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Diploma Thesis

Implementation of Robust Design in ANSYS

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Chapter 1

Introduction

1.1 Motivation

Each design of a product or process includes a large number of parameters, which decide if certain technological and economical criteria will be achieved. When bringing the defined design into real-life, it is always subject of uncontrolled variation and differs more or less significantly from the deterministic target. Robust Design is a systematic and effective way of design optimization to find most efficient and stable designs. The basic principle of Robust Design is to apply special parameter combinations to a few number of experiments, prototypes or simulations, and then to gain maximum system information from the received answers.

The term Robust Design was formed in connection with the Taguchi Method, which has already been very successful in Japan for thirty years until it reached the western industry in the 1980s. Since then it has been applied to a wide variety of problems in all kinds of engineering and scientific disciplines and achieved better designs and a great success. A negative aspect of Taguchi's approach is, that he has invented several special procedures, methods and rules, which are based on heuristic knowledge and are inconsistent with mathematical principles.

For the implementation in ANSYS a more modern, scientific way of Robust Design has been chosen. Subject of interest is also to optimize the parameter settings by only running a few number of simulations. Instead of analyzing the answers of the structure themselves (like Taguchi), a response surface is fitted through the output data as function of all input variables. Based on this response surface, which should ideally represent all the properties of the structure and behave like the structure itself, a large number of samples can be generated very quickly. This enables the automatic process of a proper statistical analysis, so that effects of noises and parameter settings for most efficient, stable designs can be identified.

1.2 Information about ANSYS

The present work has been conducted at and was sponsored by ANSYS Inc. Therefore a brief overview on the company and the product is given. ANSYS, Inc., formerly Swanson Analysis Systems, Inc. (SASI), was founded in 1970 by Dr. John A. Swanson. Its headquarter is located at Southpointe in Canonsburg, which is about 15 miles south of Pittsburgh, Pennsylvania. The company developed a finite element analysis code, named ANSYS, which became a widely used application in the computer-aided engineering field. Since those early days the employment has grown from a few to over 200 employees at Southpointe and there is a network of distributors in many countries all over the world. Today's ANSYS Version 5.6 includes capabilities such as linear and nonlinear statics, dynamic analysis, buckling, contact, topological optimization, thermal analysis, fluid dynamics, acoustics, electromagnetics, also providing combinations of those methods, only to mention some main topics. ANSYS is used as analysis tool at many universities and it has a variety of design applications in industry, ranging from such everyday items as dishwashers, cookware, automobiles, running shoes and beverage cans to such highly sophisticated systems as aircraft, bridges, farm machinery, X-ray equipment and orbiting satellites. Many users all over the world have built up a large number of high quality solutions working with the ANSYS program. A remarkable example from the field of civil engineering to be mentioned is the participation in the reconstruction of the Church of Our Lady/ Frauenkirche in Dresden, Germany, where ANSYS was used to simulate parts of the over 200-year-old sandstone masonry structure. Summarized the history of ANSYS can be considered as very successful, but still as software development in general has accelerated to enourmous speed, there is always need to implement newer and better technologies, as well as to provide more options and methods to satisfy customer needs. Therefore ANSYS started to develope a new program called ”Design Space” in 1995, where the newest software knowledge should be integrated in terms as up-to-date programming languages, graphical interfaces, internet connectivity, userfriendliness and automation. Design Space has already proved its success, but still ANSYS is the better known and more demanded program. The ANSYS Version 5.7 with several new features will be available in spring 2001. One new main topic included, is the “Probabilistic Design System” from Dr. Stefan Reh.

1.3 Outline of Thesis

The purpose of this thesis was to implement Robust Design functionalities into ANSYS. With Robust Design not being part of the usual education of engineers, a previous literature research on this subject was of essential need. Therefore the thesis starts with the theoretical background of the Taguchi method and informs about purpose and algorithms of this Robust Design approach. After a critical analysis of this method, the concepts of the implemented Robust Design using response surfaces are described and possible improvements are discussed with regard to the Taguchi method.

The third chapter gives a brief overview on the Probabilistic Design System of ANSYS 5.7. Among other things, the Probabilistic Design System provides all functionalities from creating a probabilistic model to fitting response surfaces and is the basis for Robust Design using response surfaces.

The fourth chapter is a description of the implementation itself. An overview of the programmed functionalities is given and the reader is guided through the different options. The exact syntax of all programmed commands is added. It has to be noted that this implementation of commands is an internal test-version only and not available on the official ANSYS program. But it is a prototype of Robust Design, which might more or less be picked up for further versions. So from the beginning care was taken to meet the ANSYS standard.

While programming it was always important to control if everything is working correctly and to recognize which improvements shall be made. Therefore a test example had to be run. This was the referenced Robust Design example of a “Compressed Air Cooling System” using an equation model for the costs [15]. It also allows the verification of the implemented code and is documented firstly. Then two FEM-structures from the field of civil engineering have been built and analyzed. These examples include various aspects and show the practical usability of the implemented Robust Design method.

A summary on the whole project is given in the last chapter. The possibilities and the restrictions of the implementation are discussed. The gained knowledge is outlined and the major conclusions are drawn.

Furthermore a new graphical tool is demonstrated in combination with two animations to be found on the CD at the back of the thesis (see Appendix, pages 113-114).

Chapter 2

Theoretical Background of Robust Design

2.1 Taguchi Method

2.1.1 History of Taguchi Method

Dr. Genichi Taguchi built the foundations of terminologies known as “Quality Engineering”, “Robust engineering” and “Robust Design” [10, 14]. In the 1950s he published a new method including several new procedures, which is today named after him. In that time he was in charge to improve Japan’s telephone system, which was poor and unsuitable for long term communication purposes. The aspects he started his improvements on, were, that too much time and money was spent on experimentation and testing, while little emphasis was given, how to minimize the expenditure of resources [12]. This motivated him to use a new type of design of experiments. He emphasized a new way of creative thinking for efficiency and quality control. The basic concepts of Taguchi’s philosophy, on which all Taguchi procedures and techniques have been developed, are listed below [12]:

- Quality should be integral part of a product, instead of trying to achieve quality by inspection.
- Quality should best be achieved by minimizing the deviation from a target, so that the product becomes insensitive to uncontrollable environmental factors.

OA	Factors	Levels	Full Factorial
L4	3	2	8
L8	7	2	128
L9	4	3	81
L12	11	2	2048
L27	13	3	1594323
L64	21	4	$4.4*10^{12}$
L81	40	4	$1.2*10^{19}$

Figure 2.1: Standard Orthogonal Arrays

- Deviations from a given design parameter should be measured in terms of the (overall life cycle) costs of a product.

2.1.2 Outline of Procedure

Today's Taguchi method ranges from simplified standard methods to very complicated procedures, which are tailor-made for certain projects. It can require detailed knowledge in technical issues, as well as in economy and management. The aim is to find the best solution and this might include considering alternative solutions, rather than using only one certain algorithm. The Taguchi method is applied in four steps [12]:

1. Brainstorming: The participation of all relevant functional organizations including marketing is recommended, to identify the quality characteristics and design parameters important to the product/process.
2. Design and conduct the experiments.
3. Analyze the results to determine the optimum parameters.
4. Run a confirmatory test using the optimum parameters.

Design of Experiments

One main part of the Taguchi method is its efficient design of experiments using Orthogonal Arrays (OA's)[1]. Orthogonal Arrays are tables used to determine the least number of experiments. For example when studying 5 parameters at 4 different levels each,

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[1] Orthogonal Arrays were discovered by R.A. Fisher and L.H.C. Tippet in England in the 1930s [15].

Expt. No.	**Column**		
	1	**2**	**3**
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Figure 2.2: L_4 (2^3) OA

Expt. No.	**Column**			
	1	**2**	**3**	**4**
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Figure 2.3: L_9 (3^4) OA

Expt. No.	**Column**				
	1	**2**	**3**	**4**	**5**
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	2	1	2	3	4
6	2	2	1	4	3
7	2	3	4	1	2
8	2	4	3	2	1
9	3	1	3	4	2
10	3	2	4	3	1
11	3	3	1	2	4
12	3	4	2	1	3
13	4	1	4	2	3
14	4	2	3	1	4
15	4	3	2	4	1
16	4	4	1	3	2

Figure 2.4: L'_{16} 4^5 OA

$4^5 = 1024$ different experiments would be possible and describe a full factorial approach. The proposed Orthogonal Array for this configuration is: L'_{16} (4^5) Orthogonal Array. The index ’16’ indicates the number of experiments. Three standard Orthogonal Arrays are shown in Figures 2.2–2.4 [10]. A comparison in the number of experiments between Orthogonal Arrays and equivalent full factorial experiments is given in Figure 2.1. The Orthogonal Arrays describe the levels on which the effects of control factors need to be evaluated. Control factors are the controllable parameters in contrast to the uncontrollable, which are defined as noise factors. Noise factors can consist of environmental conditions (e.g. temperature, load ...), manufacturing tolerances or also other unknown circumstances. Since the influence of noise factors is of interest, several noise factor settings must be simulated for each control factor set (rows of Orthogonal Arrays). This can for example be realized by using another Orthogonal Array for the noise settings. In some practical experiments just repeating the experiment automatically leads to different noises. An example for a crossed array experiment is shown in Figure 2.5. In this case altogether 36 different experiments are conducted.

Analysis of Results, Signal-to-Noise Ratios

The result of each experiment is denoted with $y_{i,j}$. In Figure 2.5 for each setting of control factors there are four results on which a statistical analysis can be performed. Aim is to

Noise Matrix

	1	**2**	**3**	**4**
$\mathbf{N_1}$	A	A	B	B
$\mathbf{N_2}$	A	B	A	B
$\mathbf{N_3}$	A	B	B	A

Control Matrix

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

$\mathbf{Y_{i,j}}$			
$y_{1,1}$	$y_{1,2}$	$y_{1,3}$	$y_{1,4}$
$y_{2,1}$	$y_{2,2}$	$y_{2,3}$	$y_{2,4}$
$y_{3,1}$	$y_{3,2}$	$y_{3,3}$	$y_{3,4}$
$y_{4,1}$	$y_{4,2}$	$y_{4,3}$	$y_{4,4}$
$y_{5,1}$	$y_{5,2}$	$y_{5,3}$	$y_{5,4}$
$y_{6,1}$	$y_{6,2}$	$y_{6,3}$	$y_{6,4}$
$y_{7,1}$	$y_{7,2}$	$y_{7,3}$	$y_{7,4}$
$y_{8,1}$	$y_{8,2}$	$y_{8,3}$	$y_{8,4}$
$y_{9,1}$	$y_{9,2}$	$y_{9,3}$	$y_{9,4}$

Mean	**Std**	**S/N**
μ_1	σ_1	S/N_1
μ_2	σ_2	S/N_2
μ_3	σ_3	S/N_3
μ_4	σ_4	S/N_4
μ_5	σ_5	S/N_5
μ_6	σ_6	S/N_6
μ_7	σ_7	S/N_7
μ_8	σ_8	S/N_8
μ_9	σ_9	S/N_9

Figure 2.5: Crossed array experiment:
4 control factors at 3 levels (L_9) x 3 noise factors at 2 levels (L_4)

find a parameter setting which leads to a certain mean value and the possibly lowest standard deviation (Equations: 2.1, 2.2. In example of Figure 2.5: $i = 1 \ldots 9$, $n = 4$.). The lowest standard deviation is here equivalent to the highest quality or the most robust result. To achieve this Dr. Taguchi developed the Signal-to-Noise ratios, which originally come from electrical control theory [15]. The simplest form of S/N-ratio is the ratio of the mean (signal) to the standard deviation (noise). The standard S/N-ratios of the Taguchi method are given in the Equations 2.3 – 2.6. Depending on the quality criterion, the corresponding Mean Square Deviation (MSD) has to be chosen and put into Equation 2.3. Then always the biggest S/N-ratio indicates the best result. The trade of the most desired mean value and the lowest standard deviation is integrated in these equations.

Identification of Optimum Parameter Levels

The identification of the most successful experiment is obvious. But as only a comparatively few number of experiments has been conducted, they most probably do not contain the optimum combination of parameter levels. This can be found by seperating out the effect of each factor. For each control variable at a certain level, the S/N-ratio mean value is calculated (Figure 2.6). These mean values are also called ”marginal means”.

Level	**C_1**	**C_2**	**C_3**	**C_4**
1	$\frac{S/N_1+S/N_2+S/N_3}{3}$	$\frac{S/N_1+S/N_4+S/N_7}{3}$	$\frac{S/N_1+S/N_6+S/N_8}{3}$	$\frac{S/N_1+S/N_5+S/N_9}{3}$
2	$\frac{S/N_4+S/N_5+S/N_6}{3}$	$\frac{S/N_2+S/N_5+S/N_8}{3}$	$\frac{S/N_2+S/N_4+S/N_9}{3}$	$\frac{S/N_2+S/N_6+S/N_7}{3}$
3	$\frac{S/N_7+S/N_8+S/N_9}{3}$	$\frac{S/N_3+S/N_6+S/N_9}{3}$	$\frac{S/N_3+S/N_5+S/N_7}{3}$	$\frac{S/N_3+S/N_4+S/N_8}{3}$

Figure 2.6: The effect of each factor at a certain level corresponding to Figure 2.5.

Mean value
$$\mu_i = \frac{1}{n}\sum_{j=1}^{n} y_{i,j} \tag{2.1}$$

Standard deviation
$$\sigma_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(y_{i,j}-\mu_i\right)^2} \tag{2.2}$$

Signal-to-Noise ratio
$$(S/N)_i = -10\log_{10}(MSD_i) \tag{2.3}$$

MSD: smaller-is-better
$$MSD_i = \frac{1}{n}\sum_{j=1}^{n} y_{i,j}^2 \tag{2.4}$$

MSD: nominal-is-best
$$MSD_i = \frac{1}{n}\sum_{j=1}^{n} (y_{i,j}-m)^2 \qquad m = \text{target value} \tag{2.5}$$

MSD: greater-is-better
$$MSD_i = \frac{1}{n}\sum_{j=1}^{n} \frac{1}{y_{i,j}^2} \tag{2.6}$$

For each control variable $C_1 \ldots C_4$ the level with the largest S/N-mean value is chosen and consequently the optimum parameter levels are identified. The values of the table can also be graphed for each control variable. Examples of such a visualization as a function of different levels are Figures 5.4, 5.6, 5.8 in section 5.1.5. Such graphs easily allow to read the significance of a certain control variable. If the results at all levels of a control variable are roughly the same, then the result is not sensitive to this parameter and a variation of this parameter can only slightly influence the result. In this case the parameter is declared as not significant. Analogously if the results from different levels of a control variable differ relatively much, then the result is sensitive to this parameter and the parameter is declared as significant.

Extensions of the Taguchi Method

In addition to the previous section, it should be noted, that the Taguchi method can also include the following procedures:

- The design of experiments could be formed by mixed level arrays [12] for efficiently analyzing control variables on a various number of levels. In addition certain array types are available for examining interactions of control parameters [10][2].
- S/N-ratios can be specially designed to meet certain experiment's quality criteria. For example other S/N-ratios can be convinient for certain static problems or dynamic problems [10][3].
- A common practice for analyzing the results is the analysis of variances approach (ANOVA). This is a table of information that displays the contribution of each factor [10, 12, 14]. An analysis of variances can be used to determine the significance of control factors. It allows for a better quantitative differentation among the factors based on their significance.
- There are several considerations about cost and quality, like for example the Loss Function [12]. Taguchi's Loss Function considers each minimum deviation from a target value as additional cost. Whereas conventionally additional costs are only included, when certain quality tolerances are exceeded.

2.1.3 Examples

The Taguchi method has been applied to many different kinds of problems; to show the wide variety without exceeding the scope of this thesis, several examples shall only be mentioned:

General use of the Taguchi method

- Tuning Computer Systems for High Performance (UNIX) [10]
- Temperature Control Circuit (Dynamic Problem) [10]
- Electrical Filter Design [10]

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[2] Keyword: Linear Graphs

[3] Keywords: Static Problems: Signed-target Type, Fraction Defective, Ordered Categorical, Curve or Vector Response; Dynamic Problems: Continuous-Continuous, Continuous-Digital . . .

- Treatment of Asthmatic Patients [10]
- Study of Crankshaft Surface Finishing Process [12]
- Automobile Generator Noise Study [12]
- Air Bag Design Study [12]
- Optimization of Inter-Cooler, Nissan Motor Company [14]
- Accuracy Improvement of a Disposable Oxygen Sensor Used for Open Heart Surgery [14]
- Optimization of Nickel-Cadmium Battery Operation, Jet Propulsion Lab, NASA and California Institute of Technology [14]
- Development of Formula for Chemicals Used in Body Warmers [14]
- Engine Idle Quality Robustness, Ford Motor Company [14]
- Temperature-Rising Problem for a Printer Light Generating System, Minolta Co. [14]
- Optimization of a Cosit Mitigation Receiver, ITT Aerospace [14]
- Critical Parameter Characterization of a Xerographic Replenisher Dispenser, Xerox Cooperation [14]
- Optimization of Bean Sprouting Condition [14]

Use of the Taguchi method in combination with FEM

- Instrument Panel Structure Optimization [12]
- Study on the Heat Flow of a Cutting Process (ANSYS)[9]
- 3-D Magnetostatic Analysis on Spring Rate of a Sealed Switch (ANSYS) [1]

2.1.4 Success

It is claimed, that the Taguchi method has been an important factor for Japan's success in electronics, automobiles, photography and other industries [10, 15]. Since the 1980s when it came into the western industries, it has expanded to many fields, especially engineering, business and management, and helped to achieve more efficient and robust solutions. Hundreds of organizations have used this methodology and saved millions of dollars [14]. Dr. Genichi Taguchi has been awarded with several awards, like for example the Deming Award and today he is Executive Director of American Supplier Institute. His son, Shin Taguchi, successfully continues his father's work.

2.1.5 Critique of the Taguchi Method

Douglas Montgomery is being critical from a mathematical viewpoint, when precisely analyzing the Taguchi method [7]. He claims, that Dr. Taguchi “has advocated some novel methods of statistical analysis and some approaches to the design of experiments that are unnecessarily complicated, inefficient, and sometimes ineffective”. Some items of Montgomery’s critique:

- Some of Taguchi’s experiment designs may lead to an incorrect answer when large two-factor interactions appear. A proposed safer, strategy would be to identify important effects and interactions first, and then to consider the curvature only in the important variables. This usually leads to fewer experiments, easier interpretation and better understanding.
- The Taguchi method uses one array for the control variables and one array for the noise variables and then a crossed array experiment is conducted (see Figure 2.5). A better strategy is to use an adequate fractional factorial design for both, control and noise variables. This will most often reduce the number of experiments and could also improve process understanding.
- The use of Linear Graphs to design Orthogonal Arrays can be inefficient. Again, the proper alternative would be to use fractional factorial designs.
- Several considerations favour the seperate analysis of mean value and standard deviation, instead of using Taguchi’s S/N-ratios. This avoids the disadvantage of the Taguchi method, that y^2 and $\frac{1}{y^2}$ are very sensitive to outliers and values near zero.
- It can not be guaranted that picking the optimum levels from marginal means (Figure 2.6) always produces the optimum. Also Taguchi’s confirmation experiment is no guarantee. Montgomery concludes: “The best way to find optimum conditions is with response surface methods, . . . ”.

From this it can be concluded, that the Taguchi method shows several inaccuracies. In addition it should be noted, that the number of examined noises per control parameter set is always relative low, which results in relative poor statistics. Montgomery acknowledges, that Taguchi’s philosophy of quality engineering and the factorial design concept are found to be very good and in any case superior to “best guess” or “one-factor-at-a-time” methods. For integration of this concept into practice, it is proposed to use more efficient methods, which are easier to learn and apply.

2.2 Robust Design using Response Surfaces

2.2.1 Introduction

The implementation in ANSYS is programmed according to the principle of Robust Design on response surfaces. Generally it follows the same idea as the Taguchi method to optimize parameters for better and more stable designs. But the algorithms are quite different. The Taguchi method uses several special procedures and tables to extract the maximum data directly from a few number of experiments. In the current work a response surface model is fitted through the experiment/simulation data, then thousands of samples are generated which allow a proper statistical analysis for identifying the best settings. While the original Taguchi method can be performed by paper and pen, this more modern approach takes advantage of today's computational power. It is expected to gain more system information and in some cases better results by Robust Design on response surfaces.

2.2.2 Implemented Robust Design

Illustration of priniciple

The principle of the implemented Robust Design on response surfaces is illustrated in Figure 2.7. The shown model consists of one control variable X, one noise variable N and the result variable Y. At first, corresponding to Taguchi's experiments a few number of simulations is processed (in this example eight simulations). The result of each simulation is marked with a star. The simulations do not necessarily need to follow a certain design, like in the graphic were randomly generated settings for X and N have been used. Then a response surface[4] is fitted through the simulation points. As the response surface is defined by an equation model, it can not turn into any shape, but the best fitting curvature possible is determined and its accuracy is given. Assuming that a good fit has been achieved, the response surface represents a good approximation of simulation results within the considered range. Now many samples can quickly be generated. For Robust Design the results of certain control variable settings under varying noises are of interest. In the graphic three different control variable settings are highlighted. For each

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[4] The functionality of creating response surfaces already existed as part of Probabilistic Design and has not been implemented.

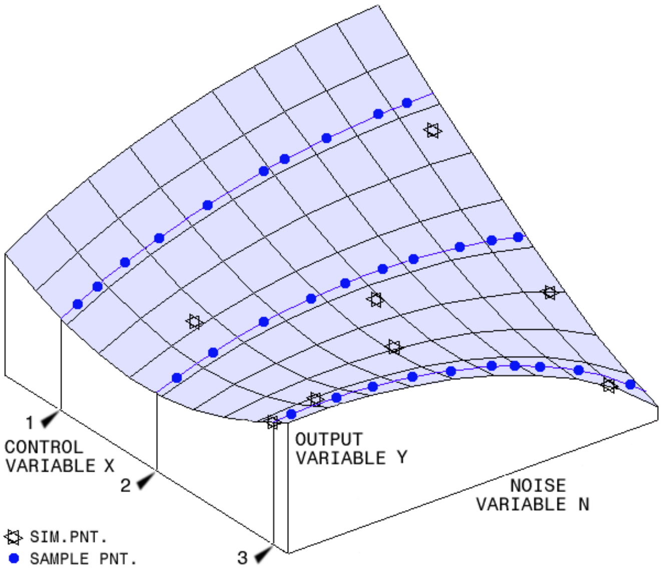

Figure 2.7: Principle of Robust Design on Response Surfaces

ten random noises are generated and the corresponding values on the response surface are calculated. These values are marked as dots. Then for each control variable set certain statistics are calculated, like for example the mean value $\mu(Y)$, standard deviation $\sigma(Y)$, signal-to-noise ratio $S/N(Y)$ or others. The statistic values of a certain control variable set equal one Robust Design sample (not shown in the graphic). So although the noises are random and are beyond control, clear data about the average, expected Y-value and its stability for each control variable set has been gained. For example in Figure 2.7, "X on position 2" indicates the most stable result of the considered levels and "X at position 1" shows the highest mean value. Depending on the objective the best Robust Design sample can be selected. Several methods are available for analyzing the Robust Design samples and achieving optimum results.

It has to be added, that this is only the low-dimension illustration example. For most systems the reponse surface is created in higher dimensional space, many control variable settings are examined and a higher number of noises should be generated.

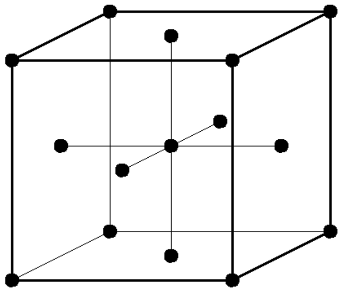

Figure 2.8: Central Composite Design for three factors

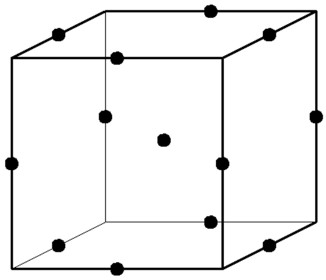

Figure 2.9: Box-Behnken design for three factors

Design of Simulations

Design of simulations describes the principle, how the input parameters combinations are created, which are used for the FEM-simulation. Depending on the dimension of the system and the chosen response surface type, a certain minimum number of simulations has to be processed. The simulation design could either be randomly generated by a Monte-Carlo simulation or a design of experiments can be selected for achieving most accurate results with a few number of simulations as proposed by Montgomery [7]. Probabilistic Design System of ANSYS 5.7 provides besides the option of user-defined designs, the central composite design and the Box-Behnken design. For three factors these designs are shown in Figures 2.8 and 2.9. The number of simulations of the full factorial central composite design in n-dimensional space is given in Equation 2.7.

$$\text{Full Factorial Central Composite Design:} \quad \underbrace{1}_{center} + \underbrace{2n}_{axis} + \underbrace{2^n}_{corners} \tag{2.7}$$

In ANSYS this number is decreased by using 2^{n-p} fractional factorials instead of the term 2^n. For example when discussing 5 factors on two levels (-1,1) the full approach would require 2^5=32 simulations. Instead all level combinations for the first four variables are created and the level of the fifth variable is always set to a linear combination of all four levels: E=ABCD. The corresponding design table is shown in Figure 2.10. The number of simulations has significantly been reduced to 2^{5-1}=2^4=16. The created experiment design is of resolution 'V', which describes the grade of linear independance. Each valid equation of linear combinations considering all experiments consists at least of 5 factors, like 1=ABCDE, A=BCDE, AC=BDE, … The maximum resolution re-

No.	**A**	**B**	**C**	**D**	**E=ABCD**
1	1	1	1	1	1
2	1	1	1	-1	-1
3	1	1	-1	1	-1
4	1	1	-1	-1	1
5	1	-1	1	1	-1
6	1	-1	1	-1	1
7	1	-1	-1	1	1
8	1	-1	-1	-1	-1
9	-1	1	1	1	-1
10	-1	1	1	-1	1
11	-1	1	-1	1	1
12	-1	1	-1	-1	-1
13	-1	-1	1	1	1
14	-1	-1	1	-1	-1
15	-1	-1	-1	1	-1
16	-1	-1	-1	-1	1

Figure 2.10: 2^{5-1} fractional factorial design, Resolution 'V'.

quired by a response surface is 'V'. So the central composite design is always reduced to resolution 'V'. Fractional factorials of resolution 'V' are: 2^{6-1}, 2^{7-1}, 2^{8-2}, 2^{9-2}, 2^{10-3}, 2^{11-4}, ... It has to be considered, that for example 2^{6-2} and 2^{7-2} are only resolution 'IV'. Generally the resolution of a particular fractorial factorial design can either be taken from tables [7] or computed as it is done in ANSYS[5]. In Figure 2.11 the number of simulations from central composite designs and Box-Behnken designs can be compared to the number of coefficients from a second-order response surface model for a different number of parameters. The increase of simulations from n=5 to n=6, as well as from n=8 to n=9 results from the fractional factorials (2^{5-1}=16, 2^{6-1}=32; 2^{8-2}=64, 2^{9-2}=128).

Response Surfaces

Response surfaces are the regression models on which the Robust Design samples are processed on. Therefore their accuracy is quite important for getting realistic results from Robust Design. Probabilistic Design System of ANSYS provides three types of response surfaces. The most accurate is the response surface based on a quadratic function including cross-terms. Furthermore there is the option of using the quadratic function without cross-terms or only using linear terms. The function of the full-quadratic response surface

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[5] The ANSYS routine for creating two-level fractional factorial designs of resolution 'V' was part of the project.

