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WAVELET ANALYSIS IN STRUCTURAL HEALTH MONITORING AND DAMAGE DETECTION

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Abstract

Wavelet analysis has been employed to numerous problems that belong to the general field of structural health monitoring in recent years. The scope of this article is to give an overview about the current state of wavelet-based applications in structural health monitoring and damage detection by means of dynamic structural tests.

A short introduction to wavelet analysis is followed by a selection of methodologies that were suggested by several authors in the context of structural health monitoring and damage detection. These algorithms concern the identification of modal system parameters, the identification of non-linear and linear time-varying systems and the detection of presence or occurrence of structural damage in a system. The results of many researchers work suggest a high potential of the mathematical algorithms based on wavelet analysis. Nevertheless, most approaches are still on an academic level. Further research is required to develop engineering tools that can be utilised for a standardised assessment and health monitoring of structures.

1 Introduction

Wavelet analysis has been increasingly applied to a great variety of engineering problems that are connected to the detection of structural damage and structural health monitoring in the last decade. A large number of publications demonstrate the versatility of these applications.

Many of these applications deal with the identification of structural damage based on vibration data that is acquired on a mechanical system. The considered systems range from rotating machinery to large civil engineering structures. Accordingly, the specific problems and respective algorithms are very manifold. While some approaches are focused on structural health monitoring analyzing data acquired under service conditions other methods were developed for specific dynamic tests which are performed, for example, in the context of regular inspections.

Section 2 of this article gives a compact introduction to wavelet analysis. The differences between continuous and discrete wavelet analyses, multi-scale analysis, wavelet-packet analysis and some essential terms that are important for the basic understanding of wavelet analysis are briefly discussed. Several methodologies that concern the identification of modal parameters, the identification of non-linear and linear time-varying systems and the detection of presence or occurrence of structural damage in a system are summarised in the following sections.

2 Wavelet Analysis – a Brief Introduction

In this section a brief introduction to wavelet analysis is given. Continuous wavelet transformation is described before passing on to discrete wavelet transformation and wavelet packet algorithms considering the one-dimensional orthogonal wavelet transformation.

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The one-dimensional wavelet transformation projects a signal $f(t)$ into a two-dimensional space.

$$W_{\varphi}^{f}(a,b) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t)\ \psi^{*}\left(\frac{t-b}{a}\right) \mathrm{d}t \quad \text{with} \quad \int_{-\infty}^{\infty} \psi(t)\ \mathrm{d}t = 0\ , \tag{1}$$

where $\psi^{*}(\cdot)$ denotes the complex conjugate of $\psi(\cdot)$. From the mother wavelet ψ the dilated and translated versions

$$\psi^{a,b}(t) = |a|^{-\frac{1}{2}}\ \psi^{*}\left(\frac{t-b}{a}\right) \tag{2}$$

are derived. These wavelets are usually oscillating, rapidly decaying functions. Generally, three types of wavelet transformation can be distinguished:

- continuous wavelet transformation,
- discrete wavelet transformation, and
- discrete wavelet packet transformation.

Their basic fundamentals are briefly described in the following sections.

2.1 Continuous Wavelet Tranformation

For the continuous wavelet transformation, the wavelets $\psi^{a,b}$ can always be described by an analytical function. Both the scaling parameter a and the translation parameter b change continuously over $\mathbb{R}$. It is excluded that a vanishes ($a \neq 0$). The continuous wavelet transform is defined by equation (1). Equation (1) becomes

$$W_{\varphi}^{f}(a,b) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t)\ \psi\left(\frac{t-b}{a}\right) \mathrm{d}t\ , \tag{3}$$

if $\psi\left(\frac{t-b}{a}\right)$ is a real function that satisfies the admissibility condition

$$0 < C_{\psi} = 2\pi \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^{2}}{|\omega|} \mathrm{d}\omega \ < \ \infty\ , \tag{4}$$

where $\hat{\psi}(\omega)$ denotes the Fourier transform of $\psi(t)$. The inverse of the continuous wavelet transform for real wavelets is given by

$$f(t) = \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{|a|}} W_{\varphi}^{f}(a,b) \psi\left(\frac{t-b}{a}\right) \frac{\mathrm{d}a\, \mathrm{d}b}{a^{2}}\ . \tag{5}$$

