5 Linear Dynamics

Linear dynamic response of slabs by excitation with harmonic and sweep loads

Keywords:

eigenvalues, mode shapes, classical modal analysis, harmonic forces, sweep forces, time series, Fast Fourier Transformation (FFT)

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1 Contents

In this lesson you will learn something about the numerical simulation with the method of modal analysis. The lesson deals with the dynamic behavior of slabs by excitation with harmonic and sweep forces. You can investigate the influence of various parameters on this behavior, e.g. geometrical and material parameters, modal damping ratios, the places of excitation and the place of response. To simulate and to learn something about these behavior is important for dynamic investigations with non-destructive tests, in the earthquake engineering and much more fields.

2 Some Basics

❑ **Equation of motion**

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = f(t)$$

k – Stiffness matrix

m – Mass matrix

c – Damping matrix

f – Load vector

❑ **Matrix eigenvalue problem**

$$\left(k - \omega_n^2 \cdot m\right) \cdot \Phi_n = 0$$

${\omega_n}^2$ – eigenvalues

ω_n – natural vibration frequencies

Φ – modal matrix

Φ_n – natural mode of vibration, eigenvector

- **Characteristic equation**

$$\det\left(k - \omega_n^2 \cdot m\right) = 0$$

- **Equation of a harmonic force**

$$f = f_0 \cdot \sin(\omega \cdot t)$$

- **Equation of a sweep force**

$$f = f_0 \cdot \sin\left(\omega(t) \cdot t\right)$$

- **Classical modal analysis, classical mode superposition methods**

✔ ***Displacements x in terms of modal contributions***

$$u(t) = \sum_{r=1}^{n} \Phi_r \cdot q_r(t) = \Phi \cdot q(t)$$

✔ ***Orthogonality of natural modes***

$$M = \Phi^T \cdot m \cdot \Phi$$

for classical damping: $C = \Phi^T \cdot c \cdot \Phi$

$$K = \Phi^T \cdot k \cdot \Phi$$

$$F = \Phi^T \cdot f$$

$$M \cdot \ddot{q} + C \cdot \dot{q} + K \cdot q = F(t)$$

K – diagonal matrix of generalized modal masses
M – diagonal matrix of generalized modal stiffnesses
C – classical damping matrix

✔ ***N-uncoupled equations, one for each natural mode (response of a SDOF-system)***

$$M_n \cdot \ddot{q}_n + C_n \cdot \dot{q}_n + K_n \cdot q_n = F_n(t)$$

$$\ddot{q}_n + 2 \cdot \xi_n \cdot \omega_n \cdot \dot{q}_n + \omega_n^2 \cdot q_n = \frac{F_n(t)}{M_n}$$

✔ ***contribution of n^{th} mode to the displacement x(t)***

$$u_r(t) = \Phi_r \cdot q_r(t)$$

✔ ***combining the modal contribution to the total displacements***

$$u(t) = \sum_{r=1}^{n} u_r(t) = \sum_{r=1}^{n} \Phi_r \cdot q_r(t)$$

3 Structure and purpose of the files

dyn-main.s	main routine which includes all subroutines and the classical modal analysis is performed
dyn-struc.s	includes the geometry, support conditions, material and physical data of the FE-model (Fig. 1)
dyn-sinforce.s	creates a harmonic force in the chosen frequency
dyn-sweepforce.s	creates a sweep force in a chosen range from a lower to an upper frequency, and the time of increasing of the frequency
dyn-eigval.s	determines the first 12 eigenvalues of the structure and shows them on screen You should study the mode shapes to understand the reaction of the slab under the loads
dyn-timeseries.s	shows the actual plot of the time history of the response point and of the load 1. displacements 2. accelerations 3. force 4. superposition of displacements and load 5. superposition of accelerations and load 6. superposition of accelerations and displacements
dyn-fourier.s	makes a Fast Fourier Transformation (FFT) of time histories of the response point and the force, the result is a spectra with effective values