Number of Input Variables	Coefficients of second-order Response Surface	Central Composite Design	Box-Behnken Design
3	10	15	13
4	15	25	25
5	21	27	41
6	28	45	49
7	36	79	57
8	45	81	65
9	55	147	121
10	66	149	161

Figure 2.11: Number of simulations - required for response surface - and provided by design.

is given in Equation 2.8 [8].

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j \qquad (2.8)$$

Corresponding to the order of the terms the equation contains $1+k+k+k(k-1)/2$ coefficents β, with k denoting the number of input parameters (control and noise variables). As the other response surface models leave certain terms out, they contain equivalent less coefficients. If the number of coefficients equals the number of simulations, the response surface is directly defined by these simulation points. If there are less simulations, no definition of the response surface is possible. If the number of simulations is not restricted by the simulation design, it is proposed to use at least 1.5 times simulations of the required minimum. Then the best response surface is fitted through the simulation points and most probably does not intersect with any simulation point. The accuracy can be checked by several goodness-of-fit measures, as for example the maximum residual and/or the error sum of squares. Often the fit can much be improved when using a transformation of Y before fitting. Then the response surface is fitted through the transformed points and whenever a value from the response surface is required as well as for the goodness of fit test, the sample is calculated on this response surface first and then modified by the inverse transformation. Some transformations available in ANSYS are: e^y, $\log_a y$, y^a. Often the fit of the response surface is improved enormously when using the Box-Cox-transformation (Equation 2.9).

$$y^* = \begin{cases} \frac{y^\lambda - 1}{\lambda} & : \quad \lambda \neq 0 \\ \ln(y) & : \quad \lambda = 0 \end{cases} \qquad \text{for } \lambda \in [-2;2] \qquad (2.9)$$

The inverse transformation is given in Equation 2.10:

$$y = (\lambda y^* + 1)^{\frac{1}{\lambda}} \tag{2.10}$$

Robust Design Sampling

The programmed Robust Design sampling works principally as already described on pages 20, 21. But it is also possible, to process adaptions for a specified search in certain regions of parametric space. This could either be achieved automatically by a previous defined quality criterion or by user selected ranges of input or output variables. A combination of both is also possible, which for example allows an automatic adaption within certain restrictions. As the quality criterion could always be changed and any selection be used for a new adaption, several criterions could be considered. There is no limit on the number of adaptions and also the number of samples to be processed per adaption can always be varied. More details are given in chapter 4 and a demonstration of this concept can be found in chapter 5. A general difference to the Taguchi method is that the implementation generally uses continuous distributions instead of discrete factor levels. The obvious advantage is, that the data analysis and also identifying the optimum is not restricted to certain levels, which could in some cases lead to significantly improved results.

Quality Measures

For each output variable, several statistical Robust Design output parameters can be defined at the same time. Possible outputs are the mean value, standard deviation and the signal-to-noise ratios discussed in section 2.1.2. For achieving accurate results it is proposed to apply not less than 50 noises per sample for these parameters. Because if the variation effects are relatively small, they could be overshadowed by the scatter of the statistic values. Then also the minimum and maximum of the sample results for a certain control variable set have been included. These two measures are of relative poor statistics and show a high variation, especially when heavy tailed distributions occur. So using these measurements can lead to incorrect solutions. But these measures can be sufficient for minor parameters or for certain kind of problems. Generally it is better to extract confidence values[6] as measures instead. The confidence value c_i is computed, so that the probability of Y being smaller than this value is of a userdefined percentage P_L : $p(Y < c_i) = P_L$ (with c_i=confidence value of control set i and $P_L \in \,]\, 0\,;1\, [\,$). The proposed minimum number of samples is defined by the following term: $\frac{10}{\min\left((P_L),(1-P_L)\right)}$. It should be noted, that Robust Design objectives can be controlled by using this single output parameter, similar to S/N-ratios. For example when mean value and standard deviation of any product cost should be minimized, either the S/N-ratio(“smaller-is-better”) can be maximized or the confidence value c_i for the confidence limit 95% can be minimized, $p(Y < c_i) = 0.95$. This measurement automatically takes both mean and standard deviation into account. One advantage is that the unit has not changed, so that the confidence values can directly be interpreted, whereas the S/N-ratios are not very informative. Furthermore, confidence values are robust to outliers or values near zero in contrast to S/N-ratios.

Identification of Optimum Parameter Levels

Since a high number of samples can be generated, the best sample directly provides the best input parameter setting. In contrast to the Taguchi method, where the effect of certain parameters are seperated first and then a new combination of levels is formed. This does not always guarantee that the opimum levels are identified. To improve and verify the best result from Robust Design on response surfaces, a new, more or less concentrated search around the input parameter settings of the best sample(s) can be processed. It is expected, that this is an effective and efficient way of identifying the optimum parameter levels. However, one condition is that an adequate high number of samples is processed in the first simulation and not too restrictive adaptions on very few samples are performed, as that might only lead to a local optimum.

2.2.3 Examples

Comparatively few literature has been found about the combination of the finite element method and Robust Design and hardly anything about FEM-examples and Robust Design based on response surfaces. One example was demonstrated at the ANSYS Conference

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[6] The function of calculating confidence values from a vector of samples for a certain probability has been adopted from Probabilistic Design.

2000 in the lecture “Design optimization by the design of experiments and Response Surface method” by Shuuichi Matsui, Japan[6]. The example was the design of a semiconductor assembly process for the stress reduction of wire bonding. The analysis was performed by a software product developed in Japan in combination with ANSYS. Another approach of Robust Design on response surfaces is published in the *Journal of Mechanics* with the title “A procedure for Robust Design: Minimizing variations caused by noise factors and control factors” [3]. The introduced example is the design of a solar powered irrigation system computed by a thermodynamic analysis software combined with an economic analysis routine. Generally the introdused Robust Design on response surfaces can also be used for the FEM-examples given on page 17, which have been solved by the Taguchi method. Furthermore, as Robust Design is applied to a wide variety of technical problems, which could be simulated by FEM, it is estimated, that there is a high potential for Robust Design integrated with FEM instead of only analyzing physical experiments.

Chapter 3

Probabilistic Design in ANSYS 5.7

The implementation of Robust Design is based on the Probabilistic Design System of ANSYS 5.7. A brief introduction to some options of the Probabilistic Design System is given through the main menu keys:

- Analysis File - `(Prob Design > (-Analysis File-) Assign)`
  At the beginning of each new Probabilistic Design session an Analysis File has to be read in. This should contain the commands to build and run the simulation model of interest and must have been previously created by the user. The Analysis File could be any input of ANSYS (FEM, Multiphysics, APDL-commands, etc.). The model parameters to be analyzed must be given as variables, which have been set to valid, deterministic values.

- Random Input - `(Prob Design > Random Input > add)`
  In the Random Input menu all variable names declared in the Analysis File will be shown. The user can select certain parameters and define them as random variables. Ten various probability density functions are applicable.

- Random Output - `(Prob Design > Random Output > add)`
  From the shown parameters those are to be chosen, which represent results of the simulation model (Stress, deflection, etc.). The output parameter values must automatically be set within the analysis file.

- Probabilistic Analysis Method - `(Prob Design > -Prob Method- > ...)`
  Before running a simulation, the user has to define the number of simulations and select the probabilistic analysis method:

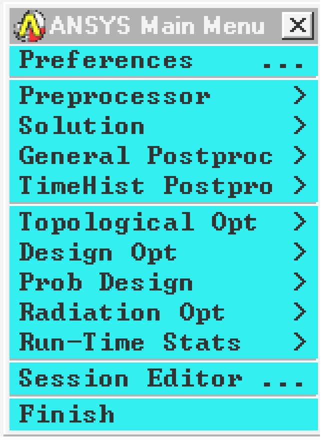

Figure 3.1:
Main Menu in ANSYS 5.7

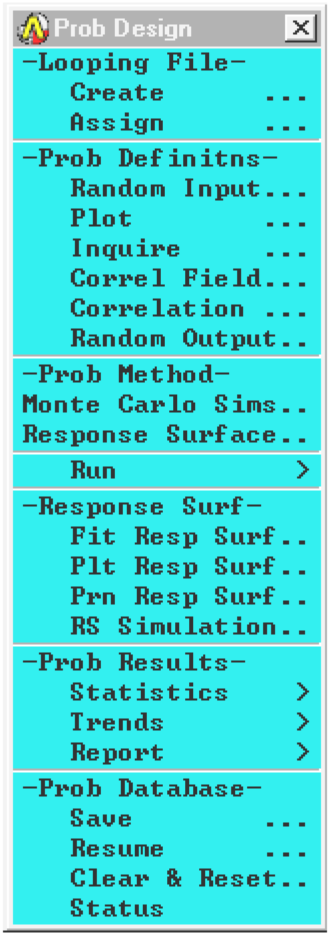

Figure 3.2:
Probability Design Menu

- Monte-Carlo Simulation type(`... > Monte Carlo Sims`)
  - Direct Monte-Carlo Simulation
  - Latin Hypercube Sampling
  - User-defined simulation data
- Response Surface type(`... > Response Surface`)
  - Central Composite Design
  - Box-Behnken Design
  - User-defined simulation data

- Run Simulations - `(Prob Design > Run > ...)`
  A solution set label has to be specified. Options: serial/parallel, tolerated failed loops, sample file option, ..

- Fit Response Surface - `(Prob Design > Fit Resp Surf)`
  A response surface can be fitted through the simulation data. This is done for each output parameter seperately. Depending on the number of simulations, different regression models can be chosen: (n =number of input parameters; m =minimum number of simulations)

  - Quadratic function including cross-terms: $m = 1 + n + \frac{n(n+1)}{2}$
  - Quadratic function without cross-terms: $m = 1 + 2n$
  - Linear function: $m = 1 + n$

  However, minimum number of simulations proposed to get a better fit and to check the goodness-of-fit is $> 1.5 \cdot m$. Response surfaces could drastically improve when using a transformation of the output parameter (only possible, if all values of the output parameter are positive). A very efficient, automatic adjusting transformation is the Box-Cox-Transformation. Also to be mentioned is the option of "Forward Stepwise Regression" to filter out insignificant terms. This can lead to more stable equation systems and better results.

On this stage either a Monte-Carlo simulation on the response surface model could be processed or a Robust Design analysis can be started. With Robust Design being the main subject of this thesis some further options of Probabilistic Design are listed only `(Prob Design > ...)`:

- 3D-Visualization of Response Surfaces `(... > Plt Resp Surf)`

- Various information of Response Surfaces `(... > Prn Resp Surf)`

- Monte-Carlo simulation on Response Surfaces `(... > RS Simulation)`

- Statistical methods on results of simulations using an Analysis File or response surfaces `(... > Statistics > ...)`:

  - Sample History Plot

  - Histogram Plot
  - Cumulative Distribution Function
  - Probabilities
  - Inverse Probabilities
- Trends of the results can be analyzed `(... > Trends > ...)`:
  - Sensitivities Plot
  - Scatter Plot
  - Correlation Matrix

Furthermore, a report of the probabilistic analysis can be generated[1]. The Probabilistic Design System includes full save/resume-capabilities. More information in detail can be found in the documentation of ANSYS 5.7.

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[1] This kind of automatic report generation of probabilistic data is protected by US patent 6,055,541.

Chapter 4

Implementation

The development of Robust Design in ANSYS started within this project. It is based on the Probabilistic Design System and fully integrates into ANSYS. All core functionalities and data structures were programmed in C. Eight complete new ANSYS-commands were developed and three existing graphic commands were extended for the use in Robust Design. The top layer of the code which parses the user's command inputs, as well as the new graphical tool to view the ranges of samples is implemented in FORTRAN for thight integration with the existing ANSYS code base.

4.1 Programming Principles

For achieving efficient C-source code, which provides flexibility, enables a good overview and quick programming progress, the following concepts were realized:

- Rooting in one global variable, a data structure was created, which expands or becomes smaller depending on the actual requirements. Number and size of variables are managed dynamically by functions which allocate, reallocate or free memory. This enables a clear and efficient data management.

- The code was modularized as deemed adequate for maximum reusabilty and ease of maintenance.

- Apart from error messages all text outputs were seperated in individual routines, which are either automatically called after certain processes or on explicit user command (RDLIST). This keeps the main functions clear and avoids repititon.

- Most of wrong user input is already blocked on the top level. Sources of program failure have been removed as far as recognized.

4.2 Development of the Robust Design Project

The development of Robust Design in ANSYS can be divided into several steps:

- The principal Robust Design sampling has been accomplished in connection with the existing ANSYS program and an appropriate data structure has been created.
- The possiblity to define a quality criterion and to process adaptive Robust Design sampling has been realized.
- Three graphic commands have been adopted from Probabilistic Design and connected to the Robust Design samples for visualizing the results.
- Routines for selecting adaptions and/or samples have been developed. This provides the possibility to perform adaptions based on selections and to consider certain restrictions. Such selections can also be examined by the graphical tools.
- Appropriate text outputs have been added for giving better control over the data. Selected samples can be written to a text file.
- A new graphic command has been developed in combination with the functionality to select samples. This enables to watch ranges of the current selection for up to fourty variables. This graphical tool turned out to be very useful.

4.3 Descriptions of Commands and Structure

The Robust Design process can be started after a Probablististic Design model including response surfaces has been created as described in chapter 3. Figure 4.1 illustrates the main functionality of each Robust Design command and puts it into context within the programmed Robust Design structure. For the full syntax of the commands it is refered to section 4.5.

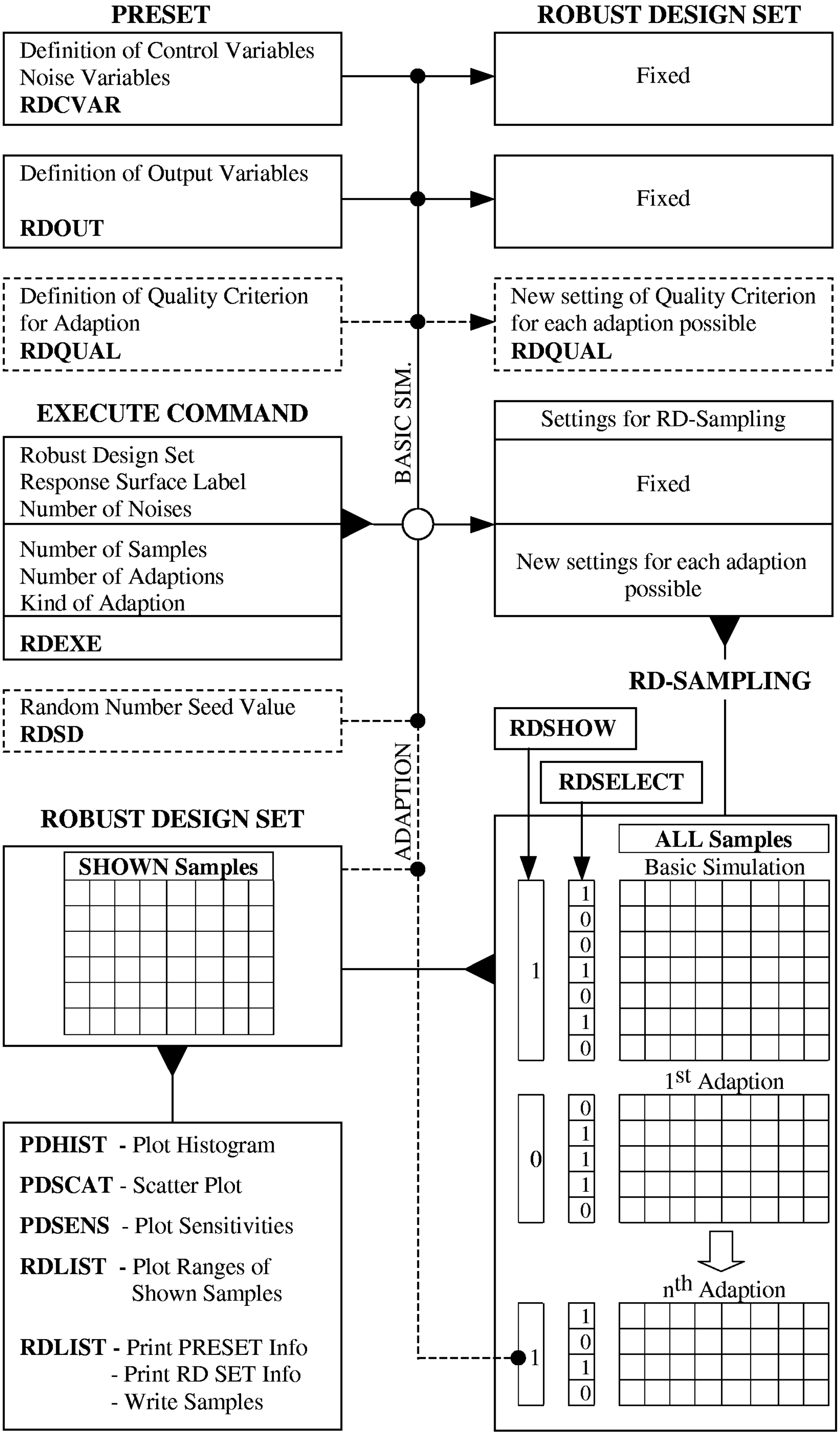

Figure 4.1: Overview on Commands and Structure

4.3.1 The “Preset”

First a Robust Design model has to be created. The control variables are selected from the defined random input variables by the RDCVAR-command. It is best to use uniform distributions as control variables for defining even distributed intervals in which the optimum solution can be searched. The other random variables are automatically interpreted as noise variables. They should be designed by a distribution type best describing the noise influence. The Robust Design output variables are defined as statistical values of the Probabilistic Design output parameters by the RDOUT-command. Mean value, standard deviation, S/N-ratio and other statistics are available. If it is intended to perform automatic adaptions a quality criterion has to be defined by the RDQUAL-command. This could be the minimum, the maximum or a target value of a certain Robust Design output parameter. These settings form the "PRESET"; they define the RD model and remain active for all the other Robust Design commands, until changed by the user.

4.3.2 The “Execute Command” - First Time

Everything is prepared to execute Robust Design sampling by the RDEXE-command. The user provides any label as “Robust Design Set” and assigns it to a response surface set by a “Response Surface Label”[1]. The number of noises to be examined for each control variable set and the number of samples is entered. With previously defined quality criterion any number of adaptions can be determined. If ’0’ or space is entered, only one block called “Basic Simulation” is performed. For each adaption a new block with the determined number of samples is created. When executing Robust Design sampling for the first time, only adaption type ’CONT’ is possible, which means that each adaption is based on the previous block.

4.3.3 The “Robust Design Set”

When executing a new Robust Design process, a data structure named “Robust Design Set” is created, in which all information of the “Preset” and the “Execute Command” is copied. The “Execute Command” starts an automatic procedure, which can be followed

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[1] Equivalent to the possibility for creating ten different response surface sets, the structure allows for ten different Robust Design sets at the same time.

by the large arrows in Figure 4.1. Using the data of the “Robust Design Set”, Robust Design sampling is performed and the corresponding samples are stored in the data structure, which is integrated in the “Robust Design Set” (lower right of Figure 4.1). The vectors containing ’0’ and ’1’ indicate that certain samples or adaptions are selected (’1’) or not selected (’0’). New blocks and samples are automatically selected. After this data structure is completed, all selected samples of selected blocks are copied in a structure named “Shown Samples”. All these processes are not visible for the user and run quickly if reasonable input has been chosen.

4.3.4 Data anaysis

The user receives text output about the actual status of samples and adaptions, which indicates the completion of the procedure. Now several tools are available for analyzing the “Shown Samples”. The PDHIST-command enables to plot histograms. The samples can be plotted in a “Scatter Plot” by the PDSCAT-command. Or sensitivities of output parameters can be examined by the PDSENS-command. These graphical tools are illustrated in chapter 5. For getting an overview on the results, it is always useful to print and plot the ranges of the shown samples by RDLIST,*“label”*,range. Furthermore the RDLIST-command provides several text outputs about the whole structure and enables to write the “Shown Samples” as table into a text file.

4.3.5 Selection of Adaptions and Samples

For selecting certain adaptions the RDSHOW-command is available at any time. Also samples matching a user-defined criterion can be selected or unselected by the RDSELECT-command. At first these commands only change the binary vectors and then the corresponding “Shown Samples” are created by an efficiently working routine. The actual ranges are automatically printed and plotted to visualize the result. RDSELECT always considers all samples independant if the refering adaptions are selected or not. This results in homogeneous selections through all adaptions, avoids data chaos and misinterpretation of results. Among other things these selection commands are useful for analyzing certain adaptions, zooming in regions with special interest[2] or for adaptions considering certain restrictions.

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[2] The graphical tools always scale to the ranges of the considered “Shown Samples”.

4.3.6 The “Execute Command” - Additional Adaptions.

Additional adaptions are possible at any time, by using the “Execute command” with the same label for the Robust Design set and an adequate adaption type. “Kind of adaption”=’NEW’ deletes the existing Robust Design structure and starts over with the data of the “Preset” and the “Execute Command”. Additional adaptions are produced by the attributes ’CONT’, ’SEL’ and ’PICK’. For producing samples, which are comparable to those of previous blocks, most data of the “Robust Design Set” is fixed. Except the quality criterion for adaption is always changeable by using the RDQUAL-command. Furthermore the number of samples, number of adaptions and kind of adaption can be varied. Depending on the used attribute the adaptions are either based on the last block or on the actual, defined “Shown Samples”. The additional blocks are always appended to the last, actual block. Information about the adaption algorithm is given in section 4.4.2.

4.3.7 Random Number Seed Value

For producing the same samples again or for assuring a complete new set of samples, the RDSD-command has been introduced. With this command the random number seed value can either be set to a user-defined value or to the system time. This value can be set at any time and is considered for the next sampling procdure of any “Robust Design Set”. If this command is not used, the seed value is initialized by the system time first and then automatically continues.

4.4 General Information about Internal Processes

4.4.1 Robust Design Sampling

For Robust Design sampling the Monte-Carlo simlation of Probabilistic Design has been adopted. The randomly generated samples are modified to meet the Robust Design layout. Always for the number of noises to be analyzed, identical control variable values are used as sample design. The Monte-Carlo simulation includes all defined response surfaces and the corresponding output values are calculated. These results are used to compute the Robust Design output parameters (statistical values) for all control parameter sets. The total number of samples on response surfaces can become very big, as it is the product of control sets and noise sets. For reducing the required memory, the Monte Carlo simulation has been broken up in many steps and only the effective Robust Design samples are saved in the data structure.

4.4.2 Adaption Types

For adaptions only the control variables are considered, as the noise variables characterize the noise of the system and can not be changed. The new adapted control variables are always created as uniform distributions in between new margins. Aim of adaptions is to find better solutions by increasing the sample density in the optimum region. Three different types of adaptions are available:

- The attribute ’CONT’ considers all samples of the last block. First the sample is identified, which meets best the actual, user-defined quality criterion. The corresponding input parameters describe the best actual control parameter set. The range of each new, adapted input random variable is reduced to a quarter size of the actual range below and above the best input value. If any new range exceeds the margins of the previous block, the interval is further decreased to this margin.

- The attribute ’SEL’ considers the “Shown Samples”. For the first adaption the actual ranges of the control variables are increased by 5% of the actual range size, or maximal to the ranges of the “Basic Simulation”. This creates more samples in the selected region and a better basis for further adaptions. The next automatic adaption with this attribute is equivalent to adaption type ’CONT’.

- The attribute ’PICK’ is also based on the “Shown Samples”. The first adaption reduces the intervals of the actual selection as ’CONT’, but the best sample is defined by the user who provides a sample identity number of the “Shown Samples”-table (created by RDLIST,*”label”*,SELECT) All further automatic adaptions are processed equivalent to adaption type ’CONT’.

These adaptions are relatively simple, but they are effective. For example, one ’CONT’-adaption in a four-dimensional space reduces the considered control parameter space at least to $0.5^4 = 6.25\%$ or increases sample density in this part by factor $\frac{1}{0.0625} = 16$. The adaptions are relatively restrictive with completely excluding all parameters, which are out of range. Therefore it is proposed to assure, that all adaptions are based on a sufficient high number of samples. An advantage of these restrictive adaptions is, that certain restrictions to output parameters can effectively be considered. This is demonstrated in the second and third example of chapter 5. Furtermore, by adding the functionality of processing adaptions to the Robust Design tool, the adaption routines can quickly be modified or new adaption algorithms can easily be added.

4.5 Command Syntax

This section shows the syntax of the implemented Robust Design commands including all available options. The commands “PDHIST”, “PDSCAT” and “PDSENS” have been adopted from the Probabilistic Design System[3] and extended for the use in combination with a Robust Design Set.

4.5.1 Command RDCVAR

RDCVAR, *Label, Attribute*

Definition of Control Variables

Label	*Label* defines the name of an input random variable.
Attribute	
ADD	Add input variable to the control variables.
DEL	Remove input variable from the control variables.

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[3] The command descriptions of the Probabilistic Design commands have been shortened and modified for this thesis.

4.5.2 Command RDOUT

RDOUT, *NewOutLab*, *Type*, *ExistOutLab*, *VAL*

Definition of Robust Design Output Parameters

NewOutLab	Name of Robust Design output parameter.
Type	*NewOutLab* will be defined as statistical value of *ExistOutLab*.
MEAN	Mean value
STDV	Standard deviation
MIN	Minimum
MAX	Maximum
CONF	Confidence value (→ *VAL*)
SNSB	S/N-ratio(smallest-is-best)
SNNB	S/N-ratio(nominal-is-best) (→ *VAL*)
SNBB	S/N-ratio(biggest-is-best)
DEL	Delete Robust Design output parameter *NewOutLab*. (no *ExistOutLab*).
ExistOutLab	Name of Probabilistic Design output parameter.
VAL	*VAL* is only to be defined in combination with **CONF** or **SNNB**. **CONF**: Confidence limit, $VAL \in \,]\,0\,;\,1\,[$. **SNNB**: Target value.

4.5.3 Command RDQUAL

RDQUAL, *RDName, RDOutLab, Criterion, NOMVAL*

Definition of a Quality Criterion for Adaption

RDName	No input assigns this command to the Preset. Or label of Robust Design Set.
RdOutLab	Name of Robust Design output parameter.
Criterion	
MIN	Minimum is set as aim.
MAX	Maximum is set as aim.
NOM	A user–defined target value is set as aim (→ *NOMVAL*).
OFF	Criterion for this output parameter is switched off.
NOMVAL	NOMVAL defines the target value and is only to be used in combination with **NOM**.

4.5.4 Command RDEXE

RDEXE, *RDName, RSSetLabel, SETS, NOISES, ADAPT, Type, ID*

Execution of Robust Design Simulation

RDName	Label of Robust Design Set.
RSSetLabel	Label of Response Surface Set on which Robust Design sampling is performed.
SETS	Number of Robust Design samples.
NOISES	Number of simulated noise combinations for each Robust Design sample.
ADAPT	*ADAPT* defines the number of adaptions to be processed. Default is ’0’ = no adaptions, only basic simulation.
Type	
NEW	A complete new simulation based on the Preset is processed (default).
CONT	The simulation is based on the last adaption and will be attached to it.
SEL	The simulation is based on the actual selection of samples and adaptions. It will be attached to the last adaption.
PICK	Based on the actual selection of samples, the user has to define one certain sample out of the actual sample table, towards which the simulation should be processed.
ID	*ID* is a user-defined number of a sample and is only to be used in combination with adaption type ’PICK’.

4.5.5 Command RDSHOW

RDSHOW, *RDName, Type, AD1, AD2*

Selection of Adaptions for further Data Analysis

RDName	Label of Robust Design Set
Type	
CLE	Unselect all adaptions (no *AD1*, *AD2*).
ADD	Add adaption(s) to actual selection (*AD2* is optional).
DEL	Remove adaption(s) from actual selection (*AD2* is optional).
ONLY	Unselect all adaptions and then select adaption(s) (*AD2* is optional).
ALL	Select all adaptions (no *AD1*, *AD2*).
AD1	Number of adaption. ’0’=basic simulation, ’1’=first adaption, ’2’=second adaption ...
AD2	If *AD2* is entered, then all adaptions from *AD1* to *AD2* are considered.

