S \end{bmatrix}$$

$L^*C^*h_{ab}$ space

- This space can be seen as CIELAB expressed in polar coordinates instead of rectangular
- Conversion from CIELAB is done as follows:

$$L^*_{ab} = L^*,$$

$$C^*_{ab} = \sqrt{a^{*2} + b^{*2}},$$

$$h_{ab} = \tan^{-1}\left(\frac{b^*}{a^*}\right)$$

  - C^* defines chroma
  - H_{ab} defines a hue (angle)
- Space is easier to understand for users
- If Lightness fixed, then colour can be expressed
  - along red-green and yellow-blue axes, or
  - As hue-chroma pair

- Colour difference can be specified by calculating rectangular hue difference:

$$\Delta H^*_{ab} = \left[(\Delta E^*)^2 - (\Delta L^*_{ab})^2 - (\Delta C^*_{ab})^2\right]^{1/2}$$

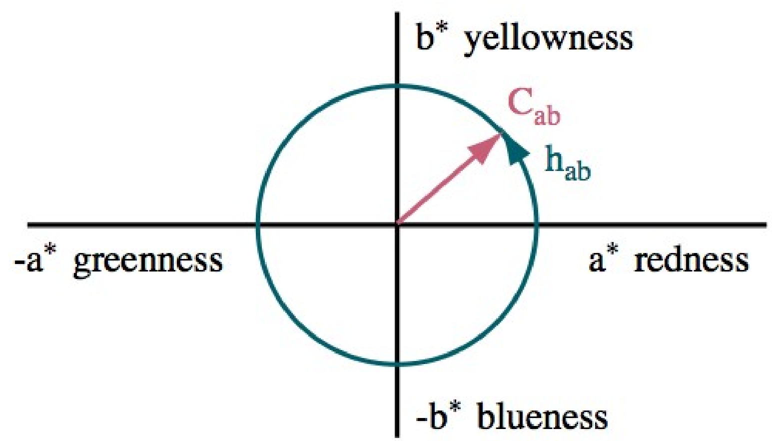
$$\Delta E^*_{ab} = \left[(\Delta L^*)^2 + (\Delta C^*_{ab})^2 + (\Delta H^*_{ab})^2\right]^{1/2}$$

- Inverse to CIELAB:

$$L^* = L^*,$$

$$a^* = C^* \cos(h_{ab})$$

$$b^* = C^* \sin(h_{ab})$$



Colour Difference Metrics: CMC (l:c)

- CIELAB and CIELUV are not as perceptually uniform as needed
- In 1984 the Colour Measurement Committee of the Society of Dyers and Colourists developed the CMC colour difference metric.
- Elaboration of CIELAB color difference, derived from LCH color space
- It uses a finer resolution for desaturated colours, to which we are more sensitive
- Colour difference is given as a ratio l:c, where
  - l scale factor for lightness
  - c scale factor for chroma
- To determine the perceptability of the difference between two colors, one sets l=c=1
  - Sometimes a ratio 2:1 is used
- To compute differences, one computes:

$$f = \sqrt{\frac{(\bar{C}^*)^4}{(\bar{C}^*)^4 + 1900}}$$

$$t = \begin{cases} 0.36 + 0.4\,|\cos(35 + \bar{H})| & \text{if } \bar{H} \leq 164^\circ \vee \bar{H} > 345^\circ \\ 0.56 + 0.2\,|\cos(168 + \bar{H})| & \text{if } 164^\circ < \bar{H} \leq 345^\circ. \end{cases}$$

- Following values define an ellypsoid in LCH colour space

$$S_L = \begin{cases} \dfrac{0.040975\,L^*}{1 + 0.01765\,\bar{L}^*} & \text{for } \bar{L}^* > 16, \\ 0.511 & \text{for } \bar{L}^* \leq 16, \end{cases}$$

$$S_C = \frac{0.0638\,\bar{C}^*}{1 + 0.0131\,\bar{C}^*} + 0.638,$$

$$S_H = S_C\,(ft + 1 - f).$$

where $\bar{L}^* = 0.5(L_1^* + L_2^*)$
and similarly C and H are averages.