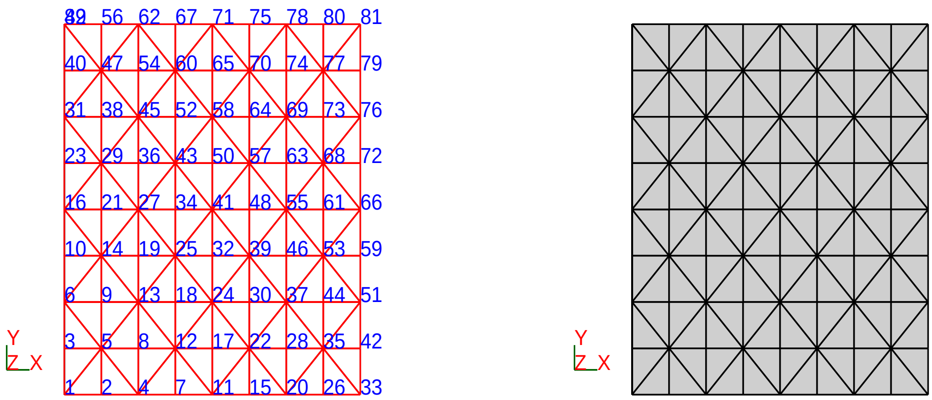


Fig. 1 FE-model (with node numbers)

4 Work

If you find some commands in this kind,

*?????????????????/

....

*?????????????????/

so it is a place where you have to **work**. You have to create some important commands or you find parameters, which can be changed.

❑ To create some important command:

Together we create some important commands of our example. You have a control message at these special commands.

In your papers you can find the command description from SLang Command Documentation.

Subject	**Command**	**Where????**
Eigenvalue analysis	compact iteigengesy	dyn-eigval.s
Classical modal analysis	dynres classical	dyn-main.s
Fast Fourier Transformation (FFT)	timeseries perigr	dyn-fourier.s

❑ Variable parameters:

These are the parameters of our exercise, which you can change

✔ **for the slab:**

Parameter	**Where????**
size of the slab in all directions	dyn-struc.s
material parameters: mass density, Young's Modulus, Poisson ratio	dyn-struc.s
damping vector	dyn-main.s
support condition of the slab	dyn-struc.s

✔ **for the load and the response of the structure:**

Parameter	**Where????**
frequency of sinus force (in an interactive modus, only for the case sinusforce)	dyn-sinusforce.s
point of excitation (in an interactive modus)	dyn-main.s
point of response, of which the results are plotted (in an interactive modus)	dyn-main.s

5 Some useful figures

If you change parameters of the slab (e.g. support conditions), so you can create various states of the slab. With this figures of analytical solutions of plates you can compare analytical and numerical results of the eigenvalues.

Auflager-bedingungen	B_1	B_2	B_3	B_4	B_5	B_6	B_7
	11.84	24.61	40.41	46.14	103.12		
	6.09	10.53	14.19	23.80	40.88	44.68	61.38
	4.35	24.26	70.39	138.85			
	5.90						
	1.01	2.47	6.20	7.94	9.01		
	10.40	21.21	31.29	38.04	38.22	47.73	
	2.01	6.96	7.74	13.89	18.25		
	6.83	14.94	16.95	24.89	28.59	32.71	
	8.37	15.82	20.03	27.34	29.54	37.31	
	5.70	14.26	22.82	28.52	37.08	48.49	
	4.07	5.94	6.91	10.38	17.80	18.85	