4.5.6 Command RDSELECT

RDSELECT, *RDName, RDParLab, Operate, Cond, LIMIT1, LIMIT2*

Selection of Samples for further Data Analysis

RDName	Label of Robust Design Set.
RDParLab	Label of input or Robust Design output parameter
Operate	
ALL	Choose all samples.
ADD	Add samples, which match user-defined condition (*Cond*).
DEl	Remove samples, which match user-defined condition (*Cond*).
CLE	Unselect all samples.
Cond	
SMAL	Smaller as ...
GREA	Greater as ...
INTE	Interval from ... to ...
LIMIT1	Limit value for **SMAL** and **GREA**. Lower limit value for **INTE**.
LIMIT2	Upper limit value for **INTE**.

4.5.7 Command RDSD

RDSD, *RDSeed*

Set Seed Value for Robust Design Simulation

RDSeed

TIME or -2	The system time will be used to initialize seed value.
INIT or -1	123457 will be used as seed value.
CONT or 0	Continue with seed value as it is. Same effect as not using this command.
> 0	This number will be used as seed value.

4.5.8 Command RDLIST

RDLIST, *RDName, Par1*

Print Out Information in ANSYS Text Window or in File.

RDName	No input assigns this command to the Preset (in combination with **CVAR, QUAL, OUTV, ALL**). Or label of Robust Design Set (any combination).
Par1	
SELE	Write selected samples as table to text file “rd_select.rd”.
CVAR	Print out the actual setting of control/noise variables.
QUAL	Print out the actual set quality criterion.
OUTV	Print out the defined Robust Design output parameters.
EXEC	Print out information about all processed Robust Design simulations.
RANGE	Show the range of the actual selection for each parameter. Plot the actual ranges in diagram.
ALL	Process all attributes except **SELE**.

4.5.9 Command PDHIST

PDHIST, *RDName, ParName, NCL, Type*

Plots the Frequency Histogram.

RDName	Label of Robust Design Set (or Result Set label of Probabilistic Design).
ParName	If a Robust Design Set label is typed in, *ParName* could be any input parameter or Robust Design output parameter.
NCL	Number of bars shown in the histogram. If this field is left blank, an adequate number of bars is automatically determined.
Type	Type of histogram.
ABS	Absolute frequency histogram.
REL	Relative frequency histogram (default).
NORM	Normalized frequency histogram.

4.5.10 Command PDSCAT

PDSCAT, *RDName, Name1, Name2, Type, ORDER, NMAX*

Plots a Scatter Graph.

RDName	Label of Robust Design Set (or Result Set label of Probabilistic Design).
Name1, Name2	If a robust design simulation is considered, any input parameter or robust design output parameter can be entered as parameter label. The parameter data for Name1 is shown on the X-axis and the parameter data for Name2 is shown on the Y-axis in the plot.
Type	Keyword for the type of trendline curve.
POLY	Polynomial trendline (default).
NONE	A trendline is not plotted.
ORDER	Order of the polynomial trendline (only in combination with **POLY**.)
NMAX	Maximum number of points plotted in the scatter plot. Default **NMAX**=10000.

4.5.11 Command PDSENS

PDSENS, *RDName, ParName, Chart, Type, SLEVEL*

Plots the Probabilistic Sensitivities.

RDName	Label of Robust Design Set (or Result Set label of Probabilistic Design).
Name1, Name2	If a robust design simulation is considered, any robust design output parameter can be entered as parameter label.
Chart	Keyword for the type of chart to be plotted.
BAR	Bar chart of the absolute sensitivities.
PIE	Pie chart of relative and normalized sensitivities.
BOTH	Both pie and bar charts plotted side by side (default).
Type	Keyword for the type of chart to be plotted.
RANK	Spearman rank-order correlation coefficient (default).
LIN	Pearson linear correlation coefficient.
SLEVEL	Significance level. The value for the significance level must be between 0.0 and 1.0 and it defaults to 0.025 (2.5%).

Chapter 5

Examples

5.1 Compressed Air Cooling System

5.1.1 Reference

This is a principal example of a simple heat-exchanger design for costs. It is adopted from the publication “Design for cost and quality: The Robust Design approach” [15]. Originally it comes from Stoecker [13].

5.1.2 System

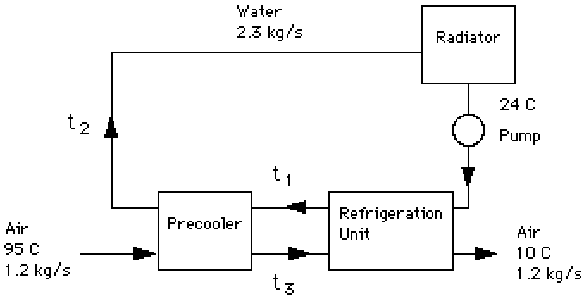


Figure 5.1: Compressed Air Cooling System [15]

Figure 5.1 shows the main units of the heat exchange system. The control variables are three temperatures (t_1, t_2, t_3) between those units. And there are three noise variables (N_1, N_2, N_3). N_1 is a noise parameter for the costs of the refrigeration unit, N_2 is the noise of the radiator output temperature and N_3 defines the input temperature of compressed air. The system and the model for the costs is reduced to three equations given in section 5.1.3. In the referenced paper all control variables are examined on three different levels and the noise variables on two levels. There the problem is solved with a L_9xL_4 crossed array experiment like in Figure 2.5 with the exception, that there is no fourth column in the first array, as there are only three control variables. The number of experiments remains 36. The objective is to find the most robust solution at the minimum level of costs.

5.1.3 From Analysis File to Response Surfaces

For computing this example by Robust Design in ANSYS, first an appropriate analysis file had to be created. The subroutine “cooling.mac” contains the cost function of the cooling system introduced in the referenced paper. The analysis file could contain any command understood by ANSYS. Several functionalities are provided for typing mathematical equations[1], so this input can easily be understood without knowing special syntax. XGES defines the total costs as sum of X1=“cost of refrigeration unit”, X2=“cost of pre-cooler” and X3=“cost of radiator”. a, b, c are given cost parameters.

```
!Analysis File subroutine: cooling.mac

a = 48
b = 50
c = 25
N1 = a
N2 = 24
N3 = 95
t1 = 28
t2 = 39
t3 = 35

X1 = 1.20*N1*(t3 - 10)
X2 = 1.20*b*(N3 - t3)/(t3 - t1)
X3 = 9.637*c*(t2 - N2)

XGES = X1+X2+X3
```

The main routine of the analysis file “pdscool.mac” first reads in the subroutine “cooling”. Then “/PDS” changes the ANSYS mode to Probabilistic Design. The command “PDANL” defines the name of the analysis file to be looped for simulation. The crossed array experiment in the paper discusses following levels:

---

[1] Described under the *SET-command of ANSYS-documentation

$$\begin{aligned} t1 &= (25;28;31) & N1 &= (48;56) \\ t2 &= (36;39;42) & N2 &= (24;27) \\ t3 &= (35;38;41) & N3 &= (95;100) \end{aligned}$$

For the analysis in ANSYS the control variables and the noise variables are defined as uniform distributed random variables using the margin levels. This is done by the PDVAR-command.

```
! Analysis File main: pdscool.mac
cooling
/PDS
PDANL,cooling,mac
PDVAR,t1,UNIF,25,31
PDVAR,t2,UNIF,36,42
PDVAR,t3,UNIF,35,41
PDVAR,N1,UNIF,48,56
PDVAR,N2,UNIF,24,27
PDVAR,N3,UNIF,95,100
```

Corresponding to chapter 3 the following steps are made:

Definition of Random Output:

```
Random Output > add > XGes
```

Definition of Simulations:

```
Monte Carlo Sims > Latin Hypercube > “Num. of Simulations:  36”
```

Run Simulations:

```
Run > Run Serial > “label=coolsol”
```

Save Simulations:

```
Save > “coolsav”
```

Fit Response Surface:

```
Fit Resp Surf > “label=coolrs (of coolsol), Quadratic w. Xterms,
Box-Cox Transf., No Filtering”
```

Text Output of Fitting:

```
Regression Analysis of Output Parameter XGES
============================================

Response Surf Set Label= COOLRS
Solution Set Label     = COOLSOL
Simulation Method      = Monte Carlo with Latin Hypercube Sampling
Regression Model       = Quadratic with crossterms
Results Transformation = Automatic Box-Cox Transformation
```

```
Filtering Input Terms  = None

Regression Equation for XGES:
----------------------------

Y* = (Y^lambda-1)/lambda  with lambda = 0.660000
TERM         COEFFICIENT
Constant    1.48441e+003
T1          7.50573e+000
T2          1.04283e+001
T3         -1.44832e+001
N1          5.03032e+000
N2         -1.92426e+001
N3         -2.38940e+001
T1 * T1     3.86790e-001
T2 * T2    -1.46429e-001
T3 * T3     2.66699e-001
N1 * N1    -6.60473e-002
N2 * N2    -2.01389e-001
N3 * N3     8.60889e-002
T1 * T2     1.54848e-002
T1 * T3    -5.59098e-001
```

< shortened > : $1+6+\frac{6\cdot 7}{2}=28$ terms (p. 30)

```
Comparison of Sampled and Approximated Output Values:
-----------------------------------------------------

 NO.   SAMPLED VALUE   APPROX. VALUE        RESIDUAL
   1   5.384450e+003   5.385023e+003  -5.730673e-001
   2   5.771116e+003   5.770242e+003   8.746468e-001
   3   4.741556e+003   4.742041e+003  -4.844830e-001
```

< shortened > : 36 simulations

```
  34   5.417097e+003   5.418198e+003  -1.101047e+000
  35   5.983870e+003   5.985124e+003  -1.253979e+000
  36   4.506132e+003   4.505351e+003   7.810245e-001

Goodness of Fit Measures:
-------------------------

Error Sum of Squares. . . . . . . . . . .  6.433208e+002
Coefficient of Determination (R-squared).  9.999106e-001
Maximum Residual (Absolute) . . . . . . .  1.036916e+001
```

As the “Coefficient of Determination” is very near to ’1’ and the “Maximum Residual” is approximately $\frac{1}{5000}$ of the absolute values, this can be considered as a very good fit. But as it was intended to use the same number of experiments as in the reference, the number of simulation is relatively low for gaining appropriate information by the goodness-of-fit: $\frac{36}{28}=1.29<1.5$.

5.1.4 Robust Design Sampling

Now it could directly be continued with Robust Design or after a new start of ANSYS, only the first two lines have to be uncommented, so that the saved file is read in and the response surface is created. Then the control variables (t1, t2, t3) are selected from the currently defined random variables by the RDCVAR-command[2]. The remaining random variables will automatically be understood as noise variables. Then the Robust Design output parameters are defined as statistical parameters from the total costs XGES by the RDOUT-command.

```
! PDRESU,’coolsav’,’pds’,’ ’
! RSFIT, ,COOLSOL,XGES,QUAX,BOX,0.01,NONE,0

rdcv,t3,add
rdcv,t2,add
rdcv,t1,add

rdout,cost_m,mean,xges
rdout,cost_std,stdv,xges
rdout,cost_80,conf,xges,0.80
rdout,cost_95,conf,xges,0.95
rdout,snsb,snsb,xges
```

Now all is set to perform the basic simulation of Robust Design sampling. After each command the corresponding text output was given. An overview on the data of the “Preset” can always be requested by: `rdli, ,all`

```
 RANDOM VARIABLES -- Preset

   Control Variables : 3
     T1
     T2
     T3

   Noise Variables : 3
     N1
     N2
     N3


 QUALITY CRITERION FOR ADAPTION -- Preset

   No criterion defined.
```

---

[2] In ANSYS generally the first four letters of the command itself are sufficient.

```
 ROBUST DESIGN OUTPUT PARAMETERS -- Preset

   COST_M   : Mean Value             XGES
   COST_STD : Standard Deviation     XGES
   COST_80  : Confidence Value       XGES -> 80 %
   COST_95  : Confidence Value       XGES -> 95 %
   SNSB     : S/N,'smallest-is-best' XGES
```

The following setting of the command RDEXE executes a Robust Design Set named “rd” on Response Surface Set “coolrs” with 2000 samples and 300 noise variations per sample. Number of adaptions is ’0’. The actual data from the “Preset” is automatically used for each new Robust Design set.

```
rdexe,rd,coolrs,2000,300,0,new
```

Based on this basic simulation further adaptions could be processed. Objective is to minimize cost_95, the 95%-confidence value of the costs. The next RDEXE-command produces three[3] adaptions with 500 samples each. “cont” indicates that the last block, here the basic simulation will be chosen for adaption.

```
rdqual,rd,cost_95,min
rdex,rd,coolrs,500,300,2,cont
```

Now automatically all samples are selected and used for three adaptions to maximize SNSB, S/N-ratio “smallest-is-best” of the costs : “sel” indicates that that the actual selection will be chosen for adaption.

```
rdqual,rd,snsb,max
rdex,rd,coolrs,500,300,2,sel
```

So altogether 7 Blocks (No. 0-6) of Samples have been generated. [4]

5.1.5 Data Analysis

An overview on the data of Robust Design set “RD” is given on the next page `(rdli,rd,all)` followed by several graphical outputs, which have been selected to visualize the result.

---

[3] One simulation is always made for ’0’, so here ’2’ equals three adaptions

[4] It is always possible and probably the usual way to analyze the data after each simulations when working interactive with the program.

```
 RANDOM VARIABLES -- Result Set RD
```

< shortened >

```
 QUALITY CRITERION FOR ADAPTION -- Result Set RD

   Maximum of Output Variable SNSB


 ROBUST DESIGN OUTPUT PARAMETERS -- Result Set RD
```

< shortened >

```
 ROBUST DESIGN SAMPLING -- Result Set RD
 Response Surface Set COOLRS

   Block Nr.   ON/OFF       Samples      Selected         Shown
           0       ON          2000          2000          2000
           1       ON           500           500           500
           2       ON           500           500           500
           3       ON           500           500           500
           4       ON           500           500           500
           5       ON           500           500           500
           6       ON           500           500           500
   ------------------------------------------------------------
   SUM:                        5000          5000          5000

   Noise Variations per Sample  :  300



 RANGE OF SHOWN SAMPLES -- Result Set RD

   T1       : [  25.000493  ,  30.998501  ]
   T2       : [  36.000113  ,  41.994841  ]
   T3       : [  35.000767  ,  40.997922  ]

   COST_M   : [  4481.8352  ,  6273.3447  ]
   COST_STD : [  203.73577  ,  242.63951  ]
   COST_80  : [  4672.7502  ,  6492.0049  ]
   COST_95  : [  4802.5416  ,  6629.5652  ]
   SNSB     : [ -75.955544  , -73.039383  ]

   Number of shown samples: 5000
```

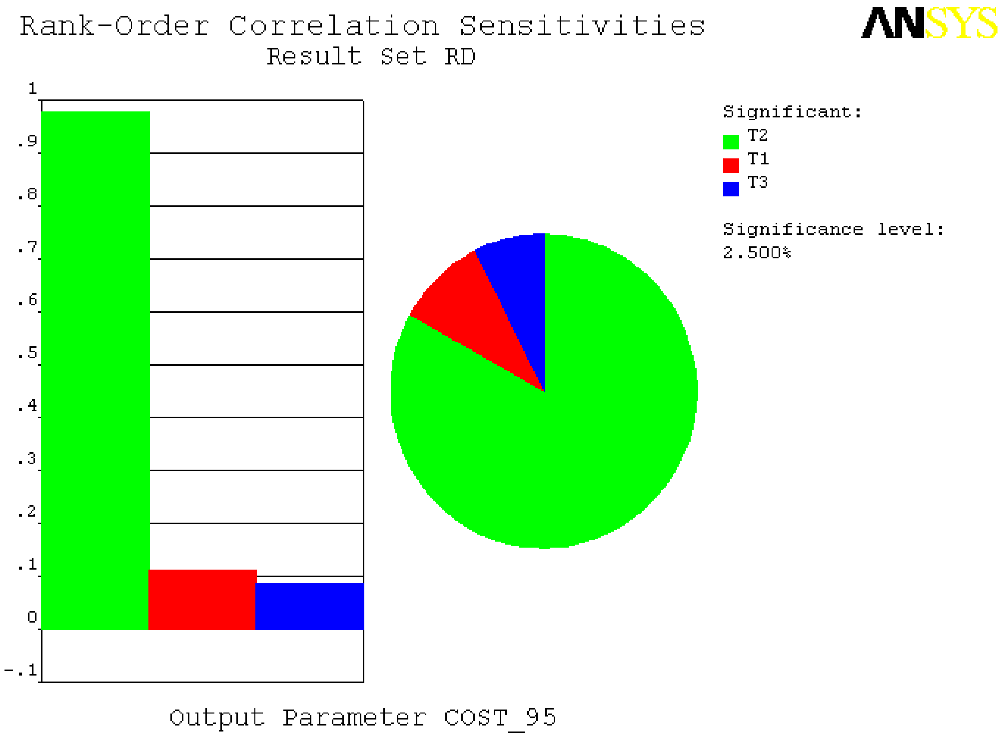


Figure 5.2: Sensitivities of COST_95, Basic Simulation.

```
Rank-Order Sensitivity of Parameter COST_95 wrt Random Input Variables
======================================================================

Solution Set Label    = RD
Simulation Method     = Monte Carlo with Latin Hypercube Sampling
Number of Samples     = 2000

The following sensitivities have been found to be significant:

Rank  Random Input  Sensitivity    Normalized
   1  T2            9.7752e-001       83.14%
   2  T1            1.1209e-001        9.53%
   3  T3            8.6079e-002        7.32%

NO sensitivities have been found to be insignificant!
(based on a significance level of   2.500%
```

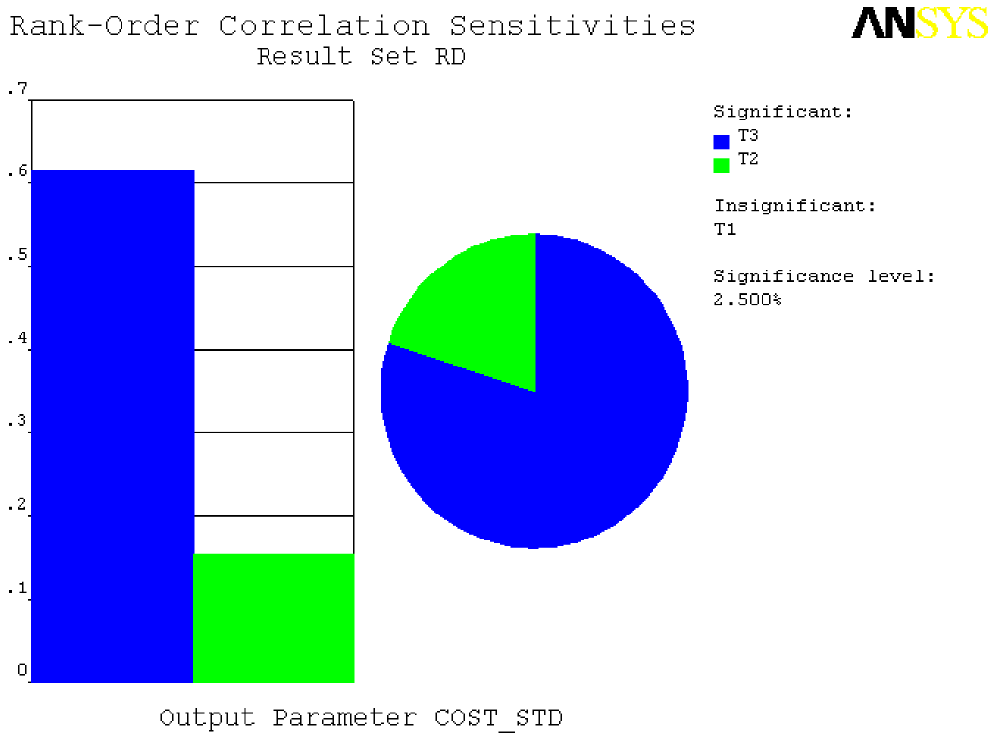


Figure 5.3: Sensitivities of COST_STD, Basic Simulation.

```
Rank-Order Sensitivity of Parameter COST_STD wrt Random Input Variables
=======================================================================

Solution Set Label    = RD
Simulation Method     = Monte Carlo with Latin Hypercube Sampling
Number of Samples     = 2000

The following sensitivities have been found to be significant:

Rank  Random Input  Sensitivity    Normalized
   1  T3            6.1499e-001       80.06%
   2  T2            1.5316e-001       19.94%

The following sensitivities have been found to be NOT significant:
(based on a significance level of   2.500%)

Random Input  Sensitivity
T1            1.6599e-002
```

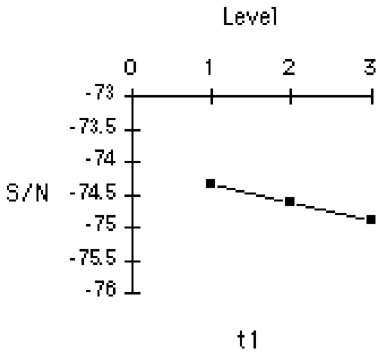


Figure 5.4: t1-S/N-Diagram [15]

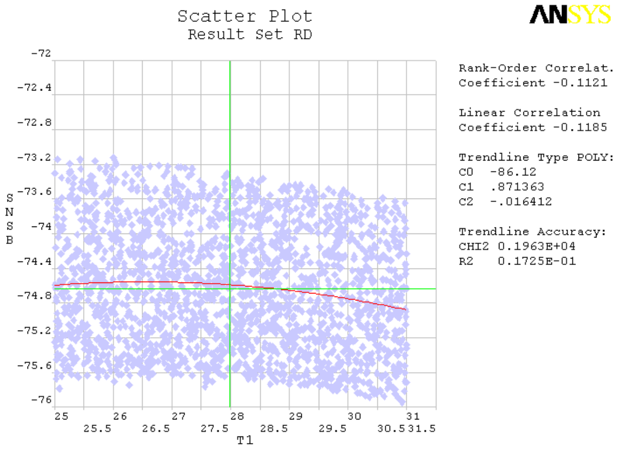


Figure 5.5: t1-S/N-Diagram, ANSYS

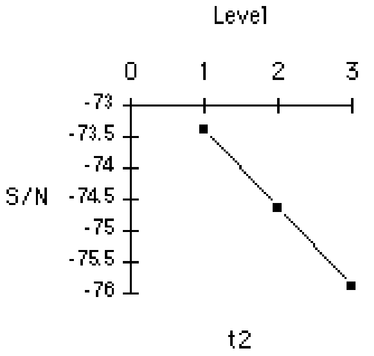


Figure 5.6: t2-S/N-Diagram [15]

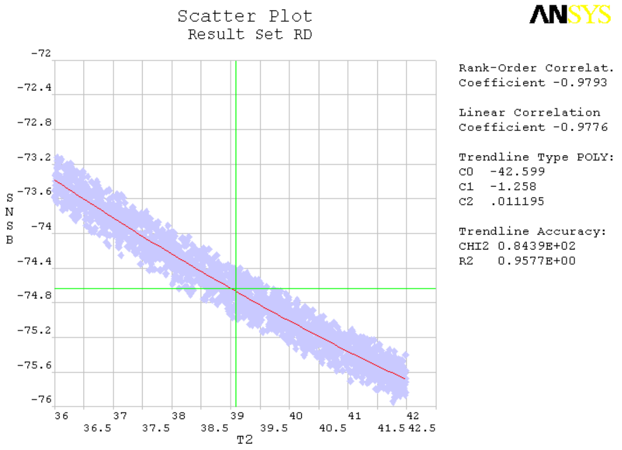


Figure 5.7: t2-S/N-Diagram, ANSYS

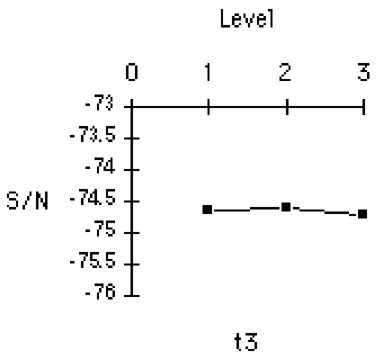


Figure 5.8: t3-S/N-Diagram [15]

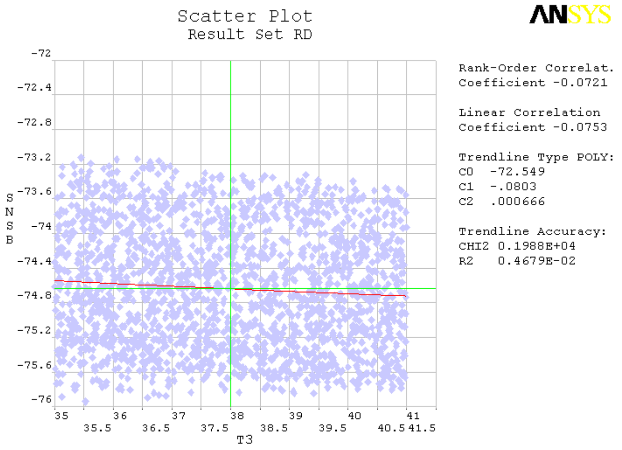


Figure 5.9: t3-S/N-Diagram, ANSYS

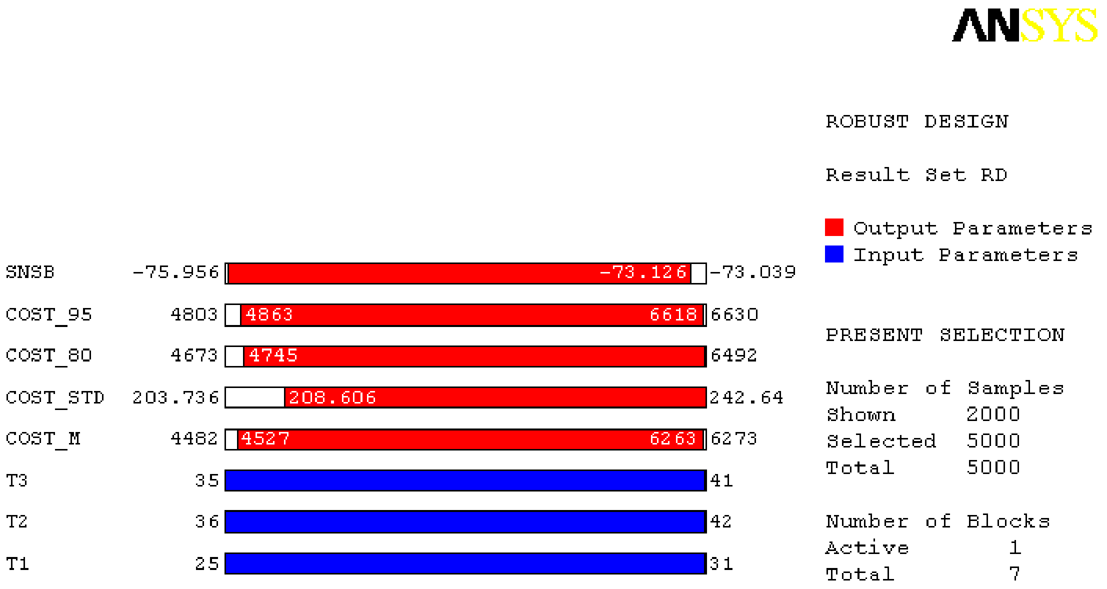


Figure 5.10: Ranges of all samples, Basic Simulation (Adaption '0').