A wavelet ψ is called of order $g \in \mathbb{N}$ [27] if its mean value and the first $g-1$ moments vanish:

$$\int_{-\infty}^{\infty} t^{k}\, \psi(t)\ \mathrm{d}t \ = \ 0,\ k = 0, 1, 2, \ldots, g-1\ , \tag{6}$$

and if the gth moment is finite and non-zero:

$$\int_{-\infty}^{\infty} t^{g}\, \psi(t)\ \mathrm{d}t \ \neq \ 0\ . \tag{7}$$

Some of the most often applied wavelets are the Haar wavelet and the Morlet wavelet. For detailed derivations and further descriptions the authors kindly refer to the literature (e.g., [12], [14], [22], [27]).

2.2 Discrete Wavelet Transformation

The multiresolution analysis is the fundamental approach of discrete wavelet transformation. There a signal $f \in \mathcal{V}_{-1} \subset L^2(\mathbb{R})$ is separated into a high and a low frequency part. The low frequency part is the projection $P_0 f$ into a lower level space $\mathcal{V}_0$. The complement of $\mathcal{V}_0$ concerning $\mathcal{V}_{-1}$ is the space $\mathcal{W}_0$, whereas the projection of f into $\mathcal{W}_0$ is $Q_0 f$. In this context, a signal $f \in \mathcal{V}_{-1} \subset L^2(\mathbb{R})$ is defined by

$$f = P_0 f + Q_0 f \quad \text{resp.} \quad \mathcal{V}_{-1} = \mathcal{V}_0 \oplus \mathcal{W}_0. \tag{8}$$

Due to the recursion algorithm for each approximation $P_j f \in \mathcal{V}_j \subset L^2(\mathbb{R})$ follows

$$P_j f = P_J f + \sum_{k=j+1}^{J} Q_k f \qquad \text{resp.} \qquad \mathcal{V}_j = \mathcal{V}_J \oplus \bigoplus_{k=j+1}^{J} \mathcal{W}_k \\ \forall\, j, J \in \mathbb{Z} \text{ and } J > j. \tag{9}$$

Consequently, the multiresolution analysis of $L^2(\mathbb{R})$ is an ascending sequence of closed subspaces $\mathcal{V}_j \subset L^2(\mathbb{R})$ [27]:

$$0 \subset \ldots \subset \mathcal{V}_2 \subset \mathcal{V}_1 \subset \mathcal{V}_0 \subset \mathcal{V}_{-1} \subset \mathcal{V}_{-2} \subset \ldots L^2(\mathbb{R}) \tag{10}$$

with the properties

$$\begin{aligned} &\overline{\bigcup_{j\in\mathbb{Z}} \mathcal{V}_j} = L^2(\mathbb{R}), \quad f(\cdot) \in \mathcal{V}_j \Leftrightarrow f(2^j \cdot) \in \mathcal{V}_0 \\ &\bigcap_{j\in\mathbb{Z}} V_j = \{0\}, \qquad f \in \mathcal{V}_j \Leftrightarrow f(\cdot - 2^j k) \in \mathcal{V}_j, \qquad \forall\, k \in \mathbb{Z}\,. \end{aligned} \tag{11}$$

Thus, the spaces $\mathcal{V}_j$ are dilated and translated versions of the basic space $\mathcal{V}_0$. $\mathcal{V}_j = \overline{\text{span}\{\varphi_{j,k}\}_{k\in\mathbb{Z}}}$ and $\mathcal{W}_j = \overline{\text{span}\{\varphi_{j,k}\}_{k\in\mathbb{Z}}}$ are defined by the scaling functions $\varphi_{j,k}$ and wavelets $\psi_{j,k}$, respectively.

$$\varphi_{j,k}(t) := \sqrt{2^{-j}}\,\varphi(2^{-j}t - k)\,, \qquad \psi_{j,k}(t) := \sqrt{2^{-j}}\,\psi(2^{-j}t - k) \tag{12}$$