Auflagerbedingungen	
	$\varphi_{1,1} = 1.57\,(1+\gamma^2)$ $\varphi_{2,1} = 6.28\,(1+0.25\gamma^2)$ $\varphi_{1,2} = 1.57\,(1+4\gamma^2)$
	$\varphi_{1,1} = 1.57\sqrt{1+2.5\gamma^2+5.14\gamma^4}$ $\varphi_{2,1} = 6.28\sqrt{1+0.625\gamma^2+0.321\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{1+9.32\gamma^2+39.06\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{5.14+2.92\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 9.82\sqrt{1+0.266\gamma^2+0.0625\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{5.14+10.86\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{1+2.33\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 6.28\sqrt{1+0.582\gamma^2+0.152\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{1+8.69\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{2.44+2.72\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 7.95\sqrt{1+0.395\gamma^2+0.095\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{2.44+10.12\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{5.14+3.13\gamma^2+5.14\gamma^4}$ $\varphi_{2,1} = 9.82\sqrt{1+0.298\gamma^2+0.132\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{5.14+11.65\gamma^2+39.06\gamma^4}$

E Young's Modulus (N/m^2)
t thickness of slab (m)
ρ mass density (kg/m^3)
a diameter / length of edges (m)
ν Possion ratio

$\varphi_1=\varphi_{1,1}$
$\varphi_2=\text{Min}\,(\varphi_{1,2};\ \varphi_{2,1})$
$\gamma=a/b$
μ= mass (kg/m^2)

$$\omega_n = B_n \sqrt{\frac{E\cdot t^2}{\rho\cdot a^4\left(1-\nu^2\right)}}$$

$$\omega_n = 2\pi\cdot\frac{\varphi_i}{a^2}\sqrt{\frac{E\cdot d^3}{12\cdot\left(1-\nu^2\right)\cdot\mu}}$$

clamped

simply supported

free

Fig. 2 Natural Frequencies of circle and square plates (left) and natural frequencies of rectangular plates [3]

During the computation you can very good observe the phase angle between load and reaction. Here for your memory the curve for phase angle and frequency ratio.

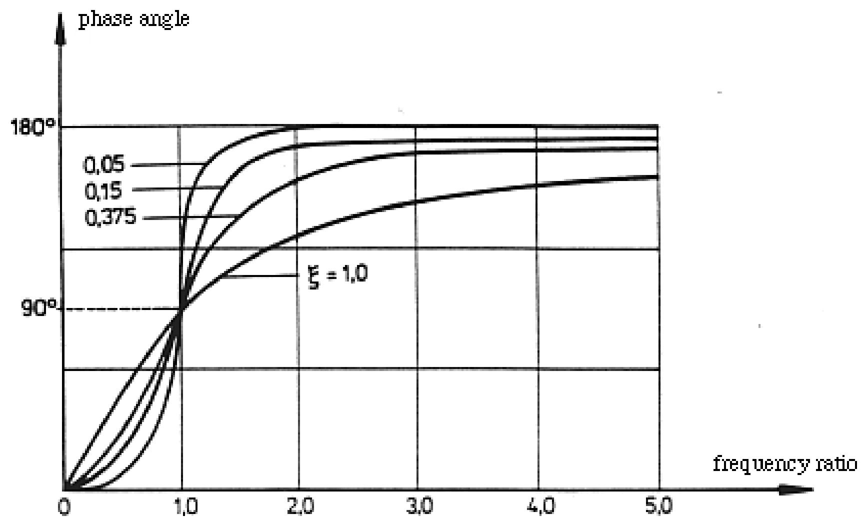


Fig. 3 Phase angle at stationary vibration

6 Results of the example

From the state of files, which you start at first, you have here the plots of the mode shapes. These help to during the modal analysis to understand what's going on.

Eigenvalues and mode shapes

see Fig. 5

Excitation by harmonic force

You can see the created harmonic force (Fig.4). We have always 1000 time steps, the same number of periods and only different time steps. So, the time step decides about the frequency of this vector.

