

2. Seminar

2 LINEARE FINITE-ELEMENTE-ANALYSE

2.1 GLEICHGEWICHTSBEDINGUNG DER ELEMENTE-GRUPPIERUNG

2.2 LÖSUNG DER GLEICHGEWICHTSBEDINGUNG

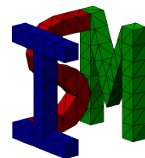
2.3 DAS ELEMENT SHELL3N

2.4 BEISPIEL

2.4.1 Struktur

2.4.2 Lineare statische Analyse

2.4.3 Spannungsberechnung



2 Lineare Finite-Elemente-Analyse

2.1 Gleichgewichtsbedingung der Elemente-Gruppierung

$$\mathbf{K}\mathbf{U} = \mathbf{R}$$

Lastvektor:

$$\mathbf{R} = \mathbf{R}_B + \mathbf{R}_S - \mathbf{R}_I + \mathbf{F}$$

Volumenkräften:

$$\mathbf{R}_B = \int_{m \ V^{(m)}} \mathbf{H}^{(m)T} \mathbf{f}^{B(m)} dV^{(m)}$$

Oberflächenkräften:

$$\mathbf{R}_S = \int_{m \ S^{(m)}} \mathbf{H}^{S(m)T} \mathbf{f}^{S(m)} dS^{(m)}$$

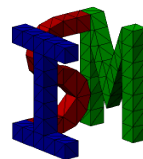
Anfangsspannungen:

$$\mathbf{R}_I = \int_{m \ V^{(m)}} \mathbf{B}^{(m)T} \boldsymbol{\tau}^{I(m)} dV^{(m)}$$

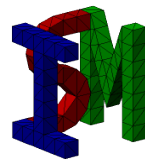
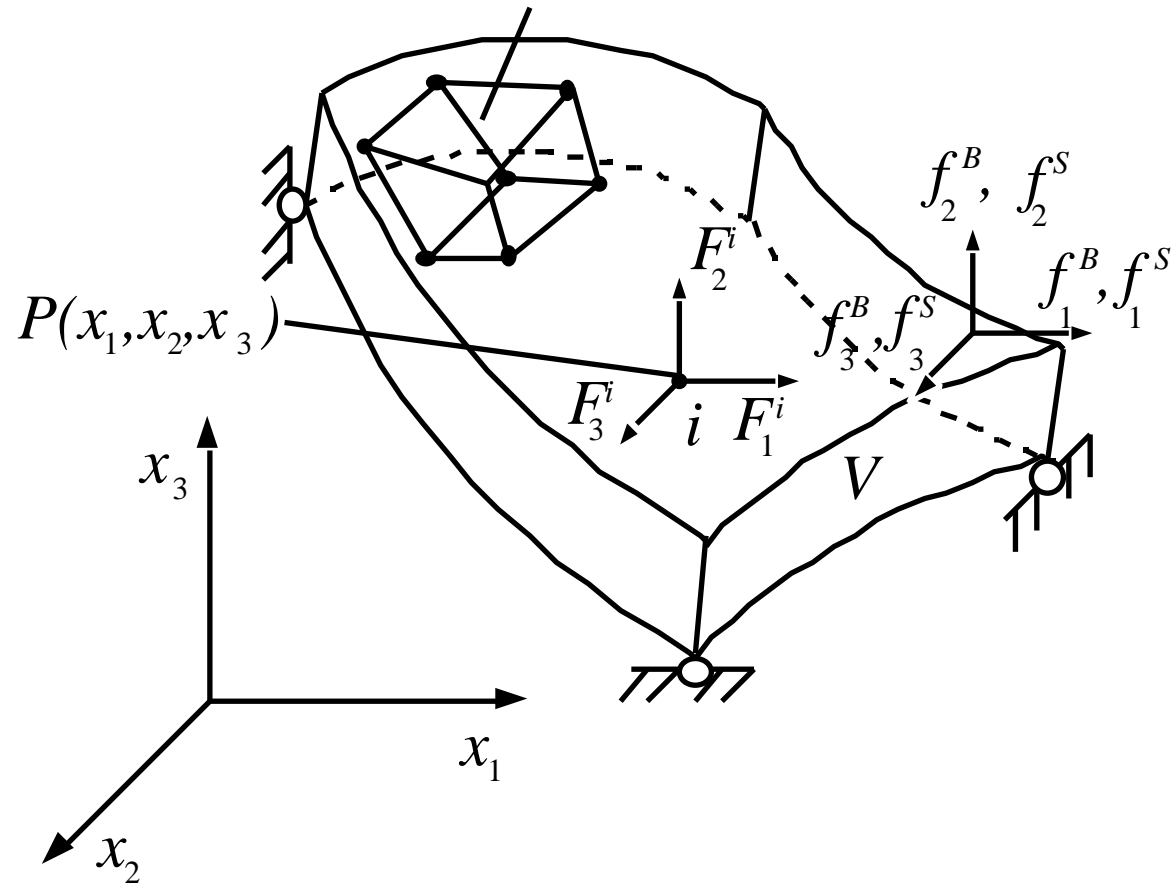
Steifigkeitsmatrix der Elemente-Gruppierung:

$$\mathbf{K} = \int_{m \ V^{(m)}} \mathbf{B}^{(m)T} \mathbf{C}^{(m)} \mathbf{B}^{(m)} dV^{(m)}$$

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finites Element



2.2 Lösung der Gleichgewichtsbedingung

Faktorisierung der Systemmatrix:

$$\mathbf{K} = \mathbf{LDL}^T$$

Lösung der Gleichgewichtsbedingung:

$$\mathbf{LDL}^T \mathbf{U} = \mathbf{R}$$

1. Lösungsstufe:

Vorwärtseinsetzen:

$$\mathbf{DL}^T \mathbf{U} = \mathbf{L}^{-1} \mathbf{R}$$

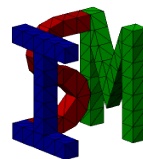
Inversion:

$$\mathbf{L}^T \mathbf{U} = \mathbf{D}^{-1} \mathbf{L}^{-1} \mathbf{R}$$

2. Lösungsstufe

Rückwärtseinsetzen:

$$\mathbf{U} = \mathbf{L}^{-1T} \mathbf{D}^{-1} \mathbf{L}^{-1} \mathbf{R}$$

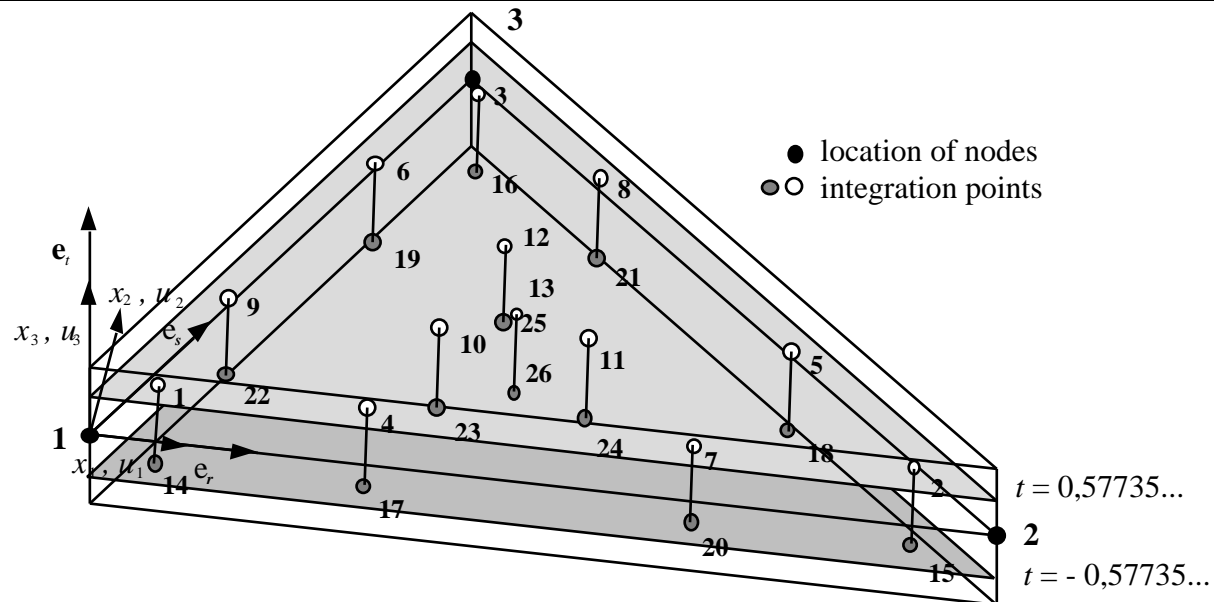


2.3 Das Element SHELL3N

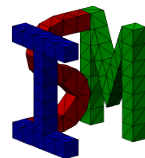
Lineares Platten-Scheiben-Element mit 3 Knoten