- Finally def. the colour difference ΔE_{CMC}, which measures perceived difference:

$$\Delta L_{\text{CMC}} = \frac{L_1^* - L_2^*}{l\,S_L},$$

$$\Delta C_{\text{CMC}} = \frac{C_1^* - C_2^*}{c\,S_C},$$

$$\Delta H_{\text{CMC}} = \frac{H_1^* - H_2^*}{S_H},$$

$$\Delta E_{\text{CMC}} = \sqrt{\Delta L_{\text{CMC}}^2 + \Delta C_{\text{CMC}}^2 + \Delta H_{\text{CMC}}^2}$$

Colour Difference Metrics: CIE 1994

- CIE 1994 is improvement of CIE L*a*b* colour difference
- It specifies a set of experimental conditions under which the formula is valid
  - D65 illumination of color patches set side by side
  - Each patch covers at least 4º
  - Illuminance at 1000 lux.
- Also derives from CIE LCH, and is parametrized by weights k_L,k_C and k_H usually set to 1.

- Computes first

$$S_L = 1,$$
$$S_C = 1 + 0.045\sqrt{C_1^* C_2^*}$$
$$S_H = 1 + 0.015\sqrt{C_1^* C_2^*}$$

- Then

$$\Delta L_{94} = \frac{L_1^* - L_2^*}{k_L S_L},$$
$$\Delta C_{94} = \frac{C_1^* - C_2^*}{k_C S_C},$$
$$\Delta H_{94} = \frac{H_1^* - H_2^*}{k_H S_H}.$$

from which the metric ΔE^*_{94}

$$\Delta E^*_{94} = \sqrt{\Delta L_{94}^2 + \Delta C_{94}^2 + \Delta H_{94}^2}.$$

- It works well, except for saturated blue and near neutral colors

Colour Difference Metrics: CIEDE2000

- Additional improvement for where CIE94 did not work well
- Derived from CIELAB
- Given two colors, one computes for each

$$C^*_{\text{ab}} = \sqrt{(a^*)^2 + (b^*)^2}$$

and the average of the two $\bar{C}^*_{ab}$

- Then

$$g = 0.5 \left(1 - \sqrt{\frac{\left(\bar{C}^*_{\text{ab}}\right)^7}{\left(\bar{C}^*_{\text{ab}}\right)^7 + 25^7}}\right)$$

- From this value, one computes for each color

$$L' = L^*,$$
$$a' = (1+g)\,a^*,$$
$$b' = b^*,$$
$$C' = \sqrt{(a')^2 + (b')^2},$$
$$h' = \frac{180}{\pi} \tan^{-1}\left(\frac{b'}{a'}\right)$$

- One computes additional intermediate values:

$$R_C = 2\sqrt{\frac{(\bar{C}')^7}{(\bar{C}')^7 + 25^7}},$$
$$R_T = -R_C \sin\left(60 \exp\left(-\left(\frac{\bar{h}' - 275}{25}\right)^2\right)\right),$$
$$T = 1 - 0.17 \cos\left(\bar{h}' - 30\right) + 0.24 \cos\left(2\bar{h}'\right) + 0.32 \cos\left(3\bar{h}' + 6\right) - 0.20 \cos\left(4\bar{h}' - 63\right)$$

from which

$$S_L = 1 + \frac{0.015\left(\bar{L}' - 50\right)^2}{\sqrt{20 + \left(\bar{L}' - 50\right)^2}}$$
$$S_C = 1 + 0.045\,\bar{C}',$$
$$S_H = 1 + 0.015\,\bar{C}'\,T.$$

Colour Difference Metrics: CIEDE2000

- Now we can finally compute the colour difference metric:

$$\Delta L_{\mathrm{CIE00}} = \frac{L_1^* - L_2^*}{k_L\, S_L}$$

$$\Delta C_{\mathrm{CIE00}} = \frac{C_1^* - C_2^*}{k_C\, S_C}$$

$$\Delta H_{\mathrm{CIE00}} = \frac{2 \sin\left(\frac{h_1' - h_2'}{2}\right) \sqrt{C_1' C_2'}}{k_H\, S_H},$$

$$\Delta E_{\mathrm{CIE00}} = \sqrt{\Delta L_{\mathrm{CIE00}}^2 + \Delta C_{\mathrm{CIE00}}^2 + \Delta H_{\mathrm{CIE00}}^2 + R_T\, \Delta C_{\mathrm{CIE00}}\, \Delta H_{\mathrm{CIE00}}}$$

Colour Difference Metrics: comparison

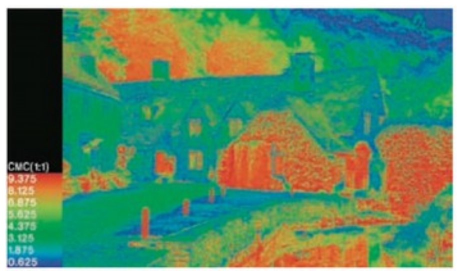
original

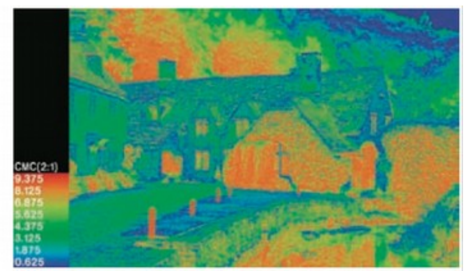
sharpened

CIE e(ab)