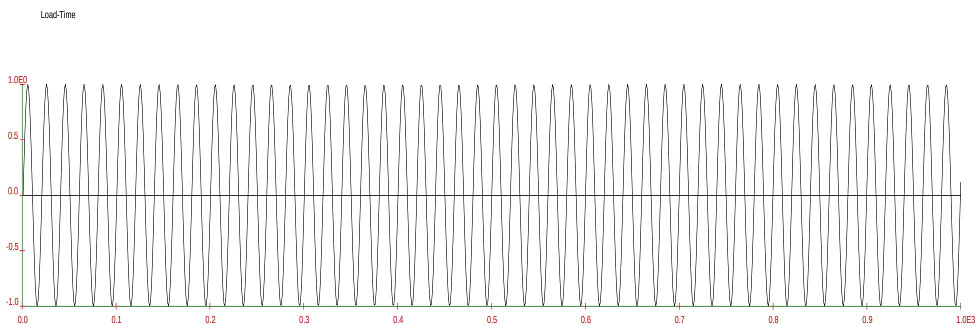
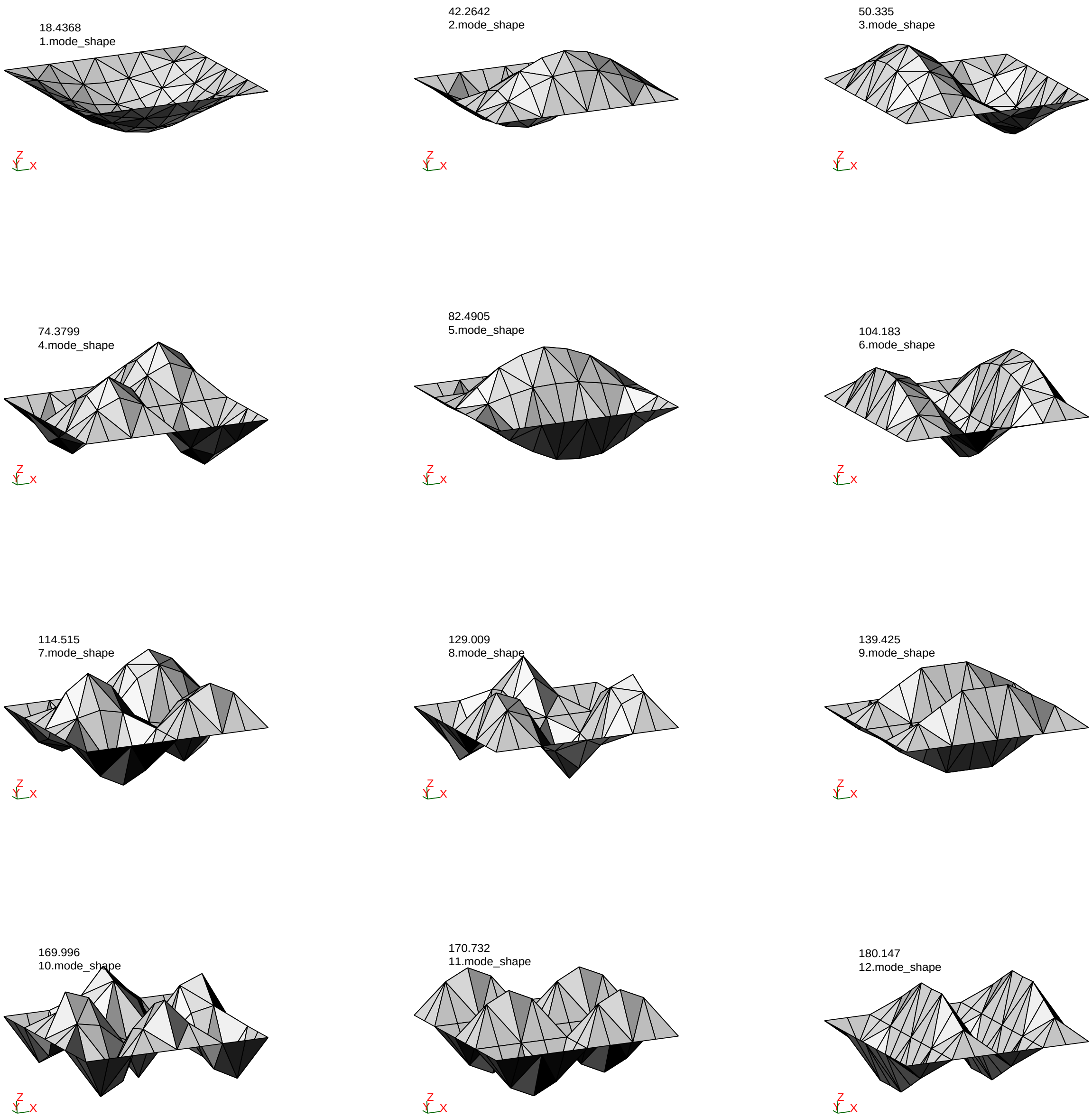