Kirchhoff-Platten-Theorie

3 Verschiebungs-DOFs und 2 Rotations-DOFs an jedem Knoten



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2.4 Beispiel

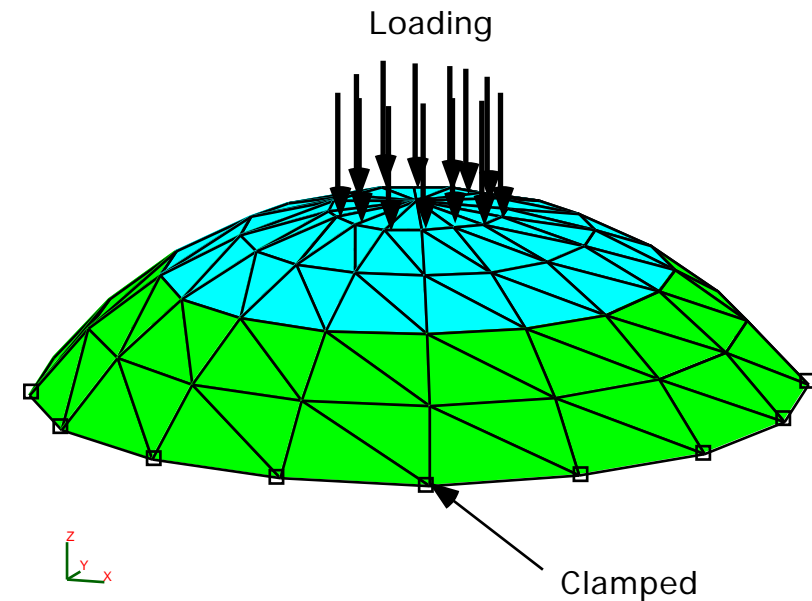
2.4.1 Struktur

PreSLang file

Sphere.s

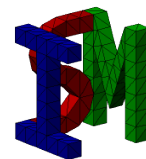
144 Shell3N-Elemente und 82 Knoten

```
* NODE DATA/
NODE ALLOCATE, REPLACE, 82,/
NODE CREATE, THREE_D, 1 0 0 5,/
* NODE RESTRAINTS/
NODE MODIFY, RESTRAINTS, 2 6
  0 1 0 0
  0 0 1 0
  0 0 0 1
  1 1 0 0
  1 0 1 0
  1 0 0 1
, /
```



```
* ELEMENT INCIDENCES/
ELEMENT ALLOCATE, REPLACE, 144,/
ELEMENT CREATE, SHELL3N, 1
  2 3 7,/
```

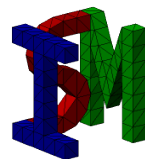
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```
* ELEMENT DATA/
OBJECT CREATE, REAL VECTOR REPLACE, 3,
  MATERIAL_DATA/
OBJECT READ, , MATERIAL_DATA
  3e+10 0.16 2500 ,/
OBJECT CREATE, REAL VECTOR REPLACE, 1,
  PHYSICAL_DATA/
OBJECT READ, , PHYSICAL_DATA
  0.4 ,/
ELEMENT MODIFY, MATERIAL_DATA, 1
  MATERIAL_DATA,/
ELEMENT MODIFY, PHYSICAL_DATA, 1
  PHYSICAL_DATA,/
ELEMENT MODIFY, COLOR, 1 YELLOW,/

31 0 0 -200 0 0 0
36 0 0 -200 0 0 0
41 0 0 -200 0 0 0
46 0 0 -200 0 0 0
51 0 0 -200 0 0 0
56 0 0 -200 0 0 0
61 0 0 -200 0 0 0
66 0 0 -200 0 0 0
71 0 0 -200 0 0 0
76 0 0 -200 0 0 0
81 0 0 -200 0 0 0
, LOADCASE1/