Since $\varphi \in \mathcal{V}_0 \subset \mathcal{V}_{-1}$ the scaling function φ and the wavelet ψ satisfy

$$\varphi(t) = \sqrt{2}\sum_k h_k\,\varphi(2t-k) \quad \text{and} \quad \psi(t) = \sqrt{2}\sum_k g_k\,\varphi(2t-k)\,, \tag{13}$$

where $\{g_k\}_{k\in\mathbb{Z}}$ and $\{h_k\}_{k\in\mathbb{Z}}$ are series that follow the conditions

$$\sum_k h_{k-2l}\,h_{k-2m} = \delta_{lm}\,, \quad \sum_k h_k = \sqrt{2}\,, \quad g_k = (-1)^k h_{p-k} \quad \text{with a fixed odd } p \in \mathbb{Z}. \tag{14}$$

Furthermore, each basis satisfies the properties

$$\int_{-\infty}^{\infty} \varphi(t)\mathrm{d}t = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} \psi(t)\mathrm{d}t = 0, \tag{15}$$

and the biorthogonality conditions

$$\langle \varphi_{j,k}, \varphi_{j,l}\rangle = \delta_{k,l} \quad \langle \psi_{j,n}, \psi_{k,l}\rangle = \delta_{j,k}\delta_{n,l} \quad \langle \varphi_{j,n}, \psi_{k,l}\rangle = \langle \varphi_{j,n}, \psi_{k,l}\rangle = 0 \quad \forall\, j \geq k. \tag{16}$$

A signal $f \in \mathcal{V}_0 \subset L^2\,(\mathbb{R})$ defined by equations (8) and (9) can be decomposed

$$f(t) = P_0 f = \sum_k a_{J,k}\widetilde{\varphi}_{J,k} + \sum_{m=1}^{J}\sum_k d_{j,k}\widetilde{\psi}_{j,k}\,, \tag{17}$$

where

$$a_{j,n} = \sum_k h_{k-2n} a_{j-1,k} \qquad \text{and} \qquad d_{j,n} = \sum_k g_{k-2n} a_{j-1,k} \tag{18}$$

are the approximation and detail coefficients, respectively.

Detailed descriptions of orthogonal discrete wavelet analysis are available, for example, in [27], [5], [14], [21], and [49]. To delve into the biorthogonal discrete wavelet analysis [13], [30], [10], and [9] are recommended.

2.3 Discrete Wavelet Packet Transformation

Compared to the discrete wavelet transformation, the wavelet packet algorithm decomposes the approximations, as well as, the details. Similar to the discrete wavelet transformation approach a finite quadrature mirror filter (low pass filter) $\{h_k\}$ and its corresponding high pass filter $\{g_k\}$, a scaling function $\varphi \in L^2$, and a corresponding wavelet function $\psi \in L^2$ are connected by equations (12), (13), and (14). $\mathcal{U}$ is a subspace of $L^2(\mathbb{R})$ spanned by an orthonormal basis $\{\eta_n\}_{n\in\mathbb{Z}}$ with

$$\eta_n(t) = \eta_0(t - n \cdot 2^q) \qquad \forall n \in \mathbb{Z} \quad \text{and a fixed} \quad q \in \mathbb{Z}, \tag{19}$$

whereas η_0 and η_n can be, but do not have to be of the type (12) or (13). The system of functions

$$\varrho_0 := \sum_k h_k\,\eta_k, \quad \sigma_0 := \sum_k g_k\,\eta_k, \quad \varrho_n(t) := \varrho_0(t - n \cdot 2^{q+1}), \quad \text{and} \quad \sigma_n(t) := \sigma_0(t - n \cdot 2^{q+1}) \tag{20}$$

can be arranged via filters of property (14). This defines a new orthonormal basis of $\mathcal{U}$. The orthogonal decomposition $\mathcal{U} = \mathcal{R} \oplus \mathcal{S}$ is valid with the definition of the subspaces $\mathcal{R} := \overline{\text{span}\{\varrho_n\}_{n\in\mathbb{Z}}}$ and $\mathcal{S} := \overline{\text{span}\{\sigma_n\}_{n\in\mathbb{Z}}}$. For any arbitrary function $f \in \mathcal{U}$ and their projections $f_R \in \mathcal{R}$ and $f_S \in \mathcal{S}$ the representations

$$f(t) = \sum_n U_n\,\eta_n(t), \quad f_R(t) = \sum_n R_n\,\varrho_n(t), \quad \text{and} \quad f_S(t) = \sum_n S_n\,\sigma_n(t) \tag{21}$$