Fig. 4 Harmonic force (1000 time steps)

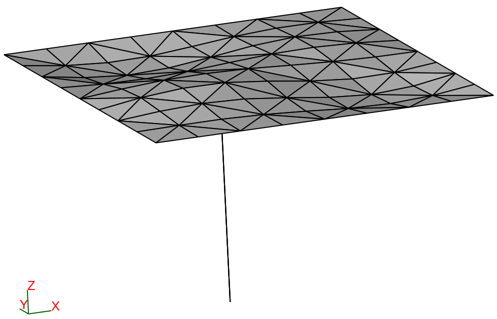


Young's Modulus E	(N/m^2)	3.0e10
thickness of slab t	(m)	0.2
mass density ρ	(kg/m^3)	2500
Possion ratio ν		0.2
modal damping		2%
support conditions		all edges are simply supported

Fig. 5 Mode shapes and frequencies (Hz)

In the next figures you can see two states of the structures in resonance. Observe the phase angle between excitation and response. The example has these parameters:

Excitation point: node number 8, response point node: number 69, excitation frequency 74.4 Hz (4. mode shape), modal damping 2%



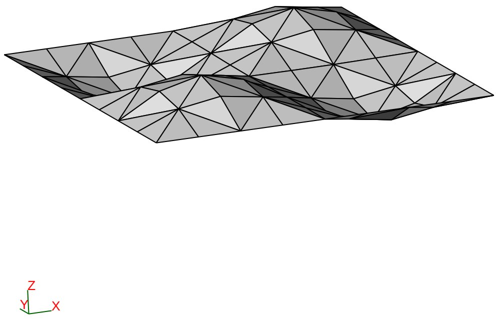


Fig. 6 Two states of the structure
left: load has a maximum, structure nearly undeformed
right: load is zero, structure maximally deformed