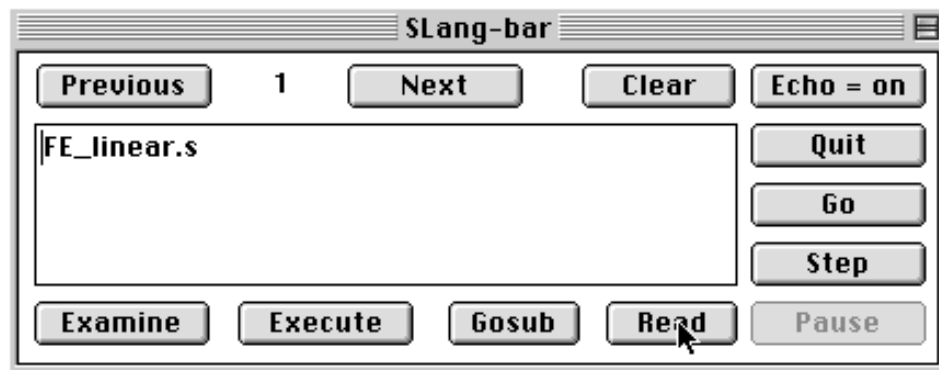
* LOAD CASES/
LOAD ALLOCATE, REPLACE, 1,/
LOAD BUILD, list, 1 16
  6 0 0 -200 0 0 0
  11 0 0 -200 0 0 0
  16 0 0 -200 0 0 0
  21 0 0 -200 0 0 0
  26 0 0 -200 0 0 0
```



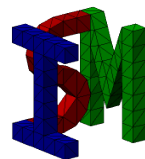
2.4.2 Lineare statische Analyse

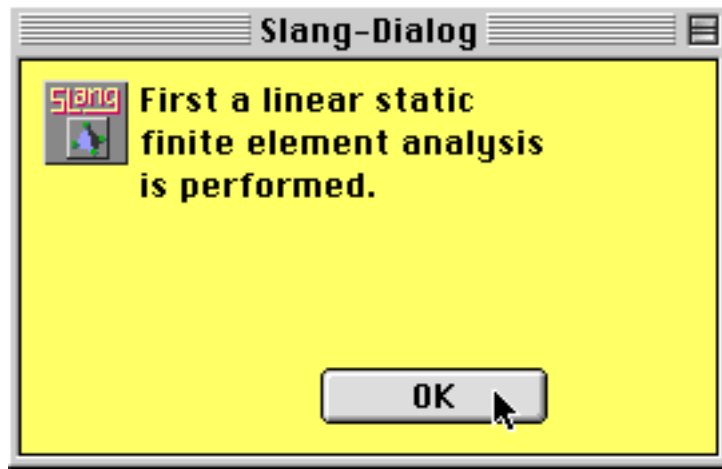
SLang file

FE_linear.s



```
#include Sphere.s  
control gosub,, linear_statics,/
```



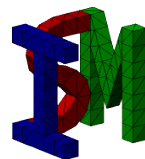


```
#label linear_statics
```

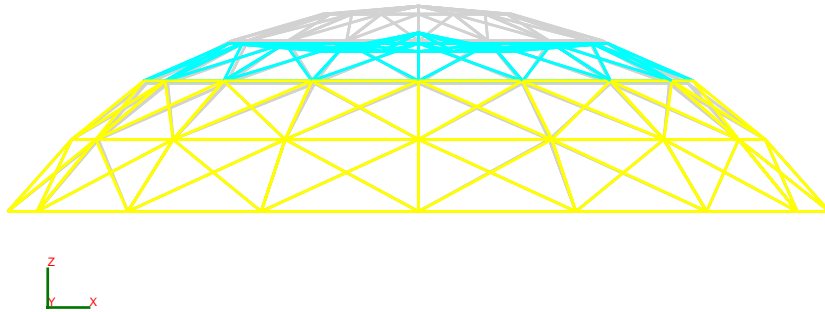
```
***** /
```