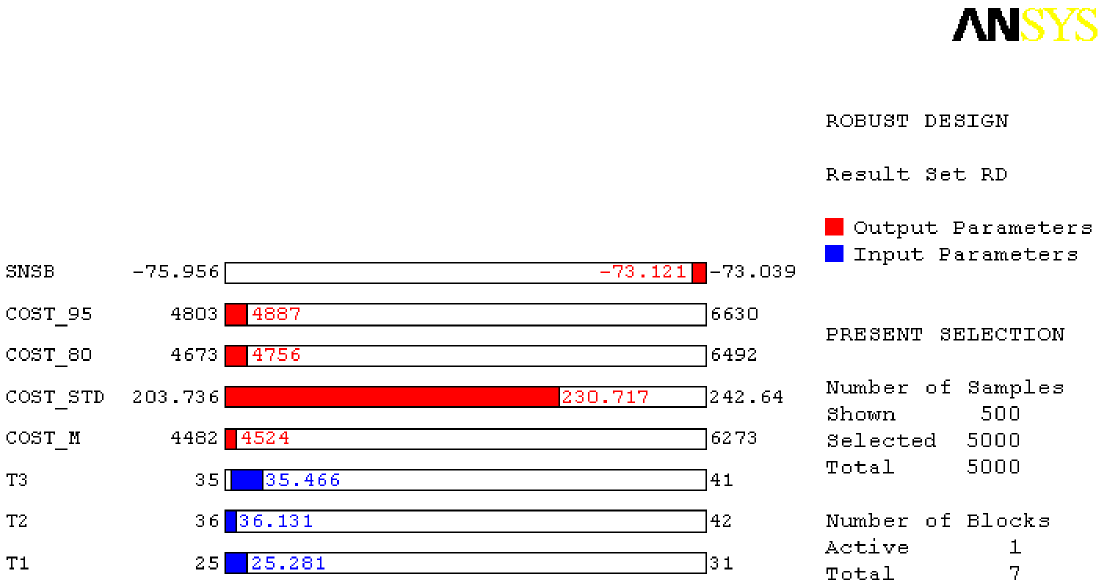


Figure 5.11: Ranges of all samples, Adaption '3'.

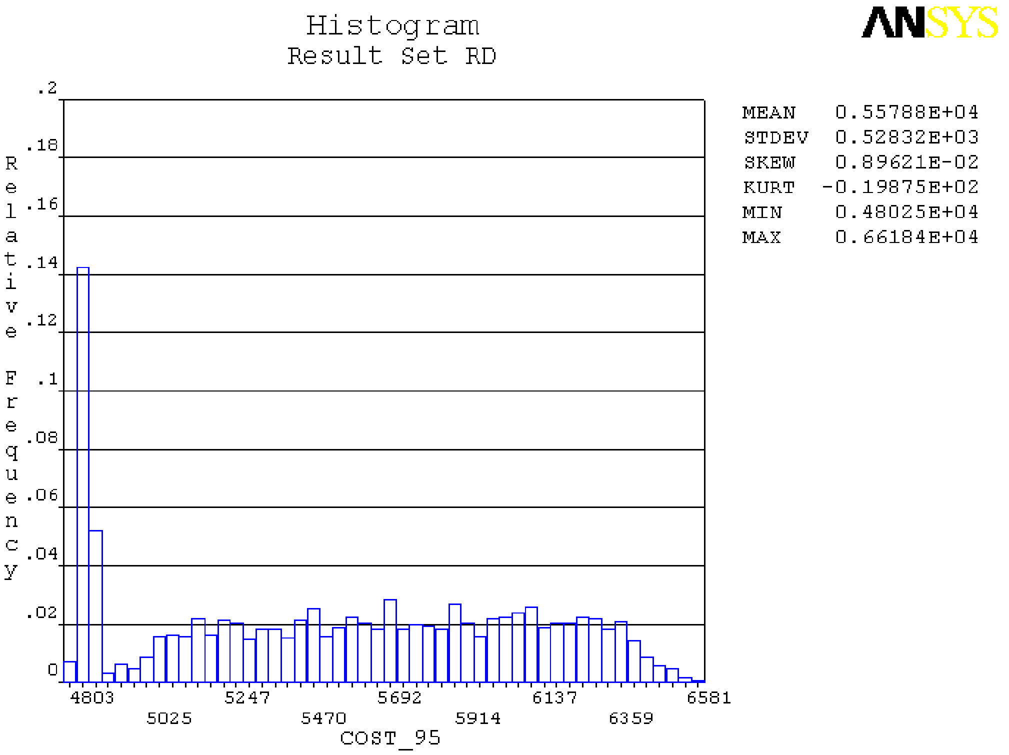


Figure 5.12: Histogram COST_95, Basic Simulation and Adaption ’3’.

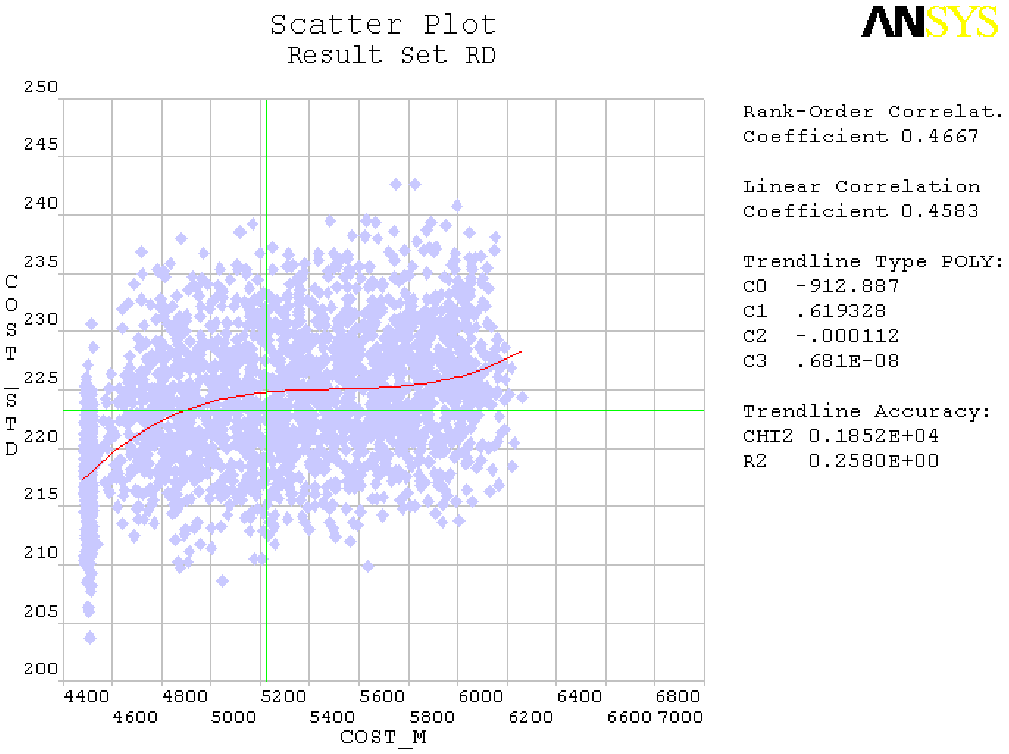


Figure 5.13: Mean and standard deviation of costs, Basic Simulation and Adaption ’3’.

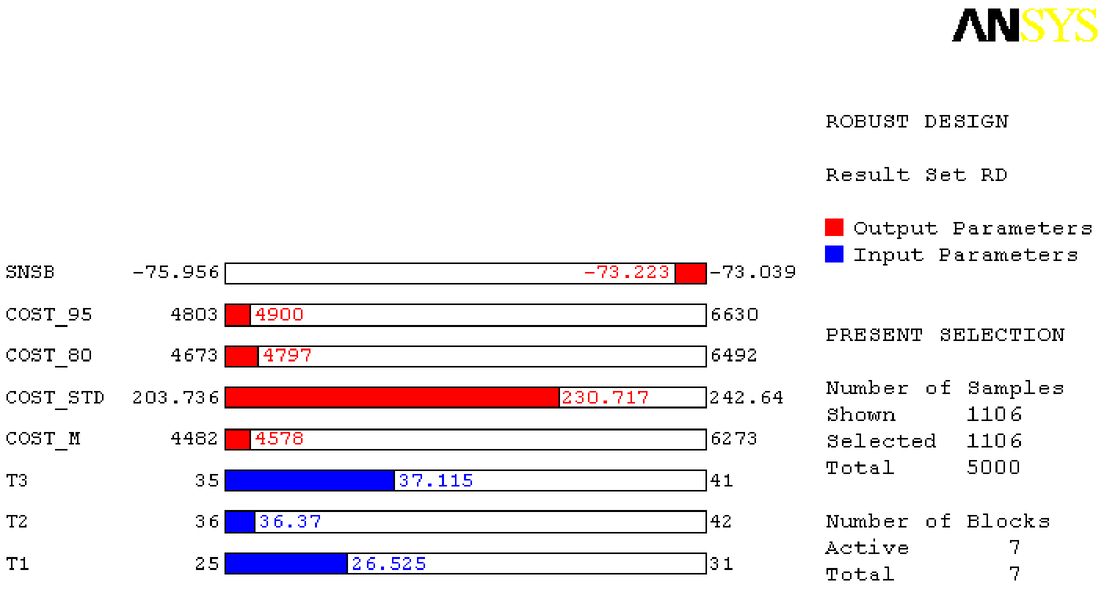


Figure 5.14: Selection on all samples of all blocks, COST_95<4900

Command: `rdsel,rd,cost_95,del,greater,4900`

```
ROBUST DESIGN SAMPLING -- Result Set RD
Response Surface Set COOLRS

  Block Nr.   ON/OFF      Samples      Selected        Shown
          0       ON         2000             4            4
          1       ON          500            35           35
          2       ON          500           260          260
          3       ON          500           500          500
          4       ON          500             1            1
          5       ON          500            27           27
          6       ON          500           279          279
  ------------------------------------------------------------
  SUM:                       5000          1106         1106

  Noise Variations per Sample  :  300
```

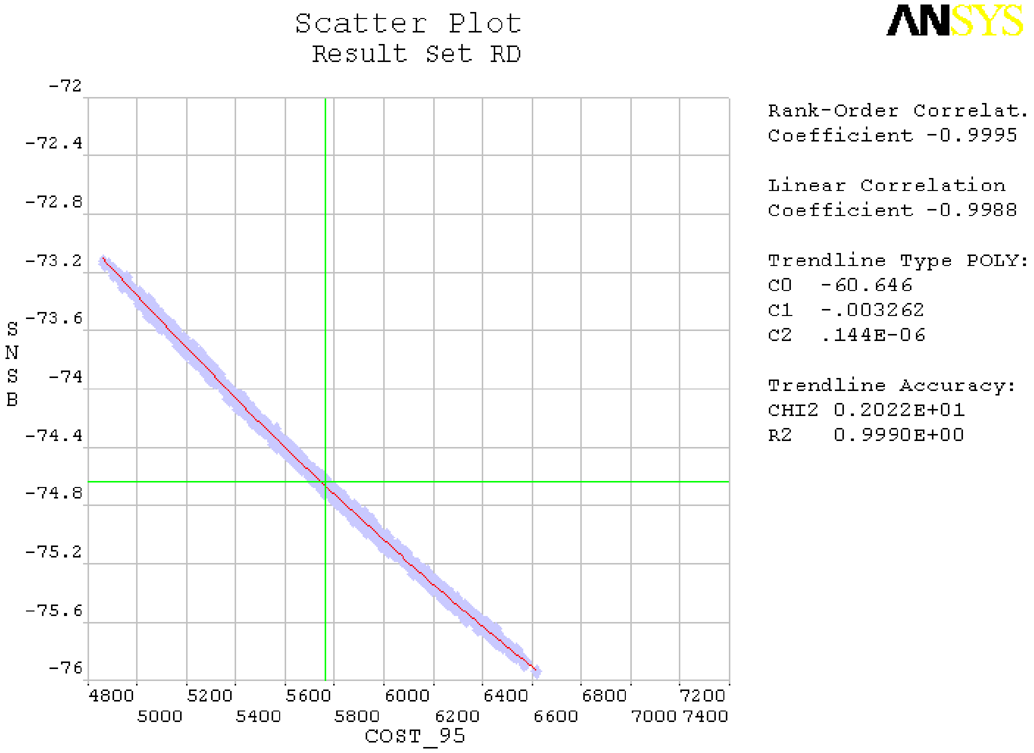


Figure 5.15: Correlation between Confidence Value and S/N-ratio.

Figures 5.2, 5.3	Sensitivities of COST_95 (95% confidence value) and COST_STD (standard deviation). These diagrams are used to identify the significant parameters.
Figures 5.4–5.9, page 59	The left column contains graphs extracted from the referenced paper [15] and on the right side the equivalent graphics from Robust Design of ANSYS are shown. These diagrams were created for seperating out the effect of each control factor on the S/N-ratio. As Robust Design of ANSYS uses continuous distributions and the reference example only examines the margin values, there are minor differences in the graphs. But none of them could be declared as wrong or right, as both methods analyze different models.

Figures 5.10–5.11

These graphics are produced by `RDLIST,RD,RANGE`. They will also be shown for any new selection by `RDSHOW` or `RDSELECT`. All exact ranges can always be found in the text window. Here for example all samples of the Basic Simulation and below those of Block No. 3 are shown. It is obvious, that the adaption has been very effective for producing more samples in the region of interest.

Figures 5.12,5.13

The effect of the adaption can also be estimated in the histogram for COST_95. Besides a wide range of many samples between 5000 and 6500, there is a concentration on the lower margin between 4800 and 4900.

This is even more visible in the scatter plot below. The wide cloud of samples shows the Basic Simulation and further there are many samples with reduced COST_M (mean value of costs) and reduced COST_STD (standard deviation of costs) in the lower left.

Figure 5.14,
page 62

This graphic shows the ranges of those samples, where the 95% confidence value of the costs is below 4900. It can be seen, that the input parameter values must be in between certain ranges to meet this criterion. The text output on this page shows how many samples from each adaption are selected.

Block number three and six show a significant difference in hits into this region, although they are both third adaptions. This can be explained, as the adaptions with “CONT”-option (Block 1,2,3) starts with a concentrated search, whereas the “SEL”-option first produces more samples in the selected region (Block 4) before concentrating (Block 5,6).

Figure 5.15 The scatter plot shows a very strong correlation between confidence value and S/N-ratio. That can be understood, as both are quality measures for Robust Design considering mean value and standard deviation at once. But this should not mislead to the opinion that this correlation must always appears like this. As these measurements are based on a different analysis, this diagram can look different for other problems.

5.1.6 Best Solutions

The optimum levels from the referenced paper can be picked from Figures 5.4, 5.6, 5.8.

t1 = 25 (Level 1)
t2 = 36 (Level 1)
t3 = 38 (Level 2)
S/N-ratio = -73.19
Mean value of costs = 4551
Std. deviation of costs = 445.4

As explained on page 63 this values could not directly be compared to the solution of ANSYS with using continuous distributions. Depending on the quality measure there are two best solutions from Robust Design of ANSYS:

t1 = 25.045
t2 = 36.003
t3 = 35.085
S/N-ratio = -73.039
Mean value of costs = 4482
Std. deviation of costs = 218.4
95% Confidence value = 4824

t1 = 25.234
t2 = 36.012
t3 = 35.145
S/N-ratio = -73.055
Mean value of costs = 4490
Std. deviation of costs = 214.5
95% Confidence value = 4802

The sample with the highest S/N-ratio (left) and the sample with the lowest 95% confidence value (right). Although the difference is very small these two measures declare different settings from the same sample data as best solutions. And it is not clear, which one is to be prefered. The sample with the highest S/N-ratio shows a lower mean value but a higher standard deviation. So compared to the other solution the costs will on an average be slightly lower. But the solution with the lowest 95% confidence value indicates a more stable result and gives more certainty, that the costs will not exceed a certain limit.

So these two measurements do not exactly carry the same philosophy. But with the temperature settings of these two solutions being very close, this is more an interesting aspect than really pointing to another solution. But there can be a more significant difference in other problems.

Generally with getting the same dimension of costs and similar graphs for the main effects of control parameters, it can be assumed, that the programmed code principally works. The results have also been checked explicitely for a small number of samples.

5.2 Braced Bending Beam

5.2.1 Idea

This example discusses a structure of a bending beam combined with two tension members. Corresponding to the principle of quality engineering the idea is to minimize the costs of the structure while achieving most stable costs and structure quality. It is not the same as minimizing the weight of the structure, as there are different material prices for the beam and the tension members. And it is different to finding the optimum geometry, because another geometry might still be strong enough but significantly reduce costs. Most probably the Robust Design solution will be near to these other optimum solutions, but the method is the design for cost and quality. Structural and economical criterions are considered in the design.

5.2.2 System

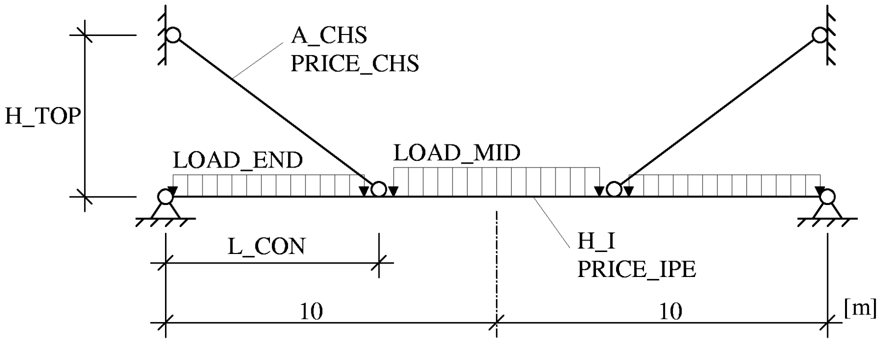


Figure 5.16: Static System

Figure 5.16 shows the static system. The diagonal tension members are defined as circular hollow sections (CHS) and the beam is an I-profile. For a realistic minimization of the costs standard sections should be used for the bending beam, as tailor-made profiles are much more expensive. Therefore an I-profile of type IPE[5] has been selected. As this profile is only delivered in certain dimensions, the solution for the height of the I-beam (H_I) will be fixed to discrete levels. Whereas the great variety of circular hollow sections allows a continuous consideration of cross-sectional area (A_CHS). The geometry of the

---

[5] Standardized I-profiles from the German industry norm DIN 1025 and EURONORM 19-57.

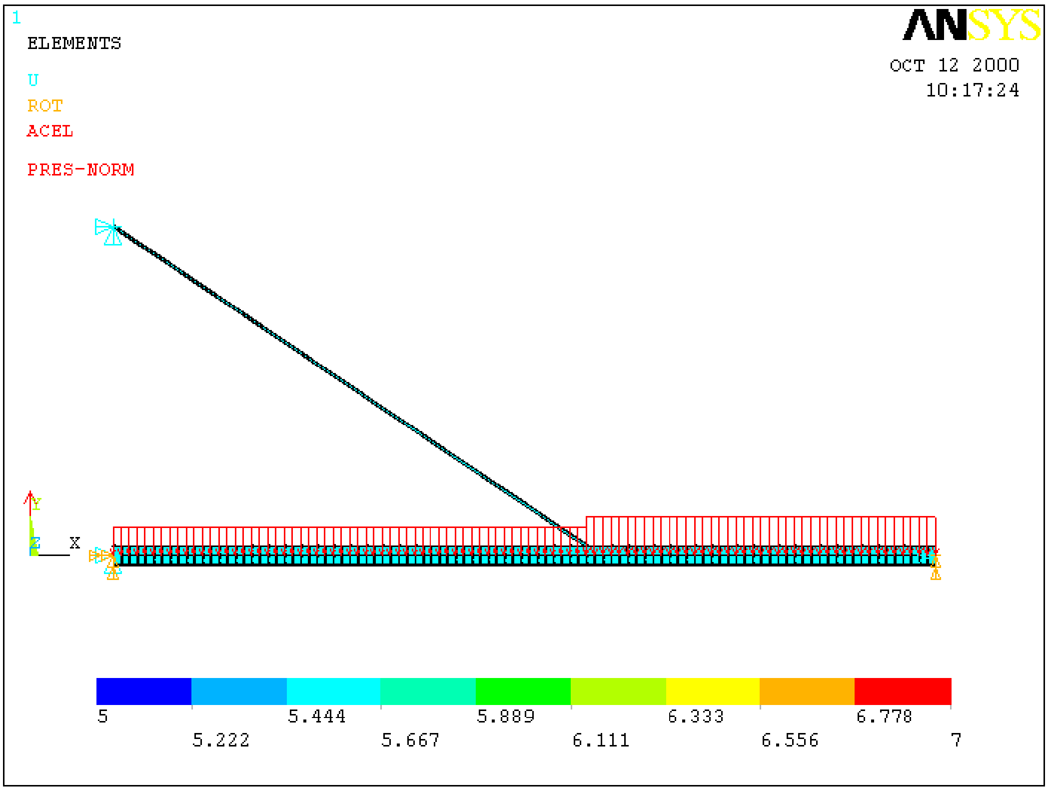


Figure 5.17: Static System

system is defined by L_CON and H_TOP. With still being in the design process the material prices for the members (PRICE_CHS, PRICE_IPE) could not exactly be determined and describe the noise of the system. Other noise parameters are the designed loads in the middle segment and the end segments of the beam (LOAD_MID, LOAD_END). Altogether this is a system of eight input parameters. For determination of invalid structures and as valuation criteria of the results the following output variables were defined:

C_CHS:	Cost of CHS	DEFLECT:	Max. Deflection	W_CHS:	Weight of CHS
C_IPE:	Cost of IPE	STRESS:	Max. Stress of IPE	W_IPE:	Weight of IPE
C_TOT:	Total Costs	TENSION:	Max. Stress of CHS	W_TOT:	Total Weight

5.2.3 From FEM Model to Response Surfaces

Taking advantage of symmetry only half of the structure is built as FEM model (Figure 5.17). The bending beam is created by 100 elements of type BEAM188 and for the tension member only one LINK8-element is sufficient. The tension member is connected to

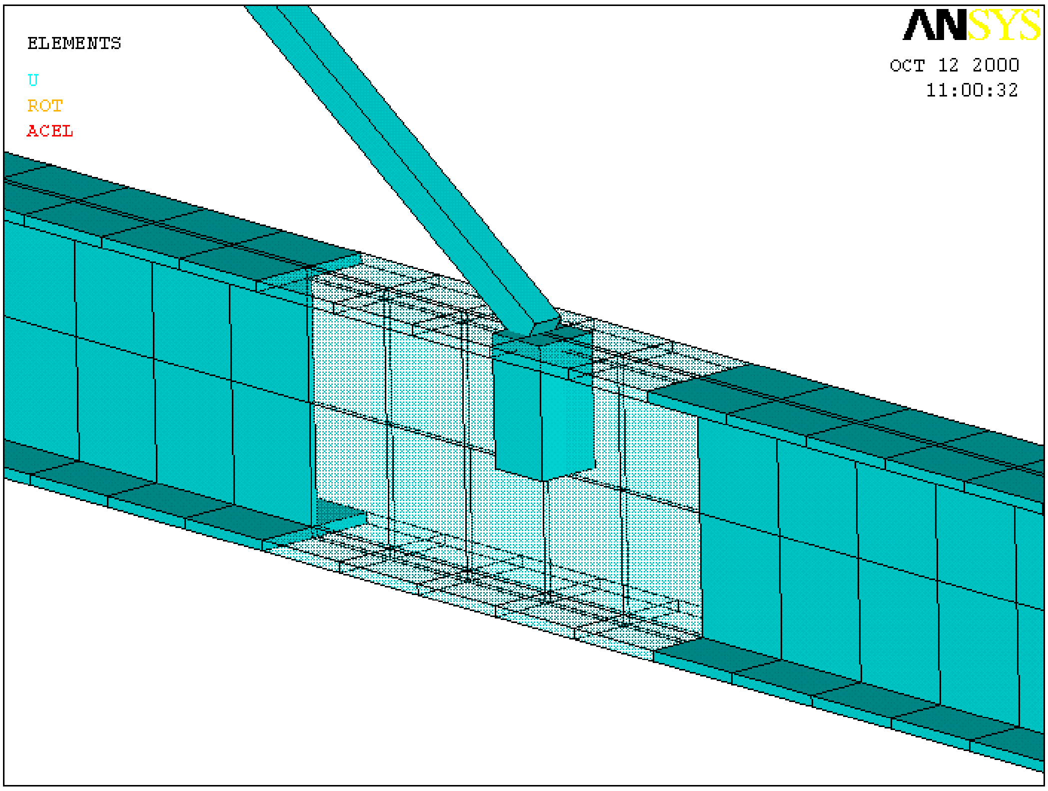


Figure 5.18: Static System Detail

the beam by a BEAM4-element (Figure 5.18). The location of the connection is defined by L_CON. To achieve any position without influencing the result, the next node is always moved to the exact position and the two surrounding nodes are put on distance of 0.1m. All the other nodes are uniformly distributed around. With Probabilistic Design only providing continuous distributions, for any height H_I a certain I-section has to be defined. This could have been reached by just choosing the next discrete level. But the consequence would have been inaccuracies and probably problems with the response surfaces. Therefore a macro has been written, which generates an appropriate I-beam for any H_I. It uses linear interpolation between the geometrical data (height, width, thickness of web and flange) of the surrounding, discrete IPE-beams. It has to be noted, that this results in a nonlinear interpolation for the moment of inertia, which is expected to be a good approximation for building the response surfaces on. So for any setting of input parameters the appropriate FEM-model can be generated.

Additional FEM model data is given below.

Elastic modulus	$E=2.1 \cdot 10^8 \frac{kN}{m^2}$	Self-weight is included.
Density	$\rho = 7.8 \frac{t}{m^3}$	Permanent dead load on complete beam: $3\ \frac{kN}{m}$
Gravity	$g = 9.8 \frac{kN}{t}$	Analysis type: linear static.

After completing the FEM model, the random variables had to be defined. The noise factors, loads and costs, were defined with regard to meet possible conditions of a realistic project. The ranges of the control factors, geometry and material dimensions, surely had to include the optimum solution. Without any previous analysis, the ranges were set extremely wide. This resulted in inaccurate response surfaces, which did not allow an adequate approximation and were only good for excluding certain settings. This first trial was declared as prestudy. The main study instead brought clear results using the following input parameter ranges:

H_I $[mm]$: [180;300]
A_CHS $[cm^2]$: [4;10]
H_TOP $[m]$: [3;6]
L_CON $[m]$: [5.5;7.5]
LOAD_END $\left[\frac{kN}{m}\right]$: [0;7]
LOAD_MID $\left[\frac{kN}{m}\right]$: [0;7]
PRICE_CHS $\left[\frac{DM}{t}\right]$: [4000;4500]
PRICE_IPE $\left[\frac{DM}{t}\right]$: [2900;3300]

Then with 8 input parameters at least 45 simulations had to be run for fitting a full quadratic response surface for each output parameter. This would propose $1.5 \cdot 45 = 68$ simulations for gaining good information about the response surface fit. For the prestudy and main study 100 Monte-Carlo simulations were executed. The accuracies of the corresponding response surfaces are given by the maximum residuals in Figure 5.19.