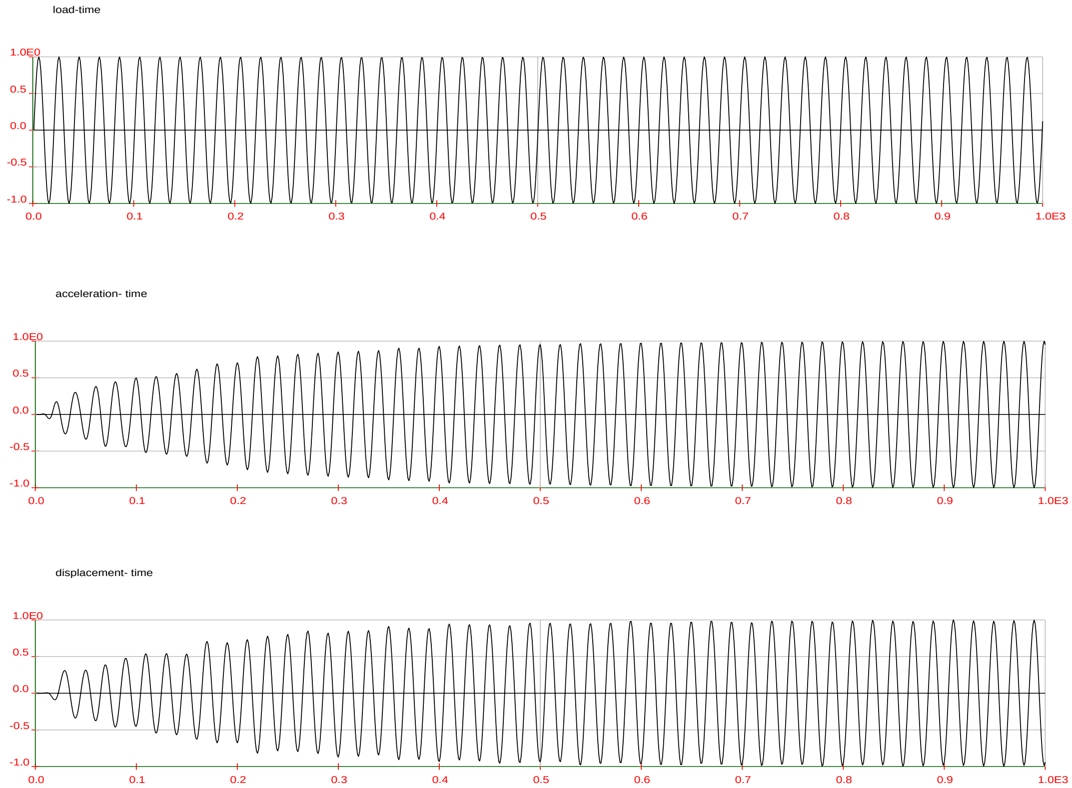


Fig. 7 Time histories of loads, accelerations and displacements (see Fig. 6)

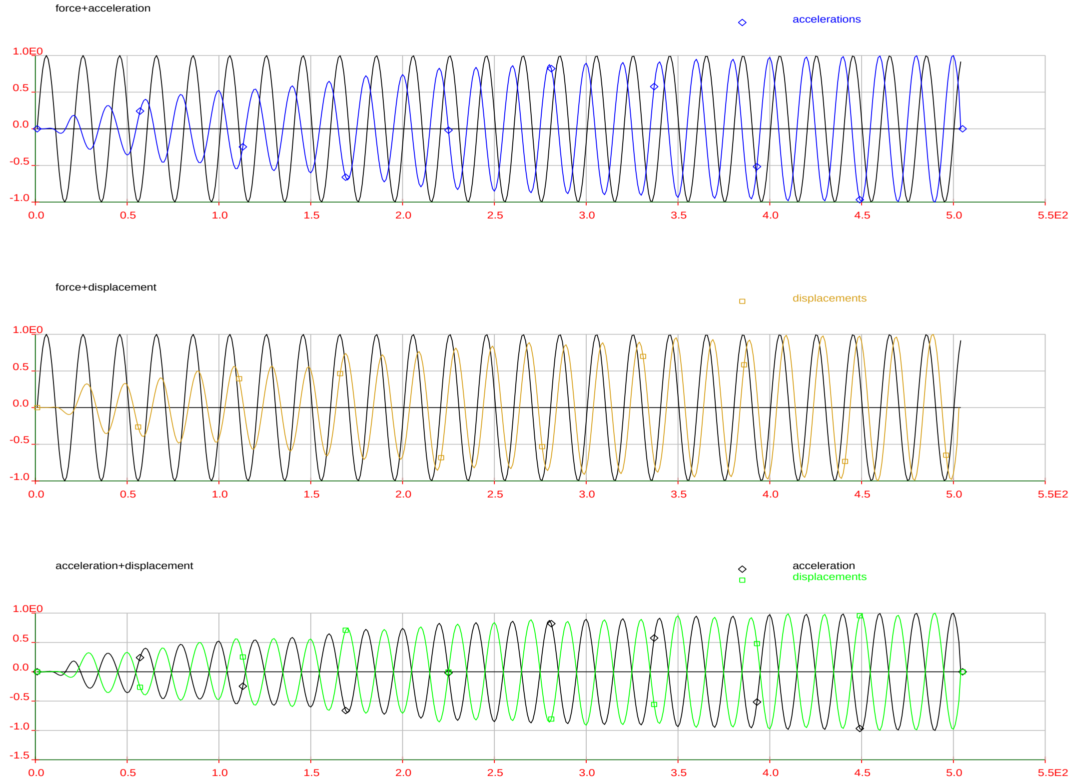


Fig. 8 Time history superposition of force - accelerations, force - displacements and accelerations and displacements (see Fig. 6)

Excitation by sweep force

This is a sweep force. The frequency of the force increases linear.