```
***** /
```

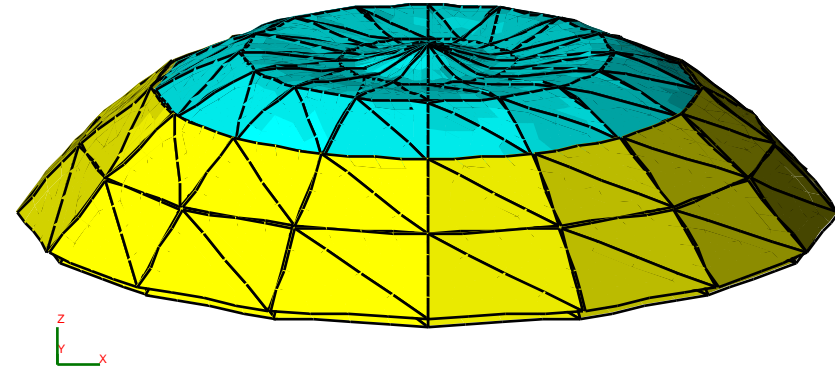
```
control return,,,/
```



structure view, reference nosymbols, 1
-90 0 0 100, /



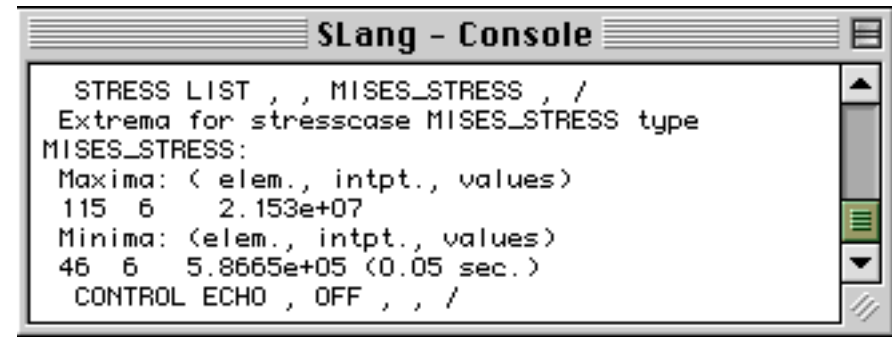
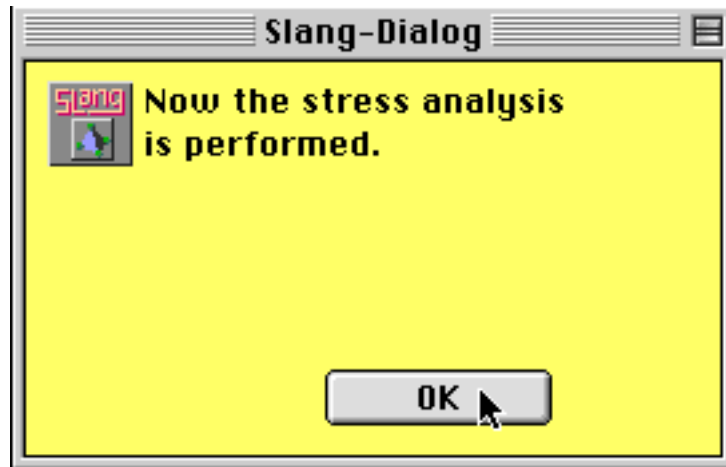
structure image, deformed detailed edges
colorshade, 1 -75 0 0 100, /



2.4.3 Spannungsberechnung

```
control gosub,, stress_analysis,/
```

```
stress build, total, mises_stress,/
stress build, total, stress_tensor,/
stress list,, mises_stress,/
```



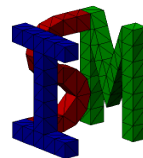
```
control return,, /
```

```
#label stress_analysis
```

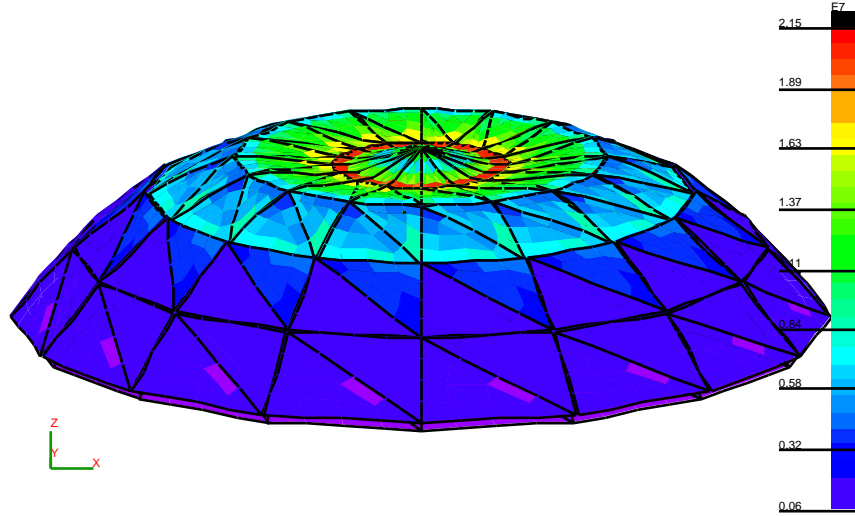
```
stress allocate, replace, 2,/
stress create, mises_stress global, 1
    1 shell3n, mises_stress/
stress create, stress_tensor local, 2
    1 shell3n, stress_tensor/
```

$$\sigma_{ij} = \begin{matrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} & 1 & 2 & 4 \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} & 2 & 3 & 5 \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} & 4 & 5 & 6 \end{matrix}$$

$$\sigma_m = \sqrt{\frac{1}{2} \left[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 \right] + 3(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2)}$$



structure stress, edges detailed
deformed, 1 -75 0 0 100 mises_stress, /



structure stress, edges deformed
detailed, 1 -75 0 0 100 stress_tensor 1, /