	PRESTUDY		MAIN STUDY	
A_CHS [cm²]	5	100	4	10
H_I [mm]	120	550	180	300
L_CON [m]	3.0	8.0	5.5	7.5
H_TOP [m]	2.0	8.0	3.0	6.0
C_CHS [DM]	112		1.0	
C_IPE [DM]	46		17.6	
C_TOT [DM]	96		16.7	
DEFLECT [m]	0.086		0.008	
STRESS [kN/m²]	86000		6500	
TENSION[kN/m²]	14800		3100	
W_CHS [ton]	0.026		0.00024	
W_IPE [ton]	0.015		0.0054	
W_TOT [ton]	0.022		0.0053	

Figure 5.19: Maximum residuals of Response Surfaces

5.2.4 Robust Design Sampling

For processing Robust Design the control variables have to be selected from the currently defined random variables. Then the Robust Design output parameters have to be defined as statistical values from the introduced output parameters of the model. For getting the best overview on the Robust Design output parameters, their number was kept reasonably low by defining the most important statistics only. After that the program outputs the following text:

```
  RANDOM VARIABLES -- Preset

   Control Variables : 4
     H_TOP
     L_CON
     H_I
     A_CHS

   Noise Variables : 4
     LOAD_END
     LOAD_MID
     PRICE_IPE
     PRICE_CHS


 QUALITY CRITERION FOR ADAPTION -- Preset

   No criterion defined.


 ROBUST DESIGN OUTPUT PARAMETERS -- Preset

   COST_M      : Mean Value              C_TOT
   COST_STD    : Standard Deviation      C_TOT
   COST_95     : Confidence Value        C_TOT -> 95 %
   STRESS_95   : Confidence Value        STRESS -> 95 %
   DEFLECT_95  : Confidence Value        DEFLECT -> 95 %
   TENSION_95  : Confidence Value        TENSION -> 95 %
   WEIGHT_TOT  : Mean Value              W_TOT
   WEIGHT_IPE  : Mean Value              W_IPE
   WEIGHT_CHS  : Mean Value              W_CHS
```

In this example the 95% confidence value of the total costs (COST_95) was the criterion for receiving a solution with stable costs at a minimum level. As there are also certain structural requirements, the following conditions have been introduced to identify valid structures:

- STRESS_95 < 214000 $\left[\frac{kN}{m^2}\right]$

- TENSION_95 < 214000 $\left[\frac{kN}{m^2}\right]$

It is emphasized, that only statistical values are applicable when using Robust Design. So although the extremest stresses are of interest, confidence values have been chosen, as they are expected to gain homogeneous results in contrast to the statistical maximum (see p. 25). These conditions have freely been assumed with regard to an allowed stress of 240000 $\left[\frac{kN}{m^2}\right]$. For real purposes it would be useful to develope a new safety concept based on these relative low statistics. As the applied stress limits automatically lead to reasonable deflections, no seperate deflection limit was introduced.

For the study always 300 noises were simulated per sample. First a basic simulation was performed with 1000 samples and then two adaptions with 500 samples each. The adaptions were always made on actual selections of the best, valid samples. This procedure is illustrated in Figures 5.26-5.35

5.2.5 Data Analysis

An overview on the data is provided by several graphics on the next pages. Corresponding explanations are added from page 83 on. Figure 5.20 shows one selection of the prestudy. It is only provided to illustrate, how the ranges were reduced and will not be discussed further. All other graphics refer to the main study.

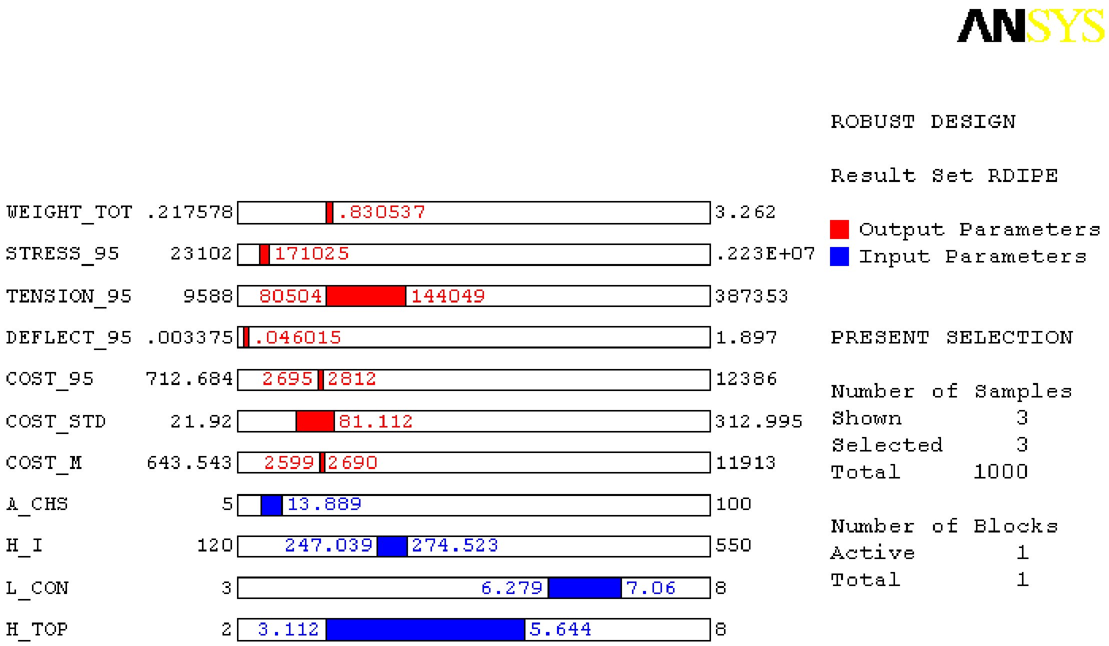


Figure 5.20: Selected samples of Prestudy.

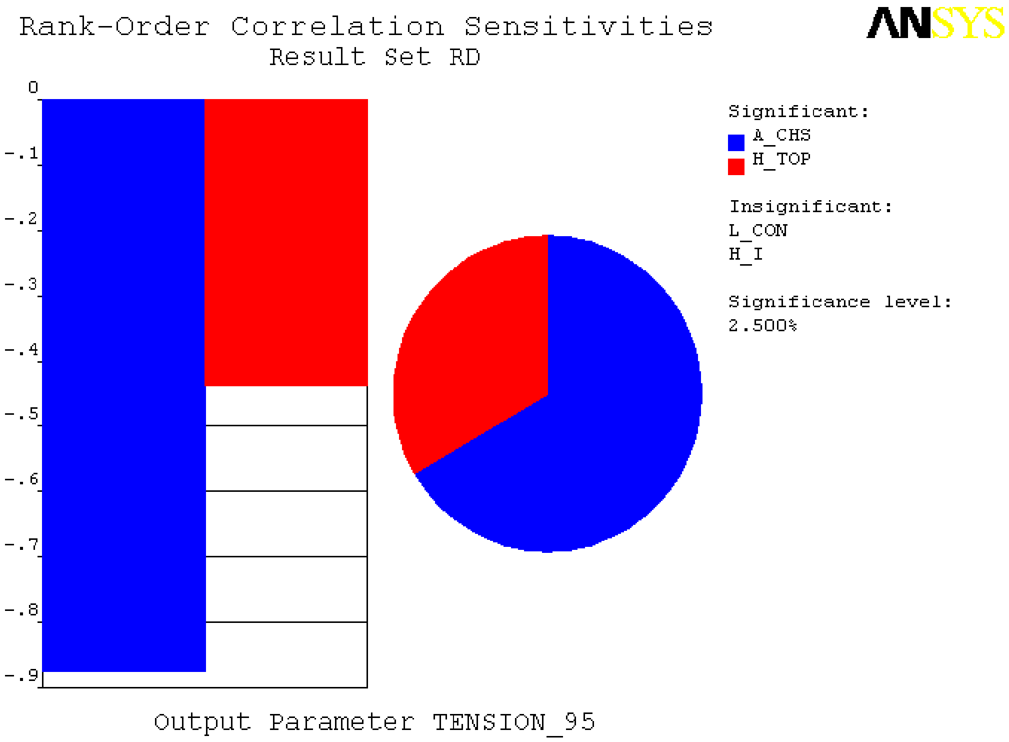


Figure 5.21: Sensitivities for Tension_95 (Basic Simulation)

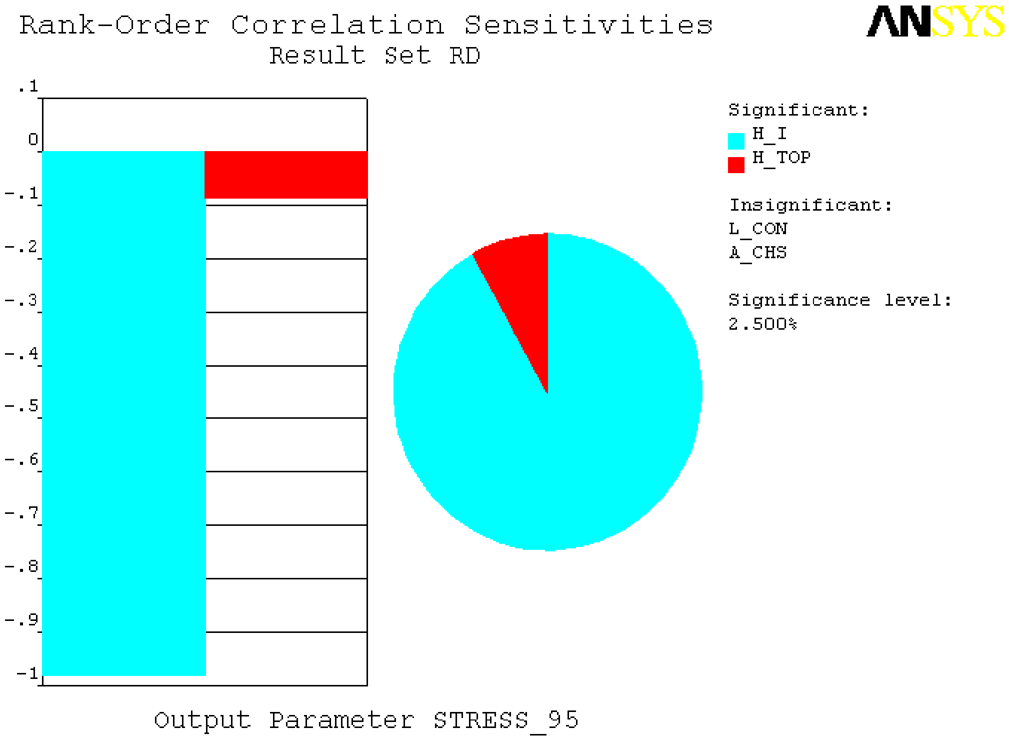


Figure 5.22: Sensitivities for Stress_95 (Basic Simulation)

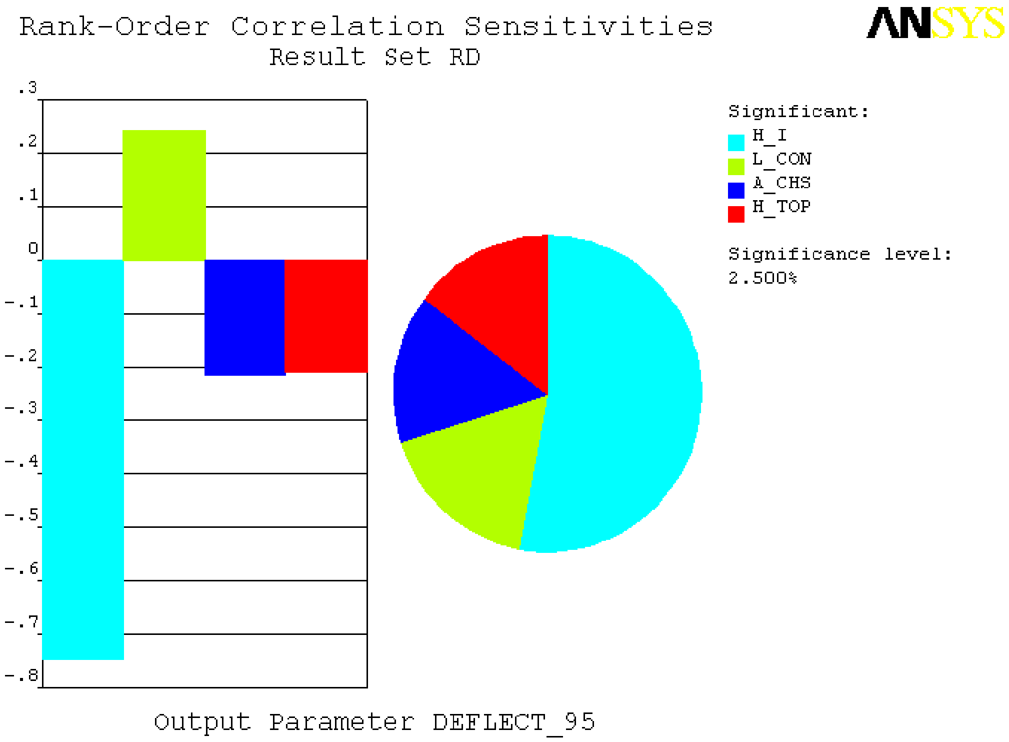


Figure 5.23: Sensitivities for Deflect_95 (Basic Simulation)

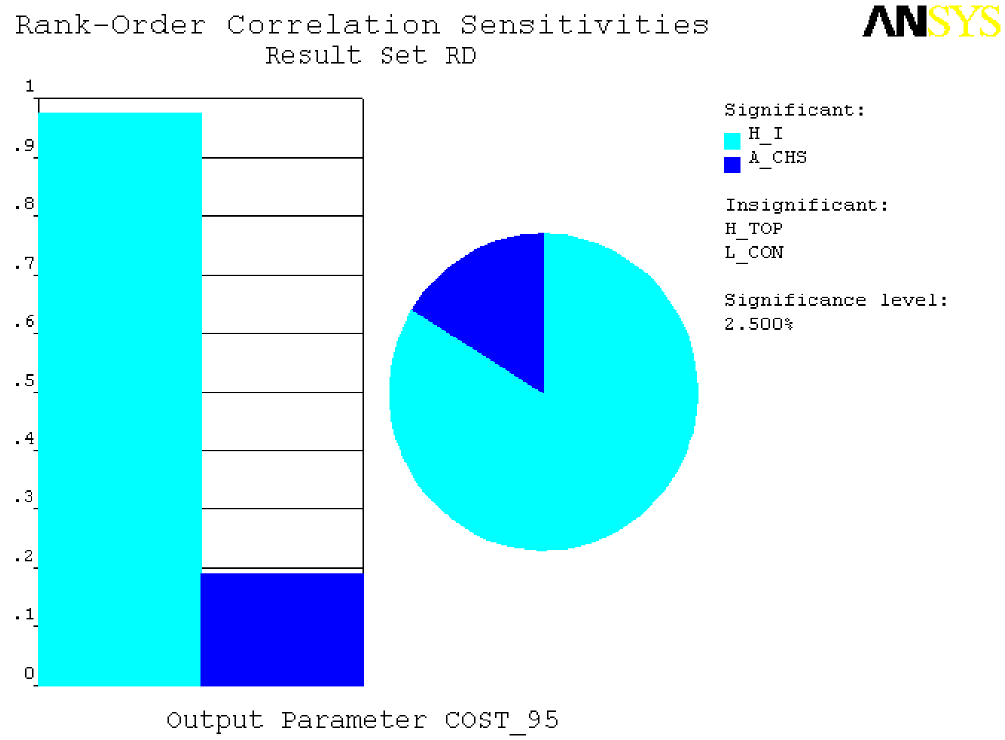


Figure 5.24: Sensitivities for Cost_95 (Basic Simulation)

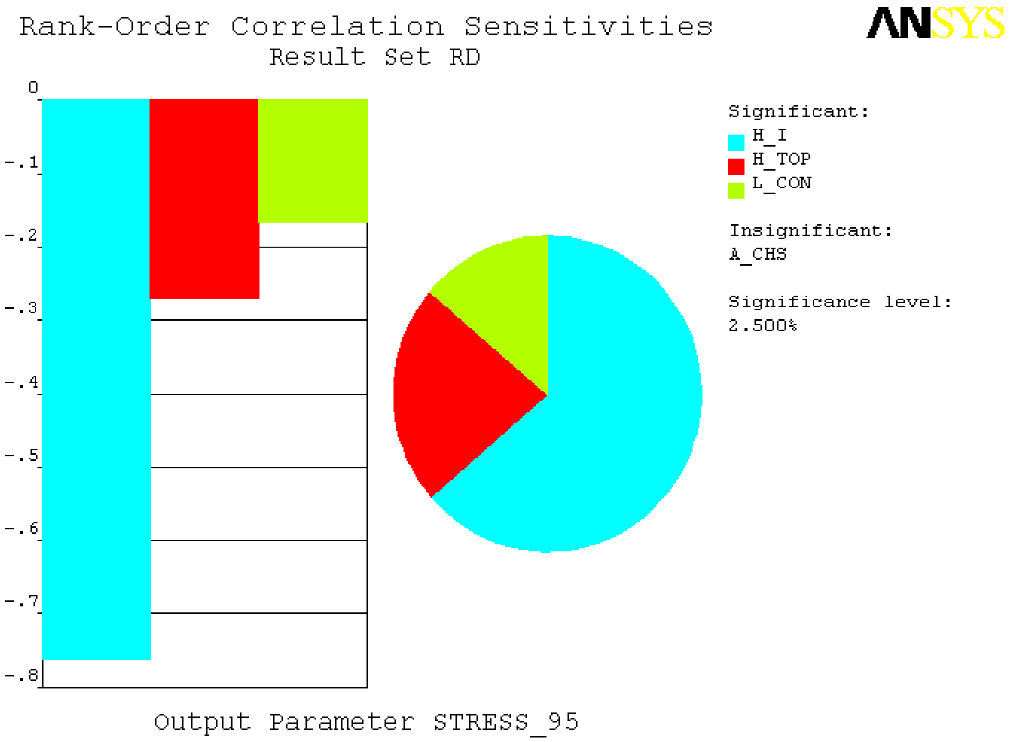


Figure 5.25: Sensitivities for STRESS_95 (FIRST ADAPTION)

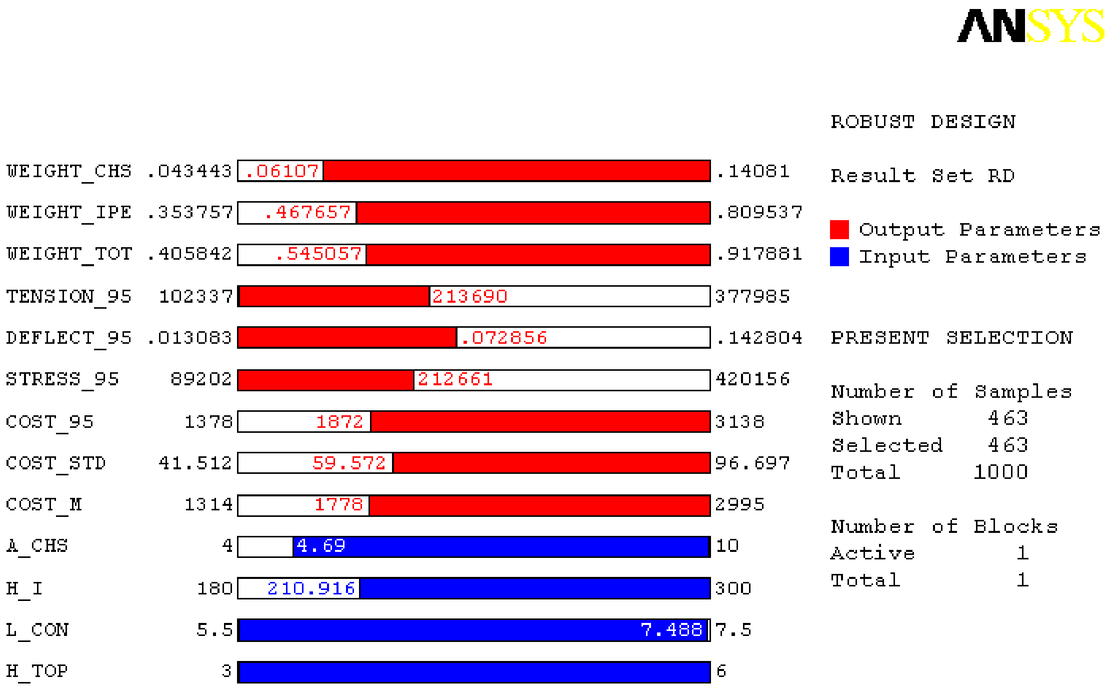


Figure 5.26: Main Study: Basic Simulation, invalid samples unselected.

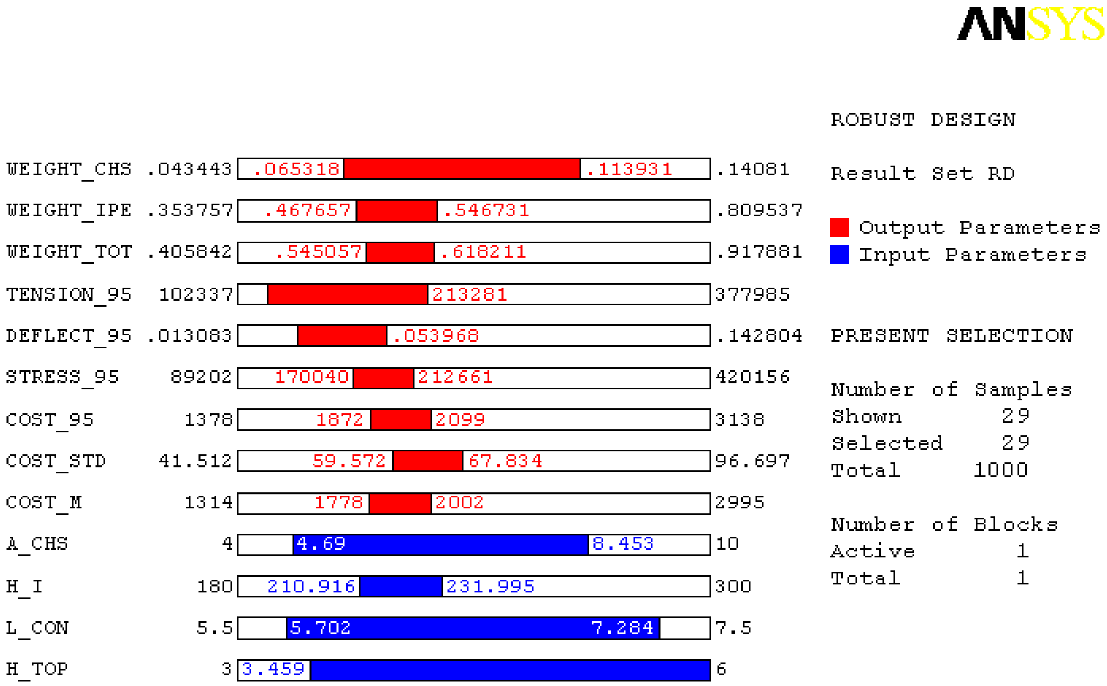


Figure 5.27: Fig. 5.26 → COST_95>2100 unselected.

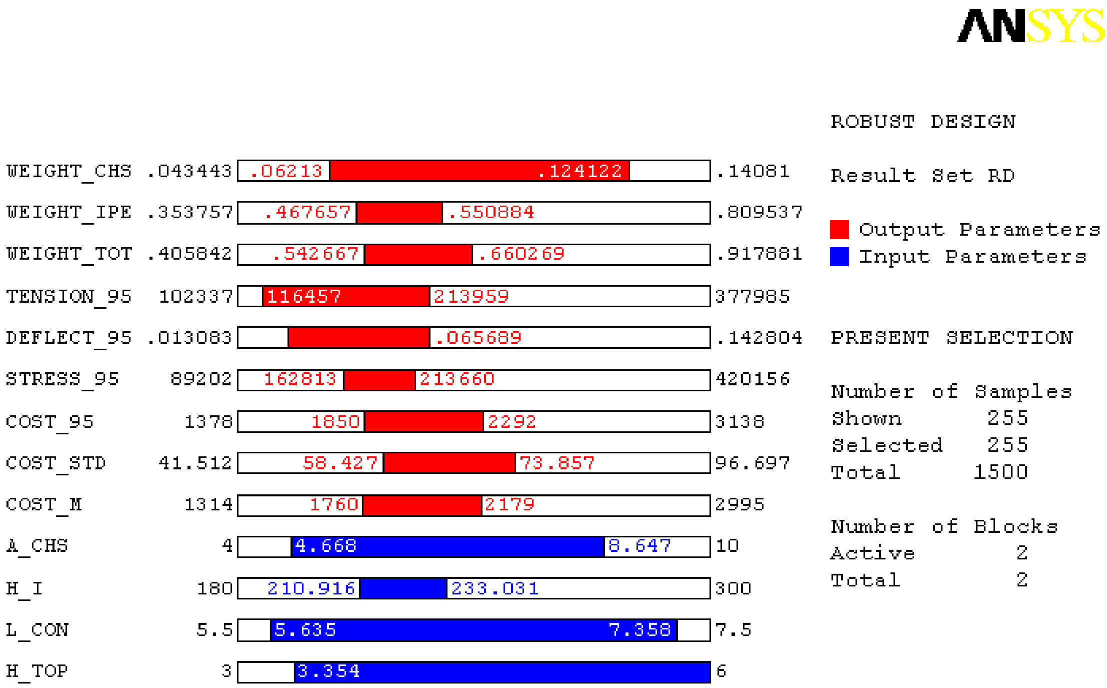


Figure 5.28: Fig. 5.27+500 adapted samples from selection; invalid samples unselected.

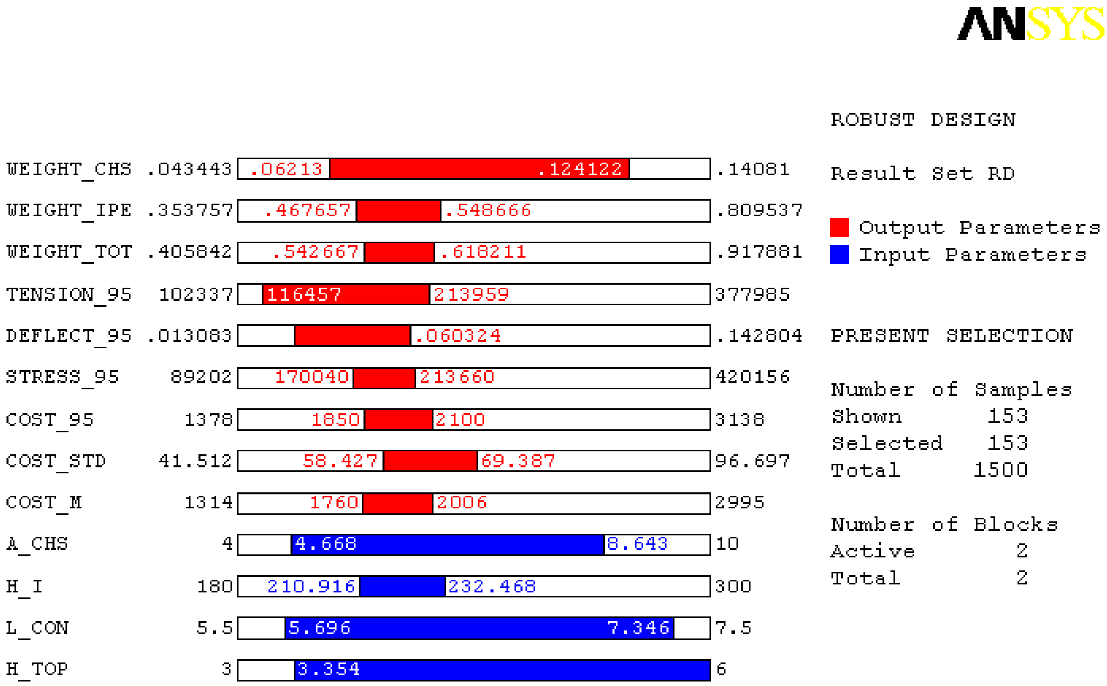


Figure 5.29: Fig. 5.28 → COST_95>2100 unselected.