can be derived from the relations

$$R_k = \sum_r h_{r-2k}\,U_r, \qquad S_k = \sum_r g_{r-2k}\,U_r \qquad \text{and} \qquad U_k = \sum_r h_{k-2r}\,R_r + \sum_r g_{k-2r}\,S_r \tag{22}$$

between the coefficients (decomposition and reconstruction formulas). Equations (21) and (22) are satisfied for each node $[q,p]$ of the wavelet decomposition tree and applicable recursively. If $U^{[0,0]}$ is the original signal, and $U^{[1,0]} := R^{[0,0]}$ and $U^{[1,1]} := S^{[0,0]}$ are the approximations and details, respectively, it follows $U^{[q+1,2p]} := R^{[q,p]}$ and $U^{[q+1,2p+1]} := S^{[q,p]}$ with $q = 0 \ldots N$ and $p = 0 \ldots 2^q - 1$. N is the maximal level of decomposition.

Recommended further reading for wavelet packets are [21], [48], [5], and [11].

3 Identification of Modal Parameters

Several attempts have been made in recent years to employ wavelet analysis for the extraction of modal parameters of linear time-invariant systems from measured data. In section 3.1 methods that are based on continuous wavelet transforms are summarised while section 3.2 is concentrated on approaches that use discrete wavelet decompositions.

3.1 Methods Based on Continuous Wavelet Analysis

An approach for the identification of natural frequencies and modal damping ratios by means of wavelet transforms of free vibration response data with respect to the complex Morlet wavelet

$\psi(t) = e^{i\omega_\psi t}\, e^{-\frac{t^2}{2}}$ was proposed in [41], [36], and [40]. Starting point is the modal decomposition of a system's free vibration response into contributions of N single modes j:

$$x(t) = \sum_{j=1}^{N} A_j\, e^{-\zeta_j \omega_{n_j} t} \sin\left(\sqrt{1-\zeta_j{}^2}\,\omega_{n_j} t + \phi_j\right) , \tag{23}$$

where A_j, ζ_j, ω_{n_j}, and ϕ_j are the residue amplitude, the modal damping ratio, the undamped natural frequency, and the phase lag, respectively. The wavelet transform of equation (23) for $\phi_j = 0$ with respect to the Morlet wavelet is [36]:

$$W_\psi^x(a,b) = \sqrt{a} \sum_{j=1}^{N} A_j\, e^{-\zeta_j \omega_{n_j} t}\, e^{-\left(a\sqrt{1-\zeta_j{}^2}\omega_{n_j} - \omega_\psi\right)^2} e^{i\sqrt{1-\zeta_j{}^2}\omega_{n_j} b} . \tag{24}$$

Depending on the wavelet's frequency ω_ψ, each scaling parameter a_j is related to the signal's frequency ω_j by

$$a_j = \frac{\omega_\psi}{\omega_j} , \tag{25}$$

assuming that both the signal and the analysing wavelet are sampled with the same frequency. For a fixed a_j, which is related to a frequency $\omega_{d_j} = \sqrt{1-\zeta_j{}^2}\,\omega_{n_j}$, equation (24) becomes

$$W_\psi^x(a_j,b) = \sqrt{a_j}\, A_j\, e^{-\zeta_j \omega_{n_j} t}\, e^{-\left(a\,\omega_{d_j} - \omega_\psi\right)^2} e^{i\omega_{d_j} b} = A_j\, e^{-\zeta_j \omega_{n_j} t}\, e^{i\omega_{d_j} b} . \tag{26}$$

For a previously chosen a_j (i.e., ω_{d_j}) the respective modal damping ratio is determined from the plot of the envelope in the semi-logarithmic scale.

A method for the estimation of the mode shapes $\phi_{k,j}$ by means of wavelet transforms with respect to the Morlet wavelet is given in [33]. If $W_\psi^{x_k}(a,b)$ and $W_\psi^{x_{ref}}(a,b)$ respectively denote the wavelet transforms of the signals at point k and at a reference point, their ratio at scale a_j

$$\frac{W_\psi^{x_k}(a_j,b)}{W_\psi^{x_{ref}}(a_j,b)} = \phi_{k,j} \tag{27}$$

is the kth component of the jth complex eigenvector.