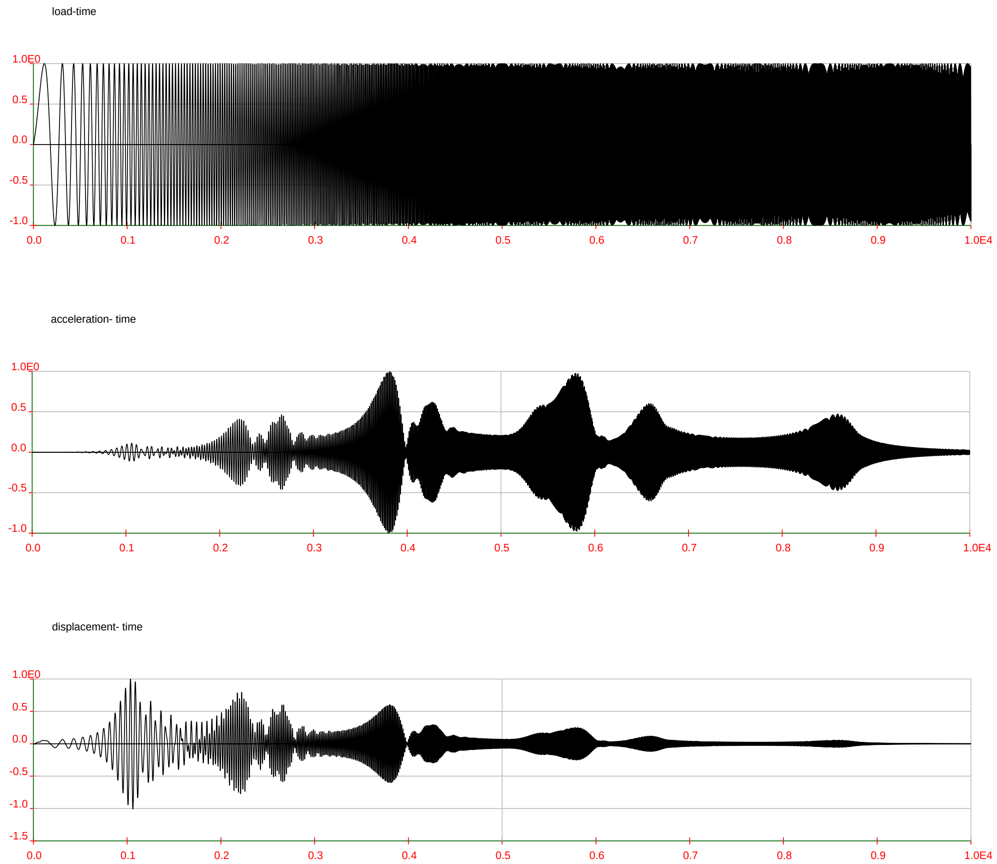


Fig. 9 Time histories of load (sweep force), accelerations and displacements

References

[1] *SLang- the Structural Language Version 3.4 (1998), User's manual*, Bauhaus-University Weimar

[2] *Chopra, A. K.* : Dynamics of structures: theory and applications to earthquake engineering, Englewood Cliffs, New Jersey: Prentice Hall, 1995

[3] Flesch, R.: Baudynamik praxisgerecht. Band 1: Berechnungsgrundlagen. Wiesbaden; Berlin: Bauverlag GmbH, 1993