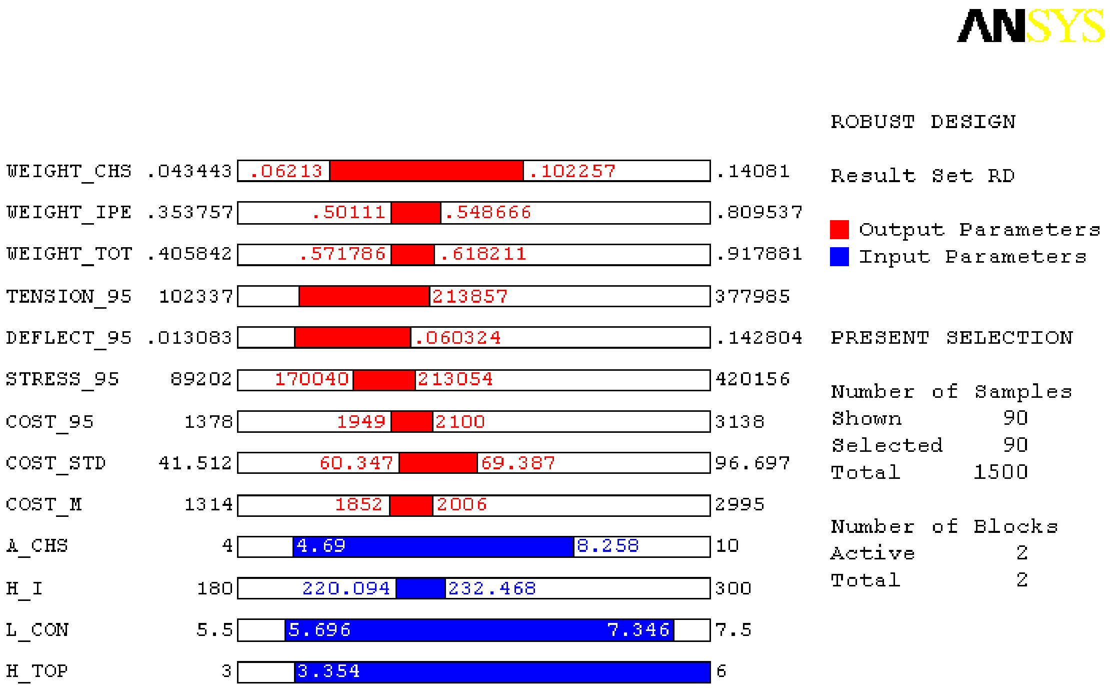


Figure 5.30: Fig. 5.29 → H_I<220 unselected.

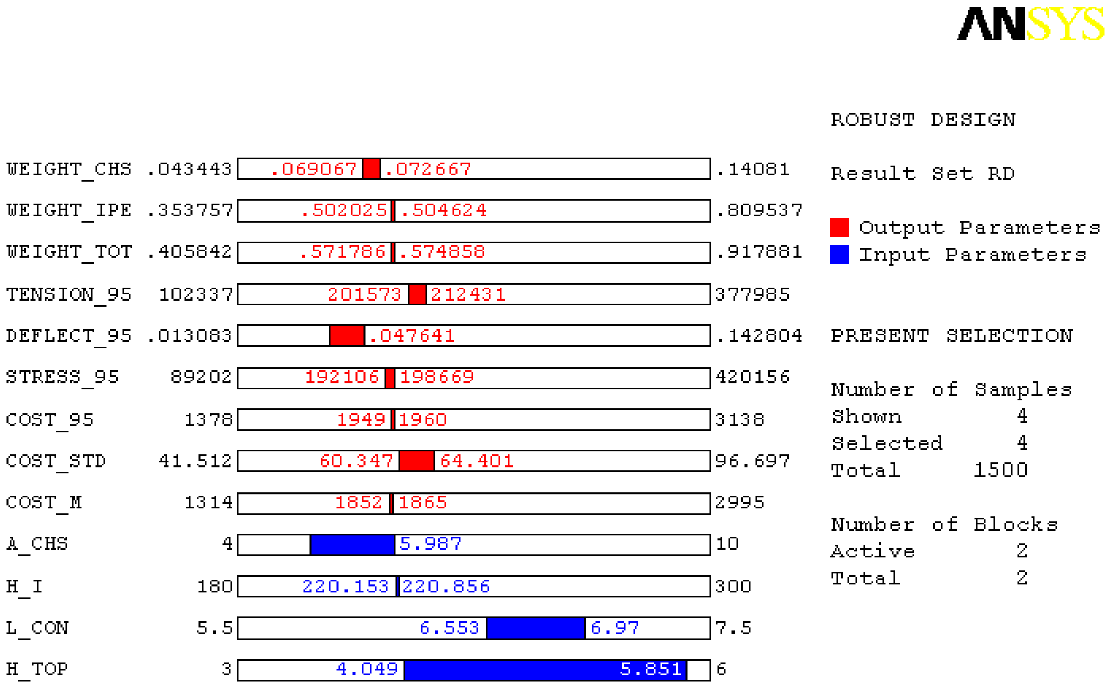


Figure 5.31: Fig. 5.30 → COST_95>1960 unselected.

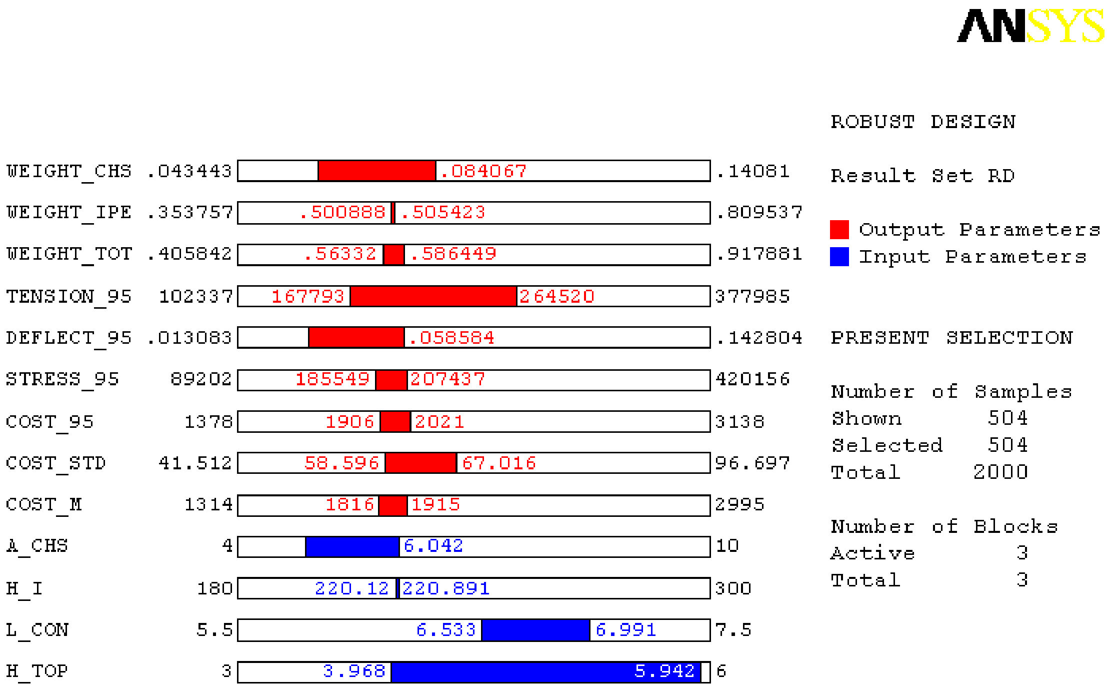


Figure 5.32: Fig. 5.31+500 samples (no selection).

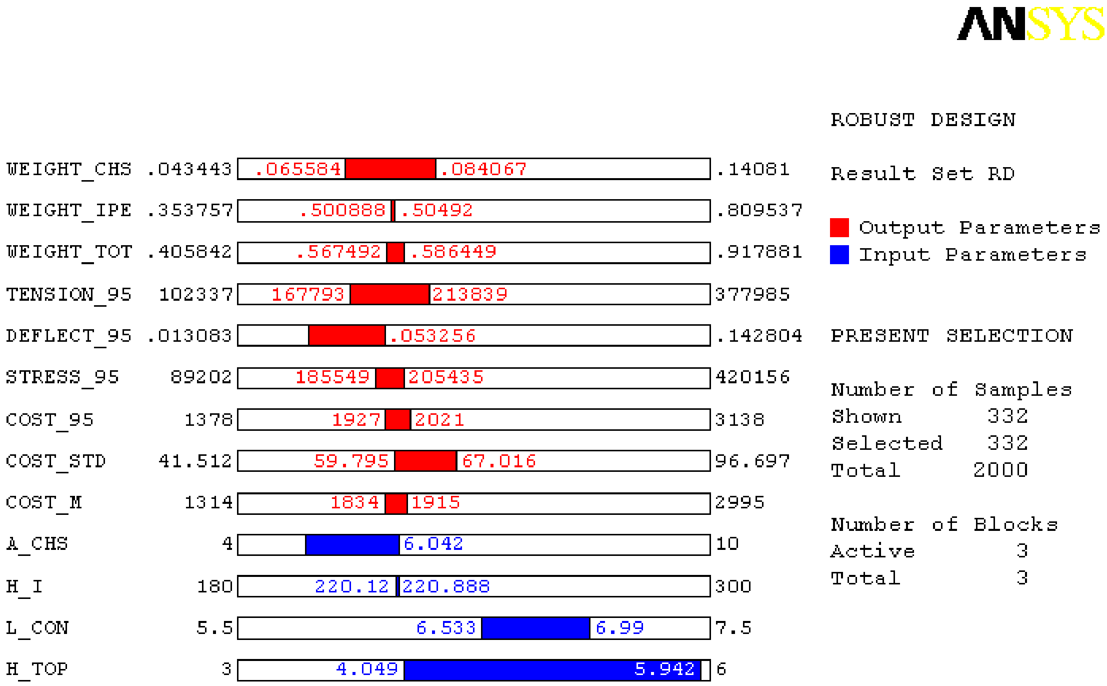


Figure 5.33: Fig. 5.32 → Invalid samples unselected.

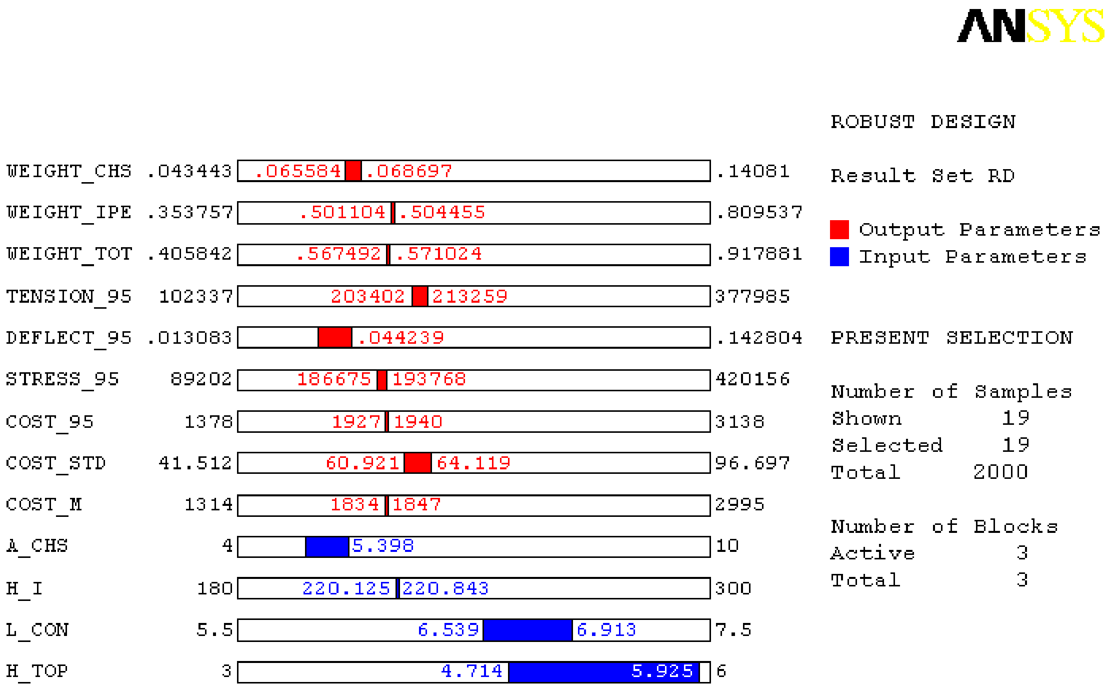


Figure 5.34: Fig. 5.33→ COST_95>1940 unselected (Best 19 Samples).

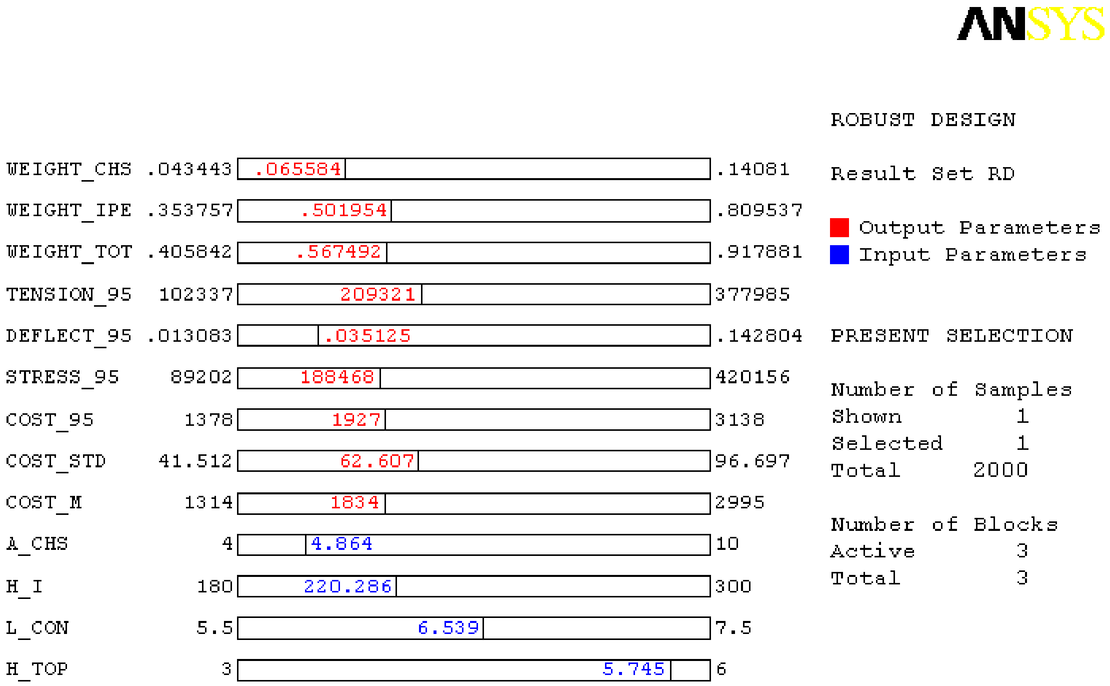


Figure 5.35: Fig. 5.34→ COST_95>1927 unselected (Best Sample).

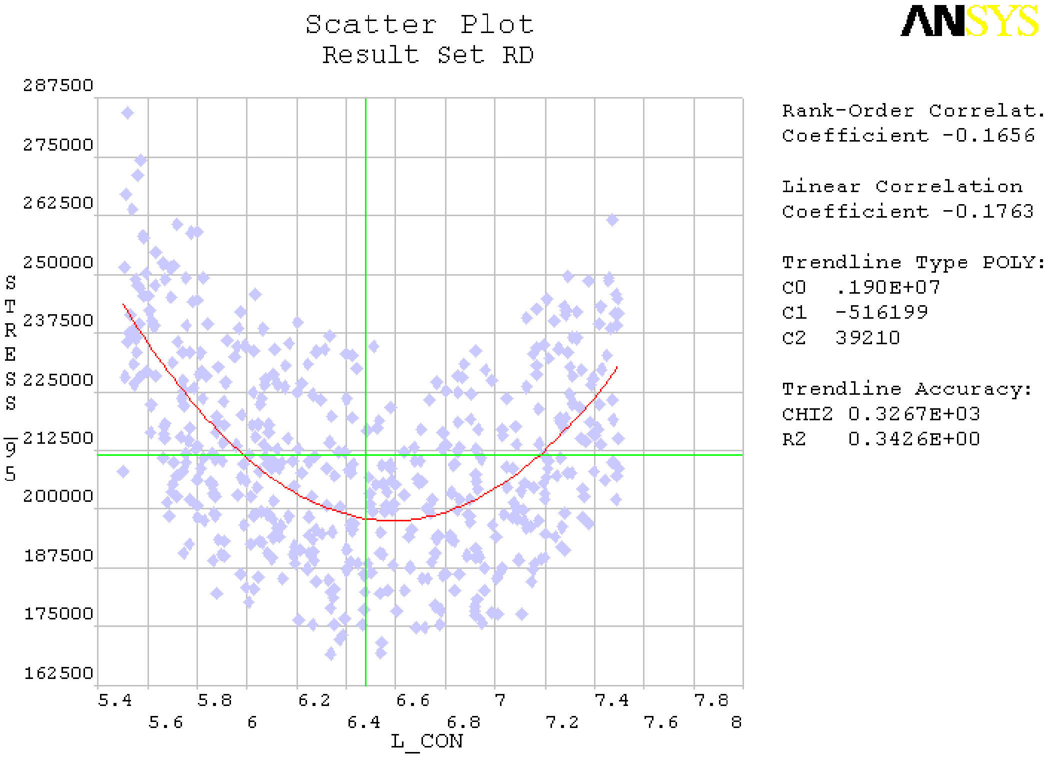


Figure 5.36: Scatter Plot: L_CON-STRESS_95: All of First Adaption

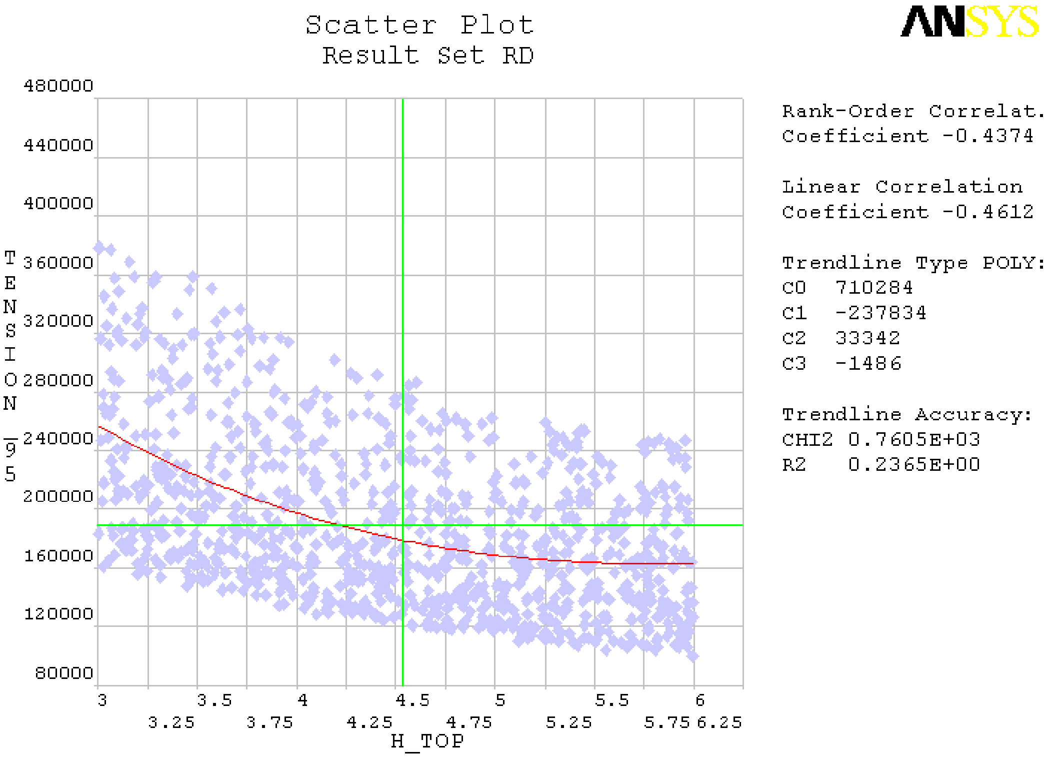


Figure 5.37: Scatter Plot: H_TOP-TENSION_95: All of Basic Simulation

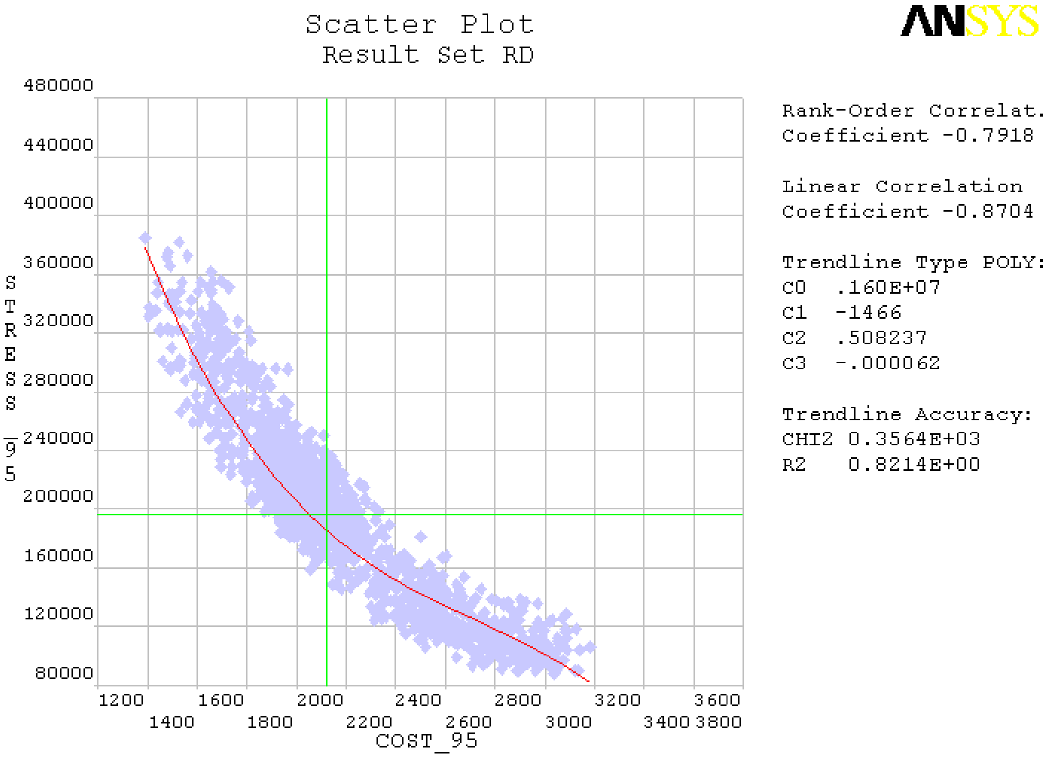


Figure 5.38: Trade-of: COST_95-STRESS_95: All samples

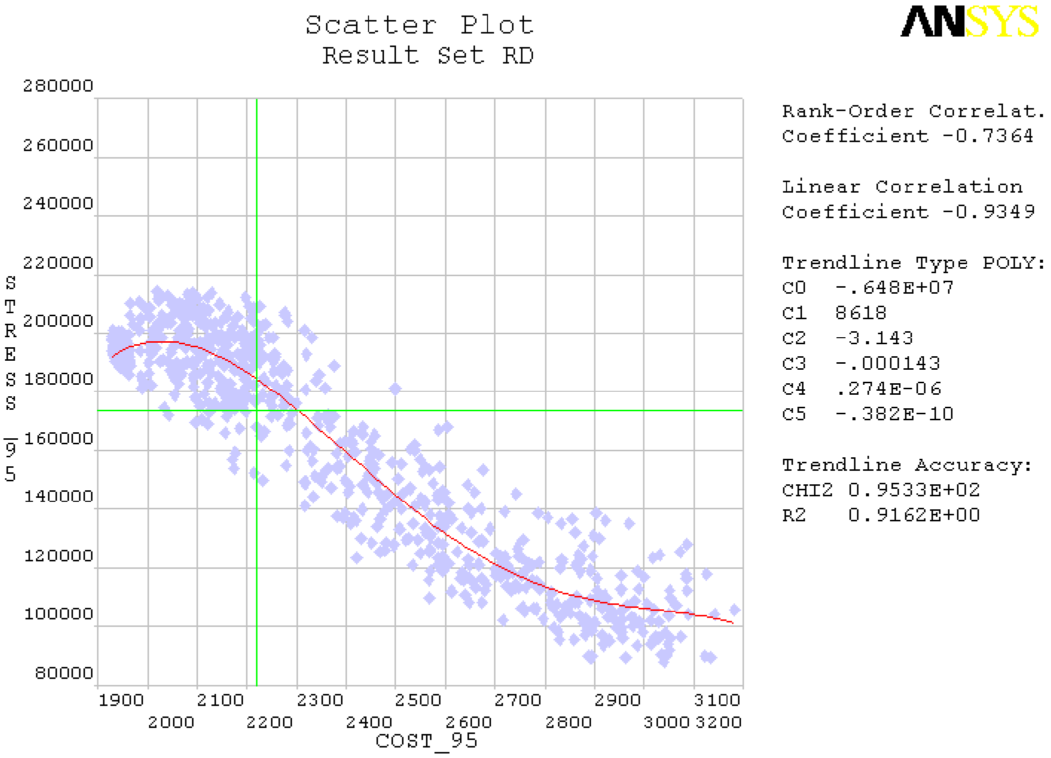


Figure 5.39: Trade-of: COST_95-STRESS_95: All valid samples

Figures 5.21–5.25	The sensitivities from all important output parameters TENSION_95, STRESS_95, DEFLECT_95 and COST_95 after the basic simulation are shown. Note that the sensitivities of STRESS_95 change after the first adaption (Figure 5.25). This illustrates that the sensitivities always have to be seen in relation to the considered ranges. So when reducing the range of one variable, another variable might become more significant.
Figures 5.26–5.35	These graphics describe the complete adaption process. Because the smallest, valid section after the first adaption has to be at least 210.916mm high, a special step is made in Figure 5.30, when all profiles smaller than 220mm are unselected. So as a IPE 200 is not possible and a beam IPE with 210mm height is not available, the search is restricted above the next discrete level of 220mm.
Figure 5.36	This diagram shows the effect of L_CON on STRESS_95. It indicates that the lowest stress is achieved when L_CON is between 6.5m and 6.7m.
Figure 5.37	This diagram shows the effect of decreasing TENSION_95 while increasing H_TOP. A greater H_TOP allows for a smaller section area of the CHS. But this does not mean that the largest H_TOP is the best solution, as the material volumes and the costs increase with the length of the tension members. Furthermore the effect of H_TOP on STRESS_95 has to be considered (Figure 5.22).

Figures 5.38, 5.39

These figures illustrate the correlation between COST_95 and STRESS_95. Here the combination of technical and economical criterion is directly visible. While the upper graphic shows all the samples, the lower one only contains valid samples (“structures”). This diagram shows the cost for reducing stress to a certain limit. Generally this kind of graphic is very useful to quantify technical properties into costs for finding most efficient solutions. This graphic also shows that the adaptions have been effective, as there is a high sample density in the low cost region and as the trend changes direction in favour of the lowest costs.

5.2.6 Best Results

Several graphics and text outputs support to gain maximum system information and identify most robust solutions. They give a better system inview and allow to control certain interaction effects. For more efficiency any number of adaptions can be processed. But all these procedures are not absolutely necessary. It is possible to pick out the best sample(s) directly after one execution of Robust Design Sampling. The best results of the different blocks are compared below:

	H_TOP $[m]$	L_CON $[m]$	H_I $[mm]$	A_CHS $[cm^2]$
Basic Simulation	5.21	6.97	220.86	5.22
1^{st} Adaption	5.85	6.94	220.63	4.91
2^{nd} Adaption	5.75	6.54	220.29	4.86

	COST_95 $[DM]$	TENSION_95 $\left[\frac{kN}{m^2}\right]$	STRESS_95 $\left[\frac{kN}{m^2}\right]$
Basic Simulation	**1960**	203000	193000
1^{st} Adaption	**1949**	205000	192000
2^{nd} Adaption	**1927**	209000	188500

The improvement from the basic simulation to the second adaption is 33DM. This absolute value seems to be small, but still it is a relative cost reduction of 1.7%. The improvement can also be seen in Figure 5.34, where 19 samples are below 1940DM. So sample density near the optimum has much been increased. On the other hand the basic simulation already brought a solution to be declared as acceptable.