CIE E(uv)

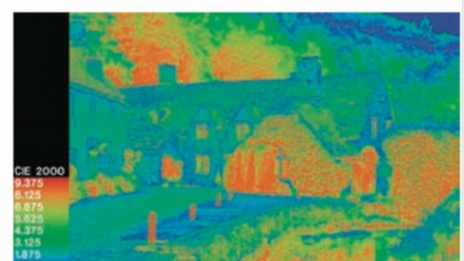
CMC(1,1)





CMC(2,1)

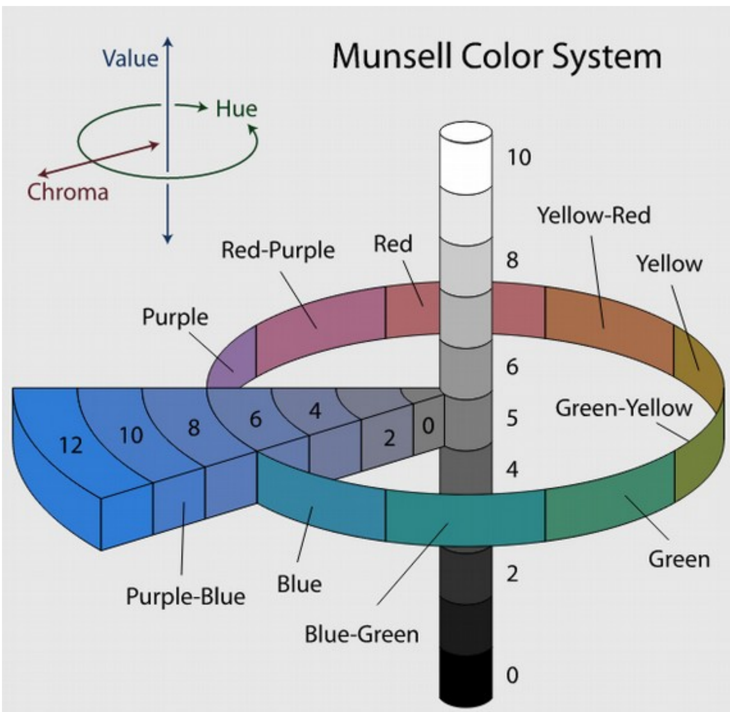
CIE 94



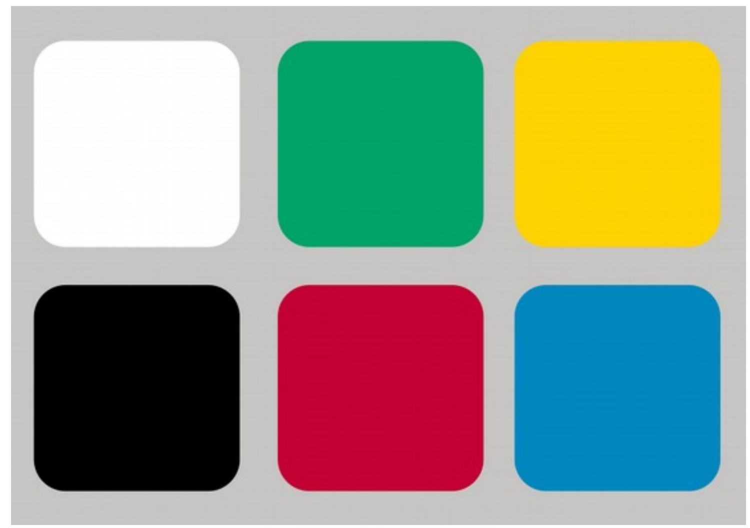
CIE 2000

Colour Order Systems

- These are a conceptual system of organized colour perception
- Munsell color system:



- Natural color system:
  - Blackness (darkness)
  - Chromaticity (saturation)
  - % of red, yellow, green, blue



- OSA colour system

Applications

- This knowledge can be used for:
  - Color matching and transfer between images
  - Principal component analysis to match colour spaces between images
  - Converting colour images to gray images
  - Rendering into complex colour spaces (not RGB)
  - Simulating painting methods
  - In image understanding, classifying edges:
    - Shadow edges
    - Reflectance edges
  - Understanding illuminance features

Thank you!

- Thank you for your attention!
- Web pages
  http://www.uni-weimar.de/medien/cg