In [4] a technique for the identification of natural frequencies and modal damping ratios from a continuous wavelet analysis of the frequency response function (FRF) is proposed. The complex function $\psi(\omega) = \frac{1}{(\omega-1)^2}$ is chosen as analysing wavelet. It is postulated and verified by two numerical examples that the natural frequencies and modal damping ratios can be estimated from the coordinates a_{max_j} and b_{max_j} of the maxima of $\mathrm{Im}\left[W_\psi^H(a,b)\right]$:

$$\omega_{n,j} = \sqrt{a_{max_j} + b_{max_j}} , \tag{28}$$

$$\zeta_j = \frac{a_{max_j}}{\omega_{n,j}} . \tag{29}$$

3.2 Methods Based on Discrete Wavelet Analysis

A wavelet-based approach for damping identification is proposed in [25]. Similar to the logarithmic decrement, that can be deduced from a free vibration $x(t)$ of an SDOF system, a wavelet-logarithmic decrement formula is derived:

$$\delta_j \simeq \frac{1}{(m-n)\,T} \ln\left|\frac{W_\psi^x(a_j, n\,T)}{W_\psi^x(a_j, m\,T)}\right| , m > n , \tag{30}$$

where j refers to the jth mode with a natural frequency ω_{n_j} that corresponds to the scale a_j.

For the identification of damping from the scaling coefficients obtained by a discrete wavelet decomposition of free vibration data with 2^M samples, that has been mapped to an interval $[0, 1]$, a discrete wavelet-logarithmic decrement is given in [25]:

$$\delta_j \simeq \frac{2^{(M-j)}}{k-l} \ln \left| \frac{a_{j,l}}{a_{j,k}} \right| \, , k > l \, . \tag{31}$$

In equation (31) the terms $a_{i,l}$ and $a_{i,k}$ refer to two maximal scaling coefficients at decomposition level j. This means, equation (31) can be interpreted as the classical logarithmic decrement formula applied to an approximation of the original signal at level j. The proposed procedure was verified by a numerical simulation in [25] and applied to data obtained from tests of an eight-storey building [18].

The response of a linear system $x(t)$ due to a given excitation $f(t)$ can be calculated via the system's impulse response function $h(t - \tau)$. It was derived in [32], that the convolution in time domain can be completely replaced for discrete response series with 2^J samples by

$$x(t_n) = \{h_{j,k}\}^T \{f_{j,k}\} \, . \tag{32}$$

The vectors $\{h_{j,k}\}$ and $\{f_{j,k}\}$ in equation (32) contain the wavelet coefficients obtained from a decomposition of the impulse response function (IRF) and the excitation.

In [34] an approach is suggested for the estimation of the discrete IRFs of a system from excitation and response data measured at several locations on the structure:

$$[x]_{m\times s} = [h_{j,k}]_{m\times rl} \, [f_{j,k}]_{rl\times s} \, . \tag{33}$$

Here m, r, s, and l refer to the number of measured response series, the number of input signals, the number of measured samples, and the number of considered wavelet coefficients, respectively. Equation (33) can be solved for $[h_{j,k}]$ as

$$[h_{j,k}] = [x] \, [f_{j,k}]^T \left([f_{j,k}] \, [f_{m,n}]^T \right)^{-1} \, . \tag{34}$$

The IRFs with respect to time are eventually obtained by wavelet reconstruction. In [35] these impulse response functions are used for the identification of state space models. The respective mode shapes, natural frequencies, and damping ratios of the system are derived from the state space representation [3].

To improve frequency response functions (FRF), that were extracted from measured data, a wavelet-based method is proposed in [7] and [6]. According to [7], the FRF is first estimated based on measured input and output data in frequency domain. Then both, the real and imaginary parts of $H(\omega)$, are smoothed by means of a selective wavelet reconstruction, in this case by soft thresholding. This procedure is slightly modified in [6] where an estimated FRF is first smoothed using a Hanning window before wavelet shrinkage is carried out.

4 Identification of Time-Varying Systems

Both, continuous wavelet transforms and wavelet coefficients of a discrete wavelet decomposition, represent the characteristics of a signal in the time-scale domain. This feature is obviously very stimulating if systems with time-varying properties are considered. Some examples for the application of wavelet analysis with respect to the identification of non-linear and linear time-varying systems are given in the following subsections.