5.2.7 Test on FEM Structure

After the best solution was found, it was interesting to know, how good it performs in the real simulation. Therefore an FEM-model with the best input parameters was built on which three loadcases have been tested:

- Loadcase 1: LOAD_END=7 $\frac{kN}{m}$; LOAD_MID=0 $\frac{kN}{m}$

- Loadcase 2: LOAD_END=0 $\frac{kN}{m}$; LOAD_MID=7 $\frac{kN}{m}$
- Loadcase 3: LOAD_END=7 $\frac{kN}{m}$; LOAD_MID=7 $\frac{kN}{m}$

The results are shown in Figures 5.40-5.45. The largest deflection of 0.0386m to be found in loadcase 1 is 10% over the 95% confidence value of deflection 0.0351m (DEFLECT_95 in Figure 5.35). This is in any case below the usability criterion of the structure. The maximum stress in the beam is 207500$\frac{kN}{m^2}$ (loadcase 3), which is 10% above the approximated stress (STRESS_95) of 188500$\frac{kN}{m^2}$. Still it is below the limit, as the I-beam was fixed to the next discrete level. The maximum stress in the tension member(loadcase 3), is 236000$\frac{kN}{m^2}$ and 9% percent over the approximated confidence value of 209000$\frac{kN}{m^2}$ (TENSION_95).

All simulation results are about 10% larger than the 95% confidence values. This is interpreted as good, as the optimum solution comes from statistics of simulations on response surfaces.

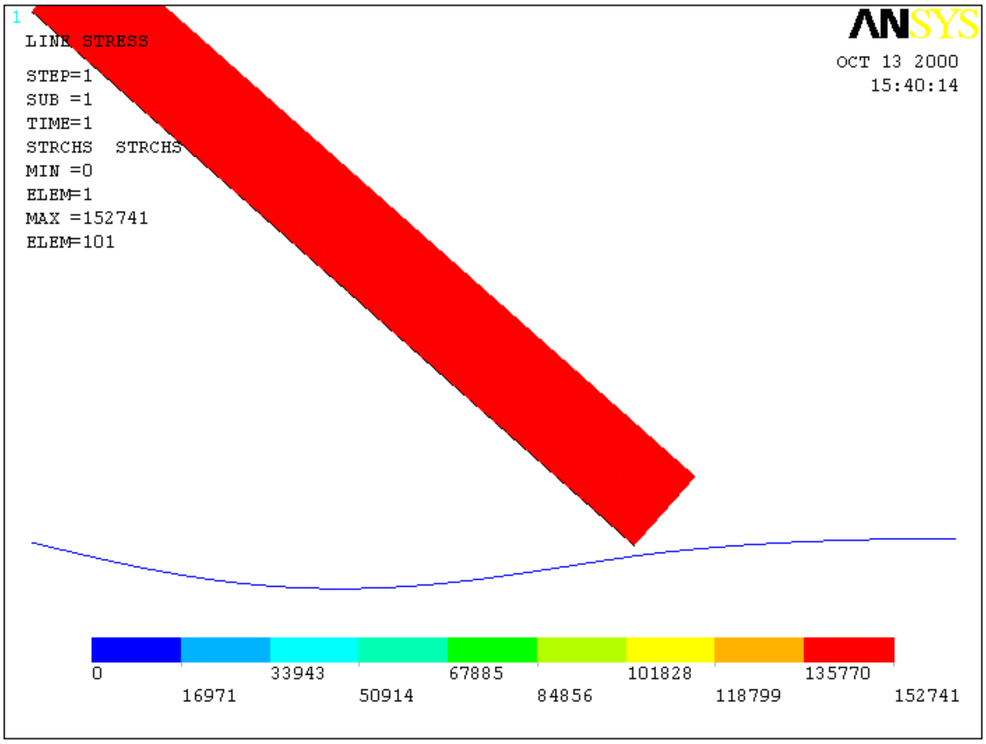


Figure 5.40: Loadcase 1: Line Stress

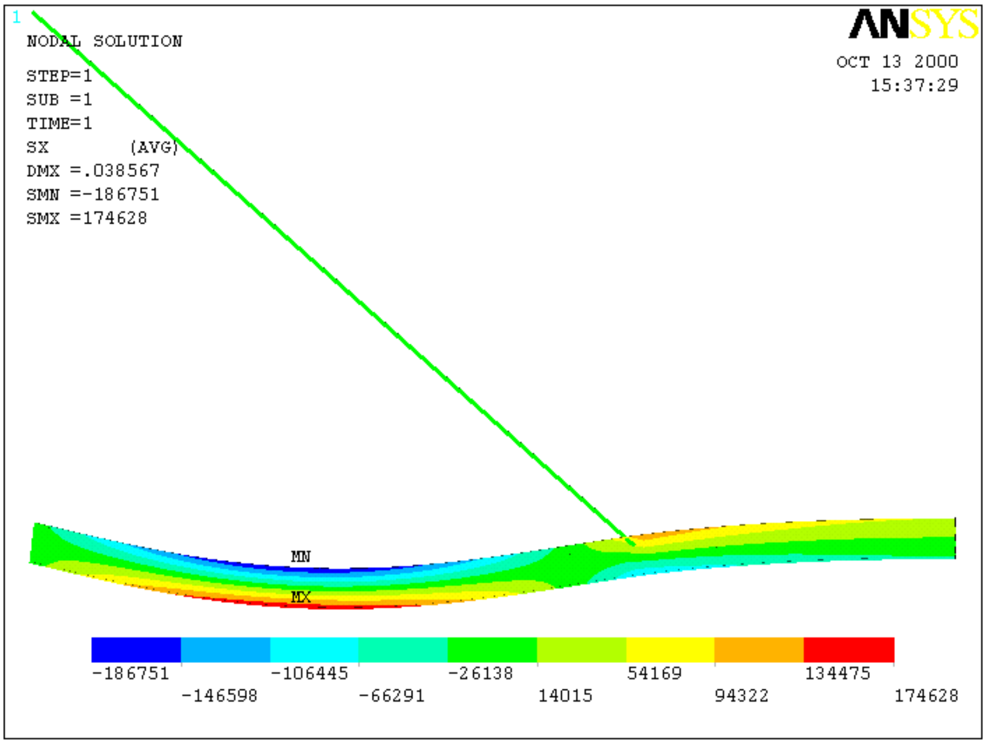


Figure 5.41: Loadcase 1: Stress Intensity (Deflection: [-0.0386m; +0.0033m])

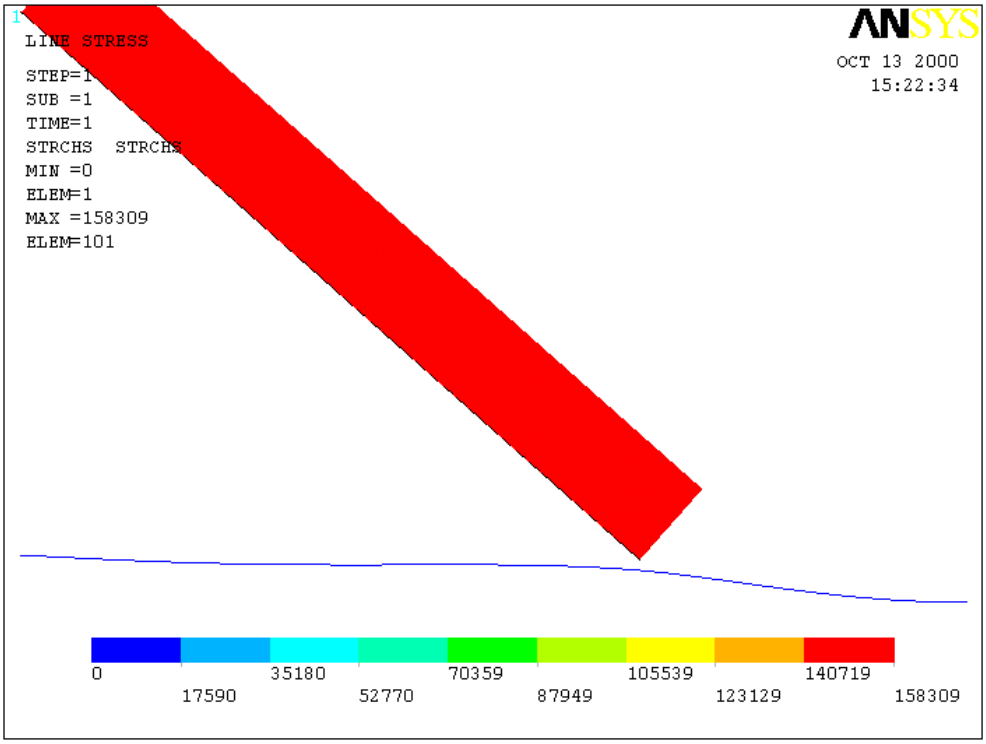


Figure 5.42: Loadcase 2: Line Stress

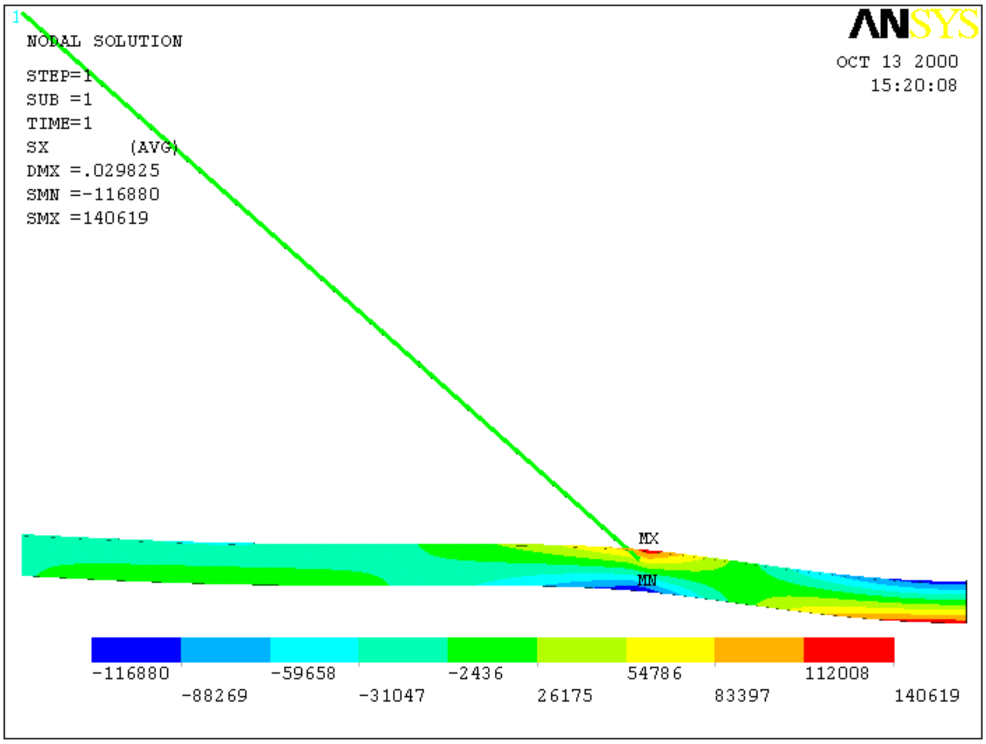


Figure 5.43: Loadcase 2: Stress Intensity (Deflection: [-0.0298m; +0.0001m])

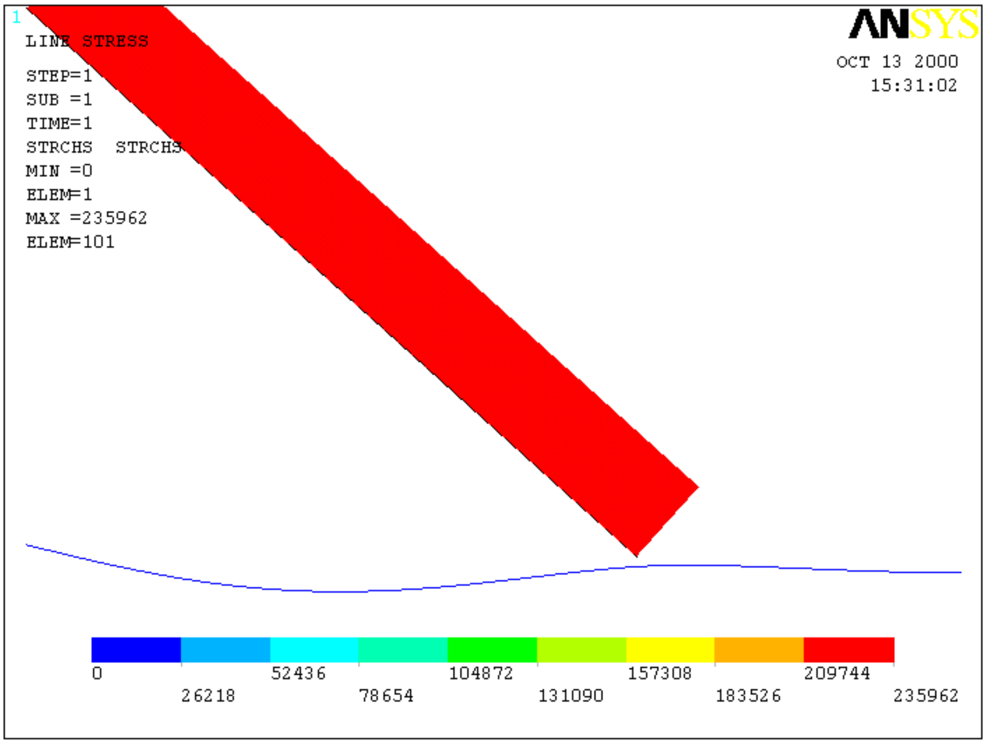


Figure 5.44: Loadcase 3: Line Stress

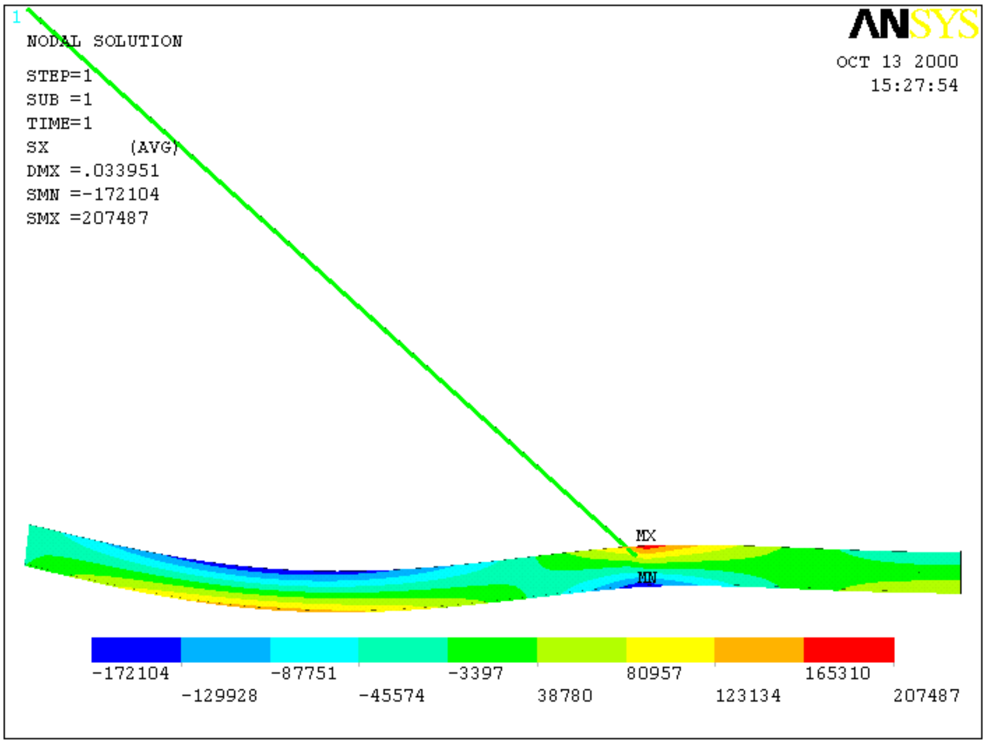


Figure 5.45: Loadcase 3: Stress Intensity (Deflection: [-0.0340m; +0.0002m])

5.3 Hollow Section Arch

5.3.1 Idea

The inspiration for this example comes from bridges with upper or lower arches (Figure 5.46, 5.47). Arches have a very high bearing capacity if they follow the ideal line of cosine hyperbolic and if they are equally loaded. The ideal arch transforms the vertical forces in normal forces only without any effective bending moment. This allows to design arches with a very high slenderness ratio using minimal material. On the other hand the minimized arch is sensitive to noises. Small deformations can influence the arch's load-carrying behaviour. Such noises could be temperature changes, geometrical inaccuracies, material inhomogenities or assymetrical loads. The test example has been reduced and only includes one varying load. Aim is to determine the arch with the lowest weight, which is robust against the influence of the load.

Figure 5.46: Rainbow Bridge (286m), Niagara Falls, USA/Canada.

Figure 5.47: Birmingham Bridge, Pittsburgh, Pennsylvania, USA.

5.3.2 System

Figure 5.48 shows the static system. Slightly varying from the optimal curvature the arch is defined as a parabolic shape. The control variables are the height of the arch (HEIGHT) and the width of the arch (SWIDTH). The cross-section of the arch is defined as quadratic hollow section with plate thickness $\frac{1}{40} \cdot \text{SWIDTH}$. The position of a virtual truck load describes the noise of the system (TRUCKPOS). TRUCKPOS can be any value between 0.0 and 0.5. The load of 7 meter length is always assigned to the next nodes. For each

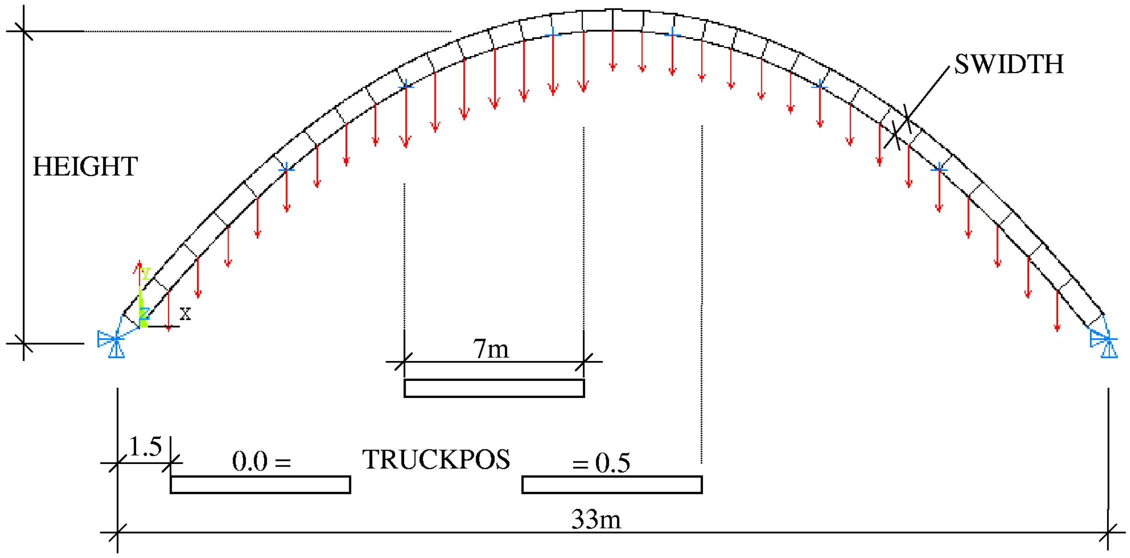


Figure 5.48: Static System

parameter setting a linear and a nonlinear static analysis is performed for identifying the geometric nonlinear behaviour of the structure. Following output parameters are defined:

WEIGHT:	Weight of arch	STRESS:	Max. Stress (nonlinear)
REACTX:	Horizontal reaction force	LSTRESS:	Max. Stress (linear)
REACTY:	Vertical reaction force	DEFLECT:	Max. Deflection (nonlinear)
REACTOT:	Total reaction force	LDEFLECT:	Max. Deflection (linear)

5.3.3 From FEM Model to Response Surfaces

The arch is formed by $4 \cdot 32 = 128$ SHELL63-elements. The inner nodes are all on a horizontal distance of 1 meter, their y-coordinates are calculated by a quadratic function. The outer nodes are put on a distance of SWIDTH rectangular to the actual slope. Both supports of the arch are pin-ended realized by 10 LINK8-elements as shown in Figure 5.49. The arch is renstrained in z-direction at six inner nodes. More data of the FEM-model is given below:

Elastic modulus	$E{=}2.1 \cdot 10^8 \frac{kN}{m^2}$	Permanent dead load on arch: $120\frac{kN}{m}$
Density	$\rho = 7.8\frac{t}{m^3}$	Permanent live load on arch: $26.4\frac{kN}{m}$
Gravity	$g = 9.8\frac{kN}{t}$	Truck load: $495kN$
Self-weight is included.		Analysis type: linear static and nonlinear static.

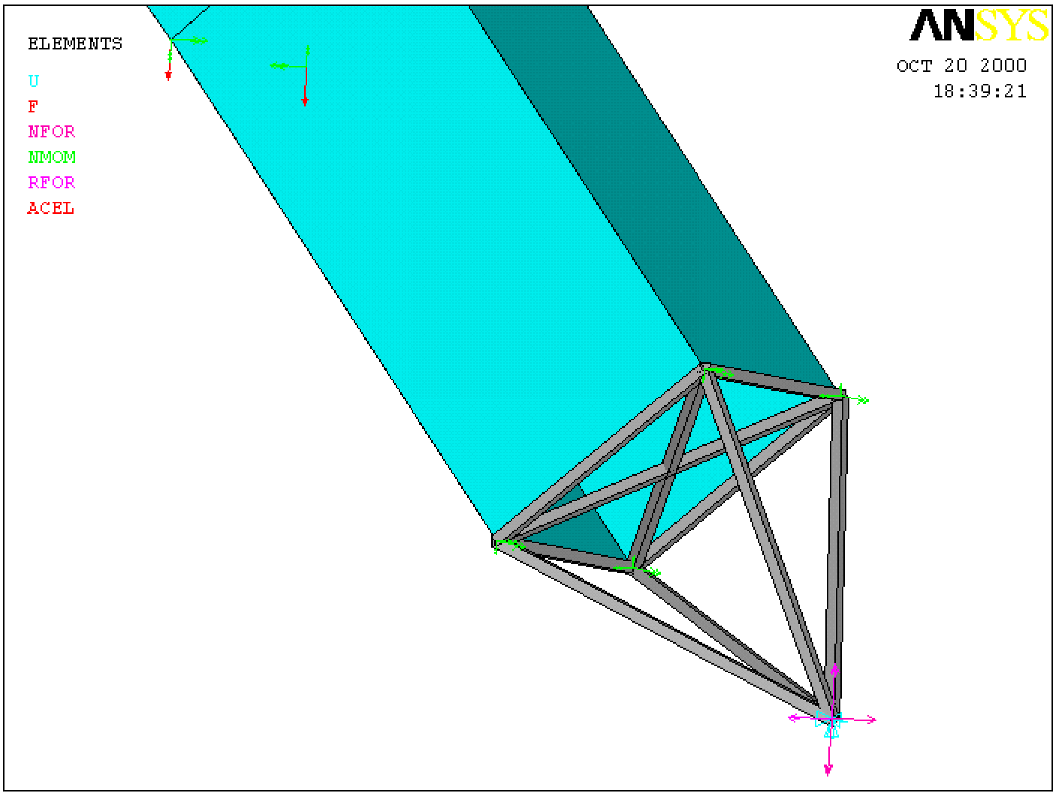


Figure 5.49: Detail: pin-ended support

With performing nonlinear static analysis convergence problems had to be considered. If an examined structure is not stable, no convenient output values are available for fitting the response surface on. This problem could temporary be solved by completely deleting this single simulation from the data table. The loss of data in unstable regions does not necessarily cause a loss of quality if other failure criterions become much earlier decisive than the stability criterion. But in some cases the stability criterion might become significant. Furthermore the response surfaces can not store if a structure is stable. Only the output values from the response surface can indicate that a structure probably is not stable anymore. The problem with non-converging parameter combinations increases with the number of input parameters. This is additional information to be considered when performing nonlinear analysis. In this example the following parameter ranges always resulted in valid simulations:

HEIGHT	$[m]$	:	[3;20]	TRUCKPOS	:	[0.00;0.50]
SWIDTH	$[m]$	:	[0.45;1.00]			

With 3 input parameters only 10 simulations had to be run for fitting a full quadratic response surface. For this example the Central Composite Design with 15 simulations was selected. The accuracies of the response surfaces are given by the maximum residuals:

WEIGHT	$[kN]$	0.47	STRESS	$\left[\frac{kN}{m^2}\right]$	37200
REACTX	$[kN]$	7.6	LSTRESS	$\left[\frac{kN}{m^2}\right]$	8900
REACTY	$[kN]$	6.2	DEFLECT	$[m]$	0.0340
REACTOT	$[kN]$	56	LDEFLECT	$[m]$	0.0078

5.3.4 Robust Design Sampling

For robust design sampling the control variables were selected and the robust design output parameters were defined. The corresponding text output of the program is given below:

```
RANDOM VARIABLES -- Preset

  Control Variables : 2
    HEIGHT
    SWIDTH

  Noise Variables : 1
    TRUCKPOS


QUALITY CRITERION FOR ADAPTION -- Preset

  No criterion defined.


ROBUST DESIGN OUTPUT PARAMETERS -- Preset

  REACTFX  : Maximum                REACTX
  REACTFY  : Maximum                REACTY
  STRS_M   : Mean Value             STRESS
  STRS_STD : Standard Deviation     STRESS
  DEFL_M   : Mean Value             DEFLECT
  DEFL_STD : Standard Deviation     DEFLECT
  DEFL_95  : Confidence Value       DEFLECT -> 95 %
  STRS_95  : Confidence Value       STRESS -> 95 %
  LDEFL_95 : Confidence Value       LDEFLECT -> 95 %
  LSTRS_95 : Confidence Value       LSTRESS -> 95 %
  WT       : Mean Value             WEIGHT
```

Aim of optmization is to minimize the weight of the arch (WT). Equivalent to the last example the 95% confidence values of deflection and stress are defined as robust design output parameters. The output values of the nonlinear analysis are authoritative. As the deflection was not found to be decisive only the following criterion was chosen to identify valid robust design samples:

- STRS_95 < 214000 $\left[\frac{kN}{m^2}\right]$

Each sample is the statistical result of 300 noise variations to assure stable 95% confidence values. First a basic simulation with 1000 samples was processed. Then two adaptions with 500 samples were performed, each on an actual selection of the best, valid samples. This process is illustrated in Figures 5.50-5.57.

5.3.5 Data Analysis

After the graphics of actual selections which demonstrate the adaption process and lead to the optimum result, several diagrams are shown for gaining more information about the structure. Explanations to the figures are added on page 103.