4.1 Methods Based on Continuous Wavelet Analysis

From the graphic representation of a continuous wavelet transform, it can be deduced how the energy density of a signal is distributed with respect to time and scale (or frequency). In [4] an example of a simulated free vibration response of an SDOF system is presented. The ridge of the corresponding wavelet transform clearly shows a frequency shift with respect to time, that indicates a non-linearity.

A more detailed investigation of the impulse response of a non-linear system based on wavelet analysis with respect to the Morlet wavelet and a method for the identification of the system's parameters are described in [38] and [40].

As shown in equation (24) the wavelet transform allows for a modal decoupling. In [38] an identification approach is suggested proposed for the extraction of the ridge, skeleton, and backbone from the mode decoupled wavelet transform. The system's parameters can be estimated by calculating the instantaneous properties along the ridges and the application of curve fitting to both the skeleton's envelope and the backbone curve. This method is applied in [38] to an SDOF system with Coulomb friction and cubic stiffness contribution and to a 2-DOF system with cubic stiffness non-linearity.

To describe the input-output relation of a system in the time-scale domain, the ratio of the wavelet transform of the system's response to that of the excitation is introduced in [42] as the wavelet-based frequency response function $(H_W(a,b))$ or scale-translation response function (STRF), as it is called in [24]:

$$H_W(a,b) = \frac{W_\psi^x(a,b)}{W_\psi^f(a,b)} = \frac{W_\psi^x(a,b)\, W_\psi^{f^*}(a,b)}{W_\psi^f(a,b)\, W_\psi^{f^*}(a,b)} \,. \tag{35}$$

In [42] the wavelet-based FRF is employed for the investigation of the non-linear behaviour of an automobile seat–passenger system under real service conditions. The wavelet-based FRF is used in the sense of a transmissibility function, this means, as a relation between the accelerations measured at a mounting bolt of the seat and those at the seat/person interface. Two resonances were deduced from the ridges of this transmissibility function. The frequencies of these resonances varied significantly over the observation interval which is interpreted as an indicator for non-linearities in the behaviour of the considered system.

An alternative method for the description of input-output relations in the time-scale domain is introduced in [24], the cross wavelet transform:

$$W_\psi^{fx}(a,b) = W_\psi^{f^*}(a,b)\, W_\psi^x(a,b) \,. \tag{36}$$

The cross wavelet transform displays the similarities of the input f with a projection of the output x at scale a and translation b. High values of the cross wavelet transform indicate a vigorous response of the system to an input with corresponding scale at the respective time instant. It is demonstrated in an example with an SDOF Duffing oscillator, that is excited by a sweep force, how non-linearities of a system can be retrieved by comparing the wavelet transform of the response with the cross wavelet transform.

An extension of the cross wavelet analysis technique for the assessment of MDOF systems is presented in [39] by means of a 3-DOF system with cubic stiffness non-linearity.

4.2 Approaches Based on Discrete Wavelet Analysis

Assuming that the displacements $x(t)$ due to a measured excitation $f(t)$ are known from a test, the equation of motion for a linear SDOF system with viscous damping can be expressed as [16]:

$$m \sum_n a_{j,n}(x)\, \Gamma_{j,n}^{j,l\,(2)} + c \sum_n a_{j,n}(x)\, \Gamma_{j,n}^{j,l\,(1)} + k\, a_{j,l}(x) = a_{j,l}(f) \ , \tag{37}$$

where $a_{j,n}(x)$, $a_{j,l}(x)$, and $a_{j,l}(f)$ respectively refer to the scaling coefficients of the displacements and of the excitation at the discrete translations n and l at level j which can be interpreted as low-pass filtered and down-sampled versions of the respective time series. The $\Gamma_{j,n}^{j,l\ (i)}$ denote the connection coefficients of the ith derivative with respect to a certain wavelet at level j [49]. The application of appropriate connection coefficients in equation (37) allows for a complete description of the response by scaling coefficients of the displacements.

For MDOF systems, equation (37) can be extended to matrix–vector relations in the same way as in time domain. Then equation (37) can be rearranged such that the system's parameters to be identified are collected in a vector $\{P\}$ while the scaling coefficients of the response are assembled in a matrix. Provided that the system's mass is known, one obtains the system of equations

$$\left[\left[\sum_k a_{j,k}(x)\ \Gamma_{j,k}^{j,l\ (1)}\right]\ [a_{j,l}(x)]\right]\{P\} = \left\{a_{j,l}(f) - [M]\left\{\sum_k a_{j,k}(x)\ \Gamma_{j,k}^{j,l\ (2)}\right\}\right\}\ , \qquad (38)$$

that can be solved for the unknown system's parameters in vector $\{P\}$.

This method's performance is demonstrated for an SDOF system and for a 2-DOF system. The capability of identifying linear parameters, that show different kinds of time-variance, is reported. It was also tested how the algorithm behaves in the case of noise-contaminated measured data.

The procedure introduced in [16] is extended for non-linear systems in [17]. For the identification of an unknown non-linearity it is suggested to try different types of non-linearity and to detect the model that results in the closest calculated response due to the experimental excitation compared with the measured values. The proposed procedure is verified by means of simulated tests of SDOF and 2-DOF systems with different types of non-linearities.

In [49], the principle for identifying the model parameters of a linear system with viscous damping described in [16] were modified such that the algorithm can be applied to wavelet coefficients (i.e. details) of measured acceleration data rather than scaling coefficients (i.e., approximations) of displacements. The identification of an FE-model of a locally damaged steel beam based on experimental data is described in [50].

All methods, mentioned so far, use the wavelet transformation more or less as a signal processing tool. In [23] a procedure is submitted that uses a wavelet decomposition to estimate a time-varying tangent stiffness of a system. The method is tested for a 5-DOF system using both simulated and experimental data. However, no results about the identified time-varying system's parameters are reported.

5 Damage Detection

There is a great variety of problems that are included within the wide subject of damage detection. They range from the detection of the presence of a certain kind of damage inside a mechanical system, over the observation of damage occurrence, up to the assessment of a structure's condition by certain indicators in the context of long time health monitoring.

This section is divided into two parts. The first part is concerned with the identification of occurrence or presence of damage using wavelet analysis, while the second subsection is devoted to wavelet-based damage indicators.

5.1 Presence and Occurrence of Damage

The detection of existing defects by means of efficient and non-destructive techniques is of particular interest in context with rotating machinery parts such as gearboxes or ball bearings. It is proposed in [15] to apply wavelet shrinkage for the identification of rubbing in a mechanical system, that results from some structural fault. This approach is based on the assumption that rubbing produces features in a measured signal which result in very large wavelet coefficients at a certain level. The identification of such features in a de-noised signal is interpreted as an indicator for present rubbing.

A related methodology is suggested in [40] for fault detection in gearboxes. Here, the damage is detected by the assessment of certain patterns in the plots of both the modulus and phase of the continuous wavelet transforms of the response data with respect to the Morlet wavelet.

The methods proposed in [1], [20], and [2] are focused on the detection of the time instant when damage occurs in a structure as might be the case during a strong earthquake. In [1] damage in a linear SDOF system with viscous damping was numerically simulated by superimposing short impulses with a random excitation. The instants of occurrence of these simulated damage scenarios were detected as spikes in the wavelet coefficient series of the response at selected decomposition levels.

Similar investigations are described in [20]. However, there the damage is simulated by abrupt stiffness reductions of the SDOF system. The wavelet decomposition was applied to response data recorded in a building during an earthquake.

Continuing the study in [20], the possibility of an improved detection of damage occurrence is investigated in [2]. It was observed that spikes in the wavelet coefficient series of the response due to an abrupt stiffness change were difficult to identify if the system was excited by a random force. Better results were obtained from the plots of continuous wavelet transforms with respect to the Morlet wavelet and of the windowed Fourier transform.

5.2 Wavelet-Based Damage Indicators

The procedures summarised in the previous subsection deduced the presence or occurrence of damage directly from certain features of wavelet transforms of measured data. In the following examples either wavelet-based functions are proposed as a measure of damage severity or damage is quantified by identified changes of the system's parameters such as stiffness or damping.

The use of wavelet coefficients' statistics for damage identification is presented in [38], [40], and [8]. It is illustrated by tests on drilling equipment [38] that the signal-to-noise ratio (SNR) of measured response data can be interpreted as an indicator of damage. Based on the assumption that the wavelet coefficients at the first i decomposition levels solely represent noise, the so-called wavelet-based statistics is defined:

$$S_j = \left(\frac{\sigma^2 \left(d_{j,n} \right)}{\sum_i \sigma^2 \left(d_{i,n} \right)} \right)^2 , \tag{39}$$

where j is the respective decomposition level. It is indicated that a large value S_j refers to a high signal-to-noise ratio.

A damage indicator, that is based on changes of variance characteristics of wavelet coefficients at a certain decomposition level j from N data series measured during tests with a random excitation in the initial (undamaged) and a damaged condition, is defined in [38] and [8].

Special wavelet filters are used in context of condition monitoring of rotating machinery in [29]. Measured response data is filtered such that only components of a single frequency or of a number of selected frequencies are retained. The condition of the considered system is then assessed by evaluating the filtered signals either in time or frequency domain (e.g., by power spectra peak ratio).

A numerical simulation of an SDOF system excited by a random force is presented in [37]. The system's stiffness is changed abruptly. It is estimated from a slightly modified version of the equation of motion in the time-scale domain:

$$\frac{m}{k} W_\psi^{\ddot{x}} \left(a, b \right) + \frac{c}{k} W_\psi^{\dot{x}} \left(a, b \right) + W_\psi^{x} \left(a, b \right) = \frac{1}{k} W_\psi^{f} \left(a, b \right) + W_\psi^{e} \left(a, b \right) , \tag{40}$$

where $W_\psi^e \left(a, b \right)$ is the wavelet transform of an error. It is reported that the parameters $\frac{c}{k}$ and $\frac{m}{k}$ are determined such that $\left| W_\psi^e \left(a, b \right) \right|^2$ is minimised.

Stiffness and damping coefficients of a 3-DOF lumped mass system are identified in [31]. It is assumed that accelerations were only measured at two degrees of freedom. To overcome this lack of information, a neural network approach is studied. As input for the neural network, formulations are used that were calculated by means of continuous wavelet transforms with respect to the Meyer wavelet.

Two damage indices that are based on wavelet packet analysis are proposed in [44]. Repetitive identical testing within a structural health monitoring scheme should then provide information about the structure's condition. The damage indicators are calculated based on wavelet packet component energies $\left(E_j^k = \int \left(d_j^k(t)\right)^2 \mathrm{d}t\right)$:

- Sum of absolute differences SAD:

$$SAD = \sum_{k=1}^{N} \left| E_j^k - \tilde{E}_j^k \right| \tag{41}$$

- Square sum of differences SSD:

$$SSD = \sum_{k=1}^{N} \left(E_j^k - \tilde{E}_j^k \right)^2 , \tag{42}$$

where $\tilde{E}_j^k$ refers to the wavelet packet component energies computed for the undamaged (reference) condition.

Refinements of the analysis of the suggested indicator are described in [43], [45], and [46]. The change of distribution of vibration energy with progressing structural damage of a reinforced concrete beam and a prestressed concrete bridge were investigated in [49] and [51]. In both experimental studies relations between the IRF's and transmissibility function's energy at specific wavelet decomposition levels and structural damage were observed.

Discrete wavelet analysis is employed in [26] for crack location in a simply supported beam. Here the displacements at a certain time instant as a function of the beam's length are decomposed rather than response data with respect to time. A discontinuity in the wavelet coefficient series at a certain level is interpreted as an indicator for the crack location. A numerical example with 1024 measurement points is presented, which seems to be rather impractical for a real test application. Similar or related studies are described in [28], [19], and [47].

6 Conclusion

This article is concerned with a compact overview about some applications of wavelet analysis in the fields of structural health monitoring and damage detection. In this context it was concentrated on approaches that are based on the analysis of vibration test data.

The considered mechanical systems are of a great variety. They range from rotating machinery to large civil engineering structures. Accordingly, the specific problems and respective algorithms are very manifold. While some approaches are focused on structural health monitoring analysing data acquired under service conditions other methods were developed for specific dynamic tests which are performed, for example, in the context of regular inspections.

Many researchers results suggest a high potential of mathematical algorithms based on wavelet analysis. Nevertheless, most approaches are still on an academic level. Therefore, further research is required to develop engineering tools, that can be utilised for a standardised assessment and health monitoring of structures.

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