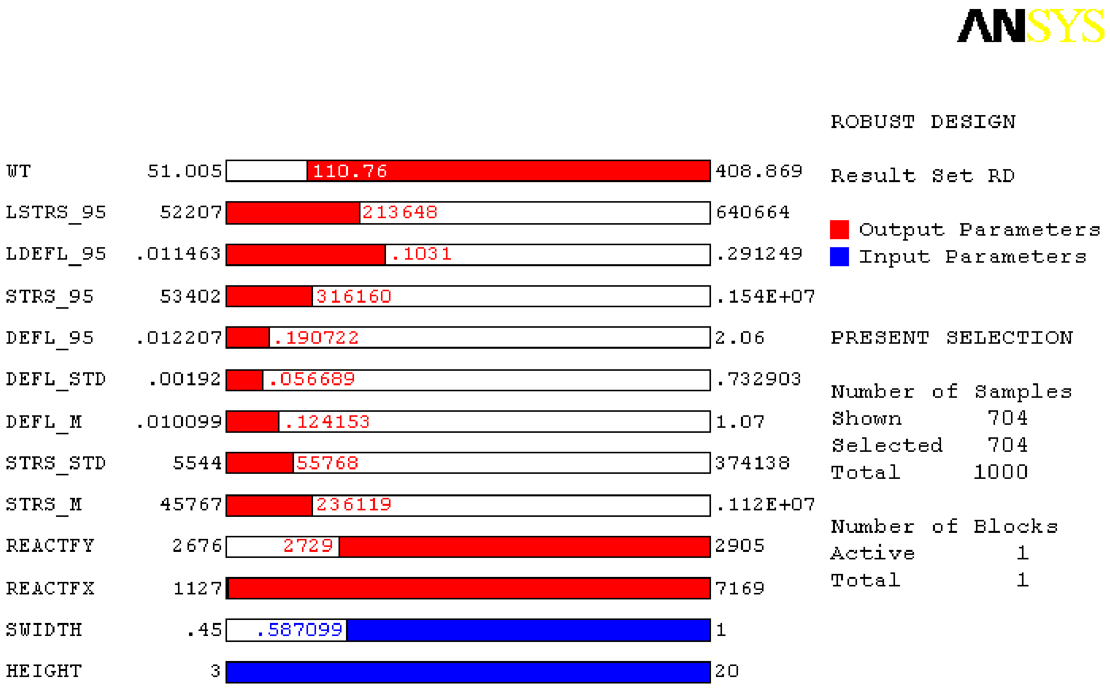


Figure 5.50: Basic Simulation: LSTRS_95>214000 unselected (Linear Stress).

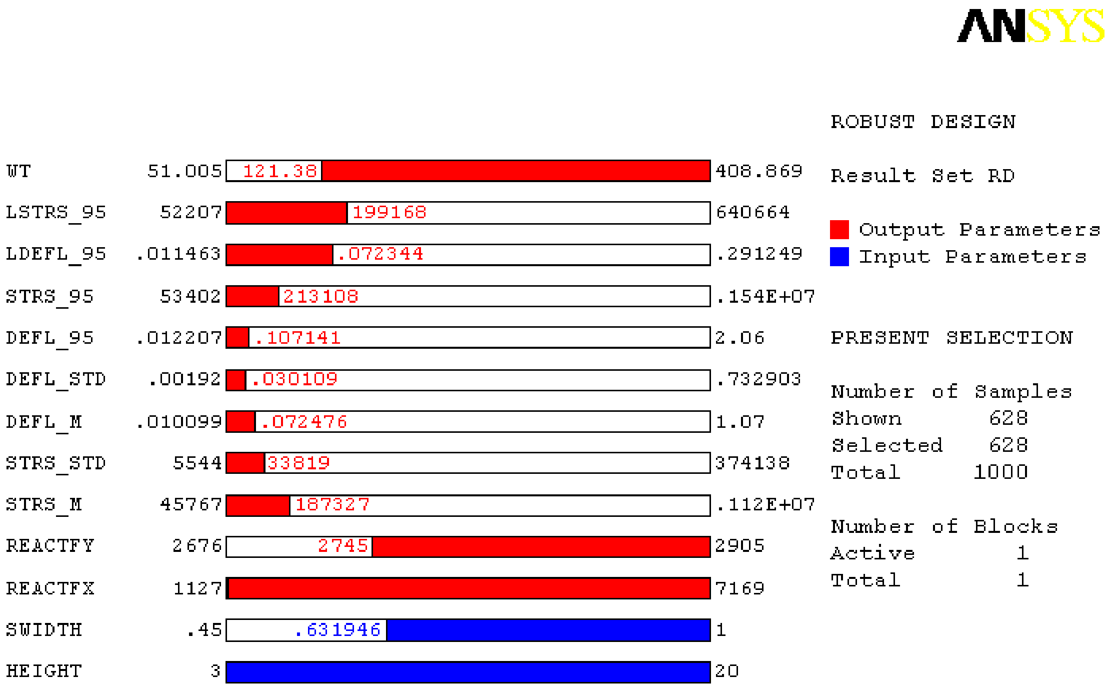


Figure 5.51: Fig. 5.50→ STRS_95>214000 unselected (Nonlinear Stress).

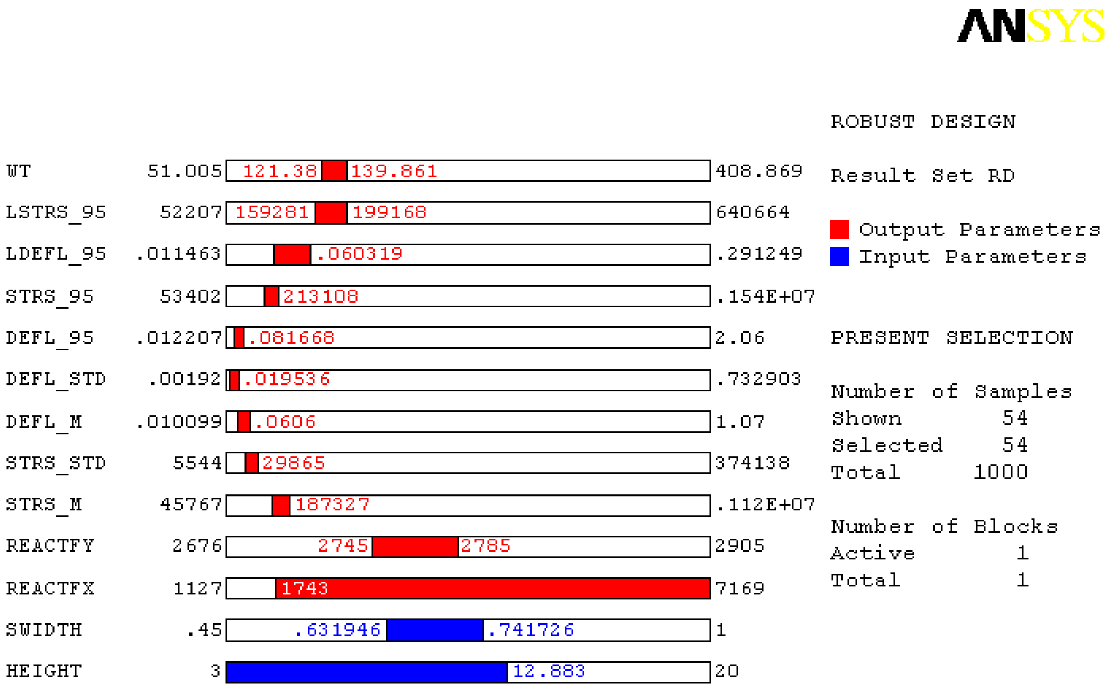


Figure 5.52: Fig. 5.51→: WT>140 unselected (WEIGHT[kN]).

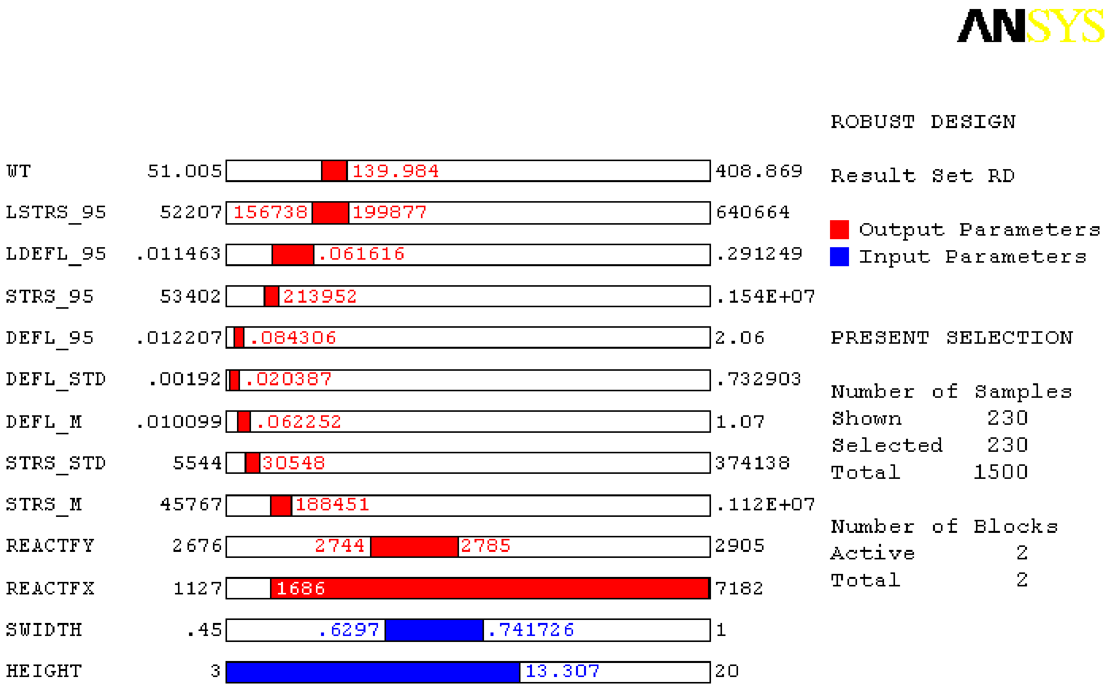


Figure 5.53: Fig. 5.52+500 samples, invalid samples unselected.

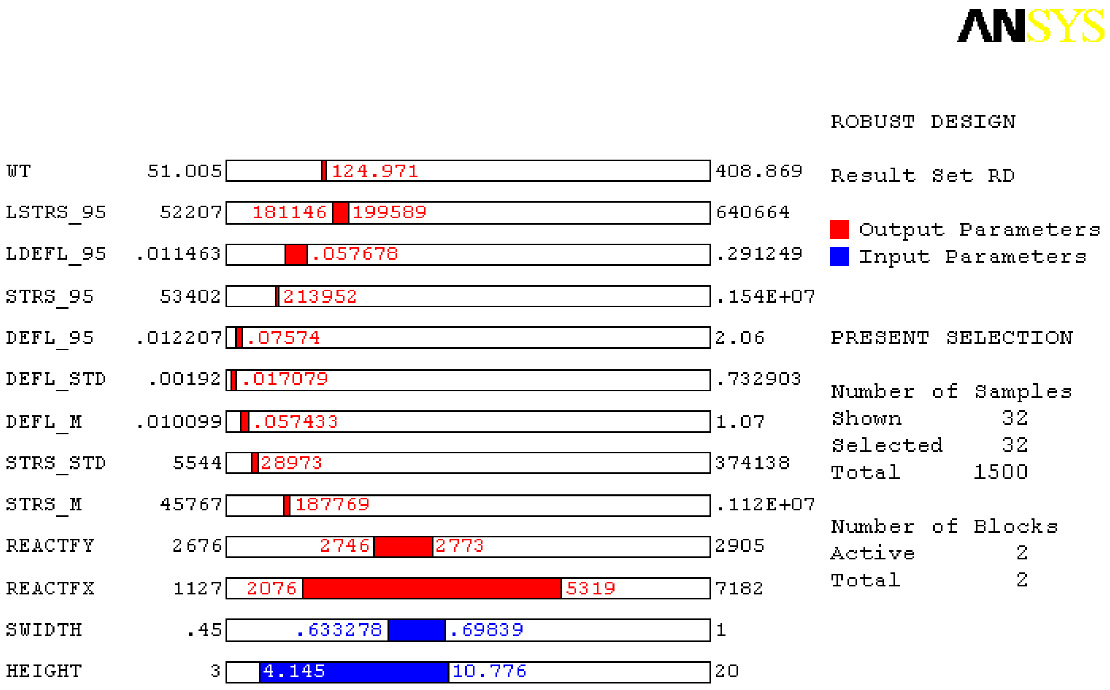


Figure 5.54: Fig. 5.53→: WT>125 unselected.

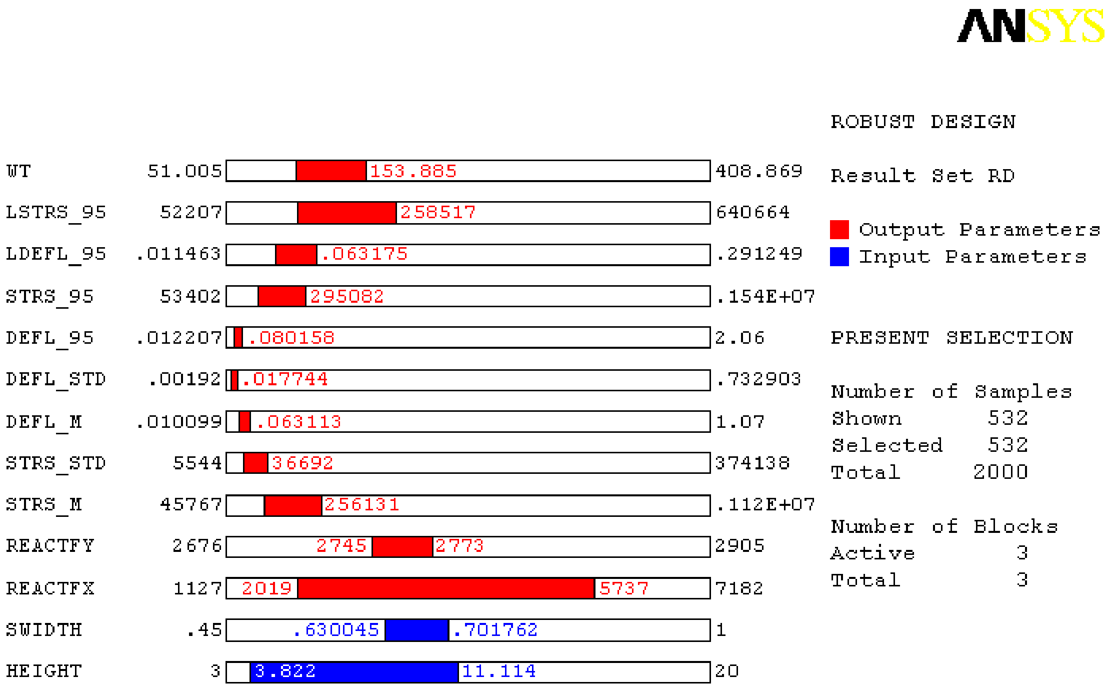


Figure 5.55: Fig. 5.54+500 samples, (no selection).

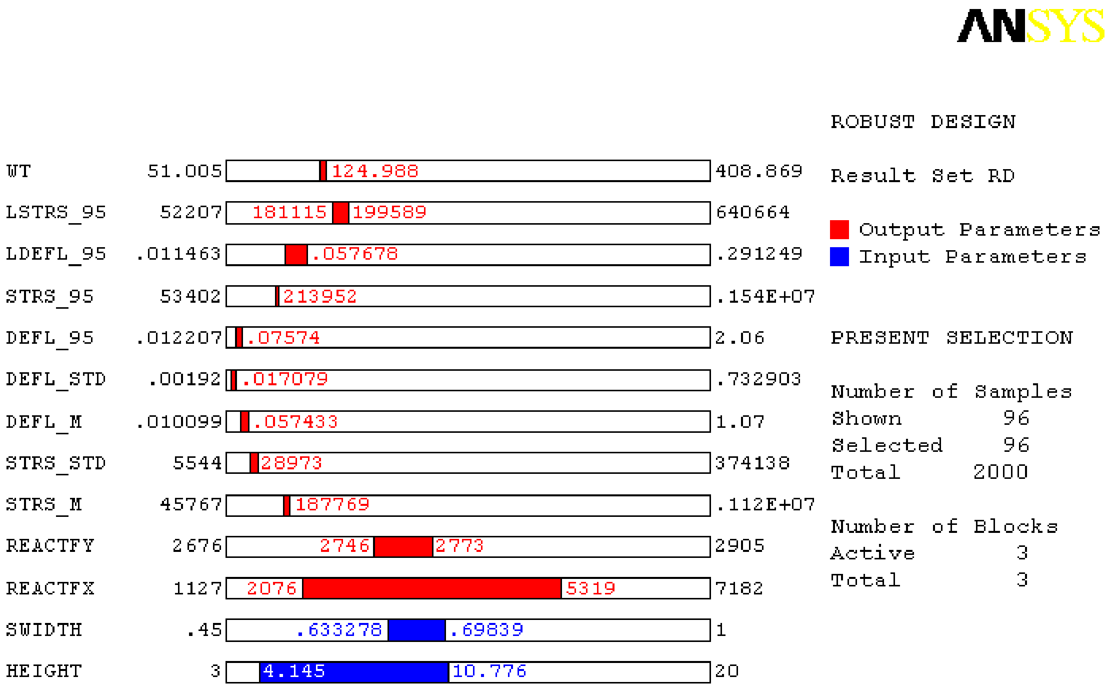


Figure 5.56: Fig. 5.55→: WT>125 unselected.

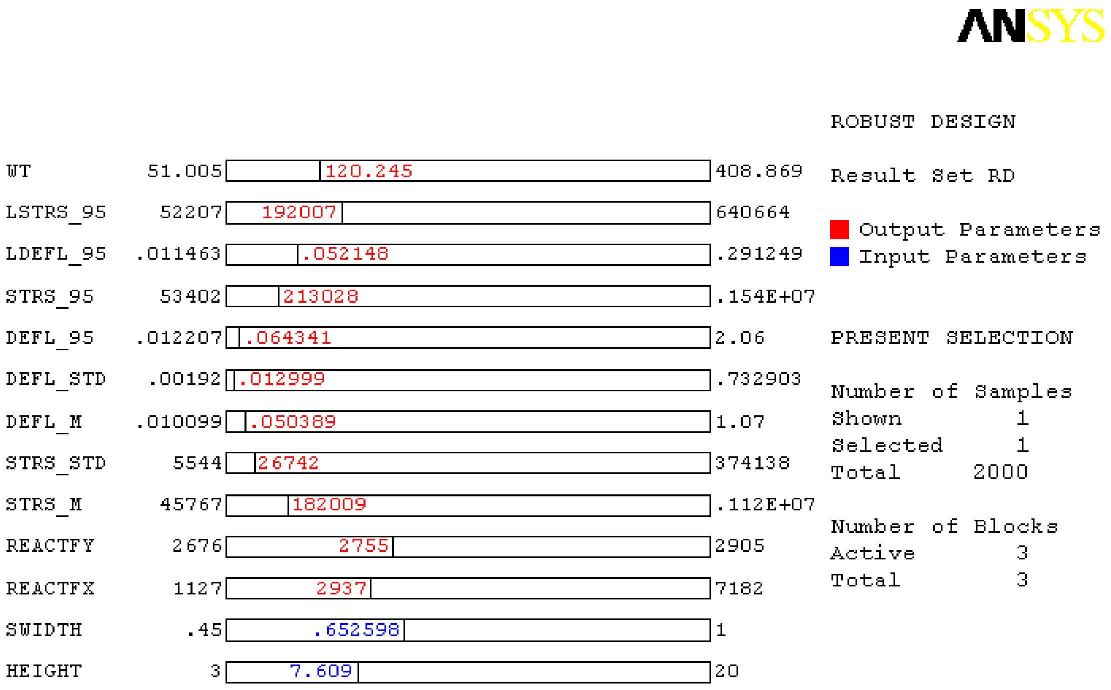
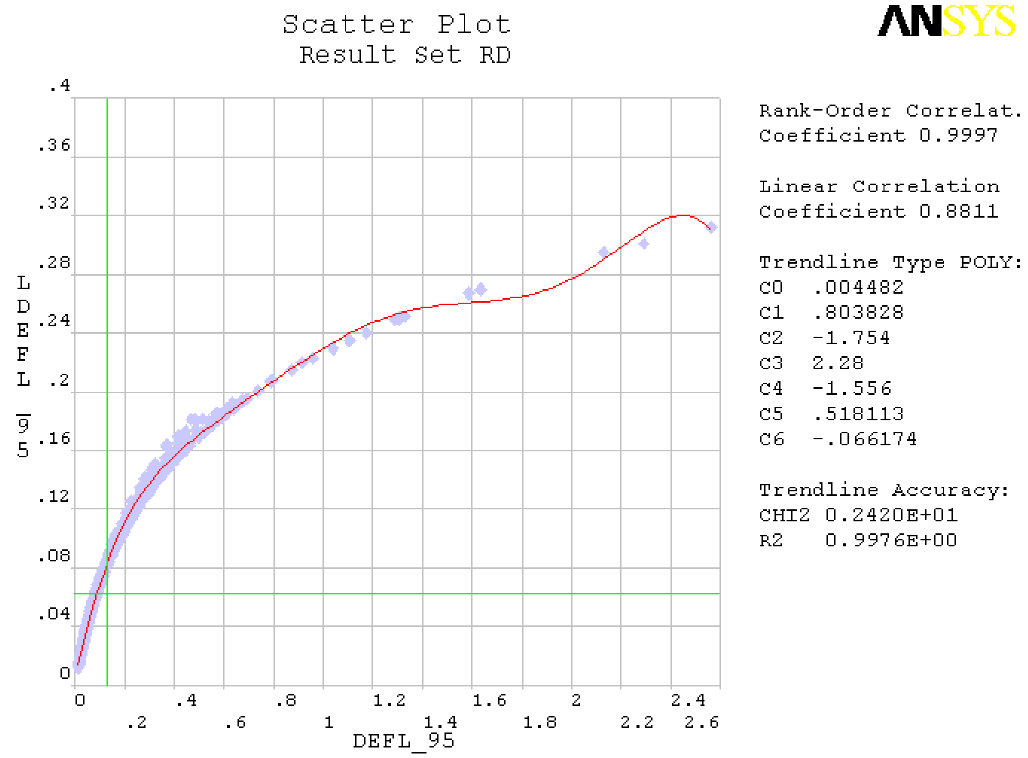
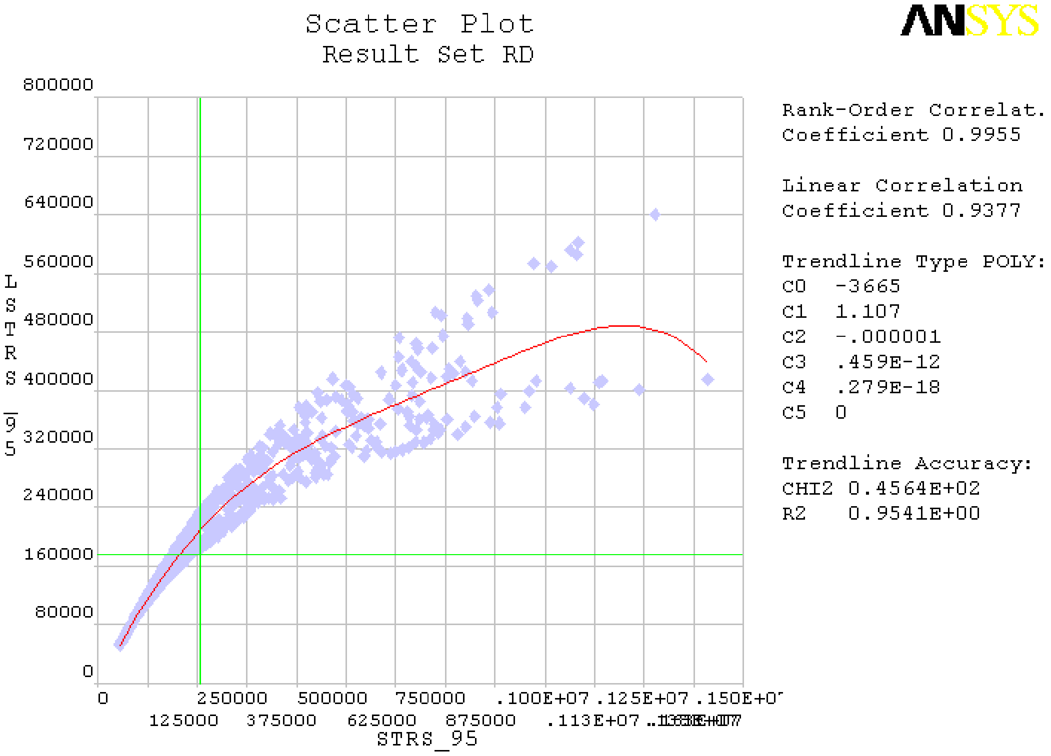


Figure 5.57: Fig. 5.56→: WT>120.246 unselected (Best Sample).



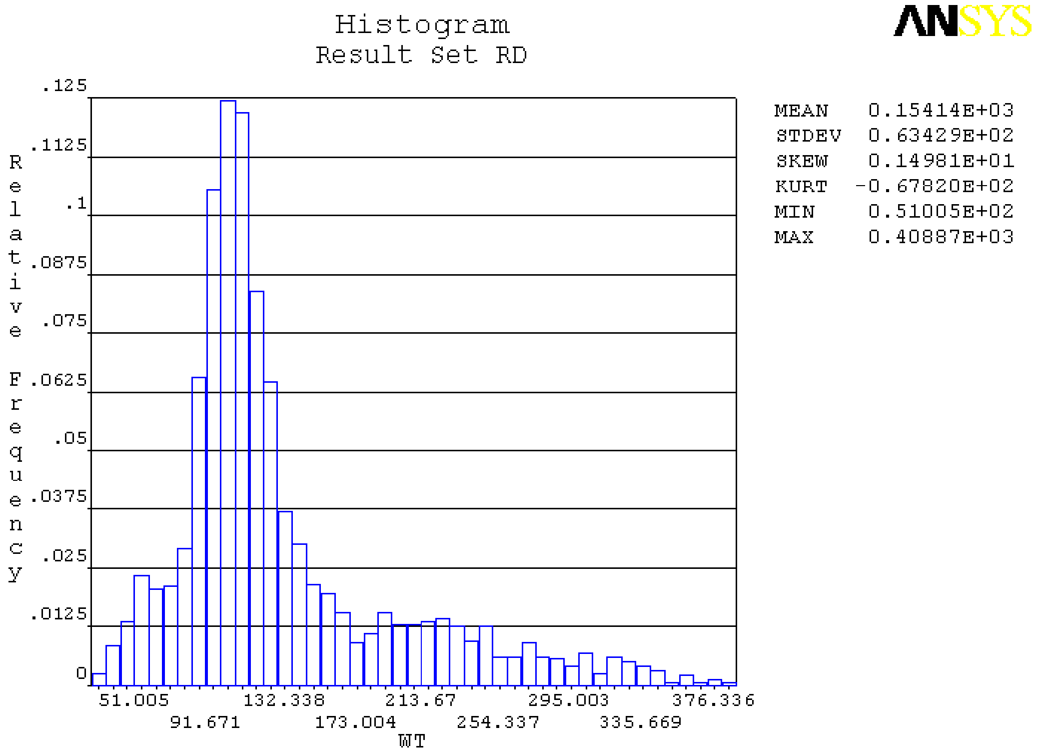
```
Rank-Order Correlat.
Coefficient 0.9997

Linear Correlation
Coefficient 0.8811

Trendline Type POLY:
C0   .004482
C1   .803828
C2   -1.754
C3   2.28
C4   -1.556
C5   .518113
C6   -.066174

Trendline Accuracy:
CHI2  0.2420E+01
R2    0.9976E+00
```

Figure 5.58: Nonlinear Deflection DEFL_95 - Linear Deflection LDEFL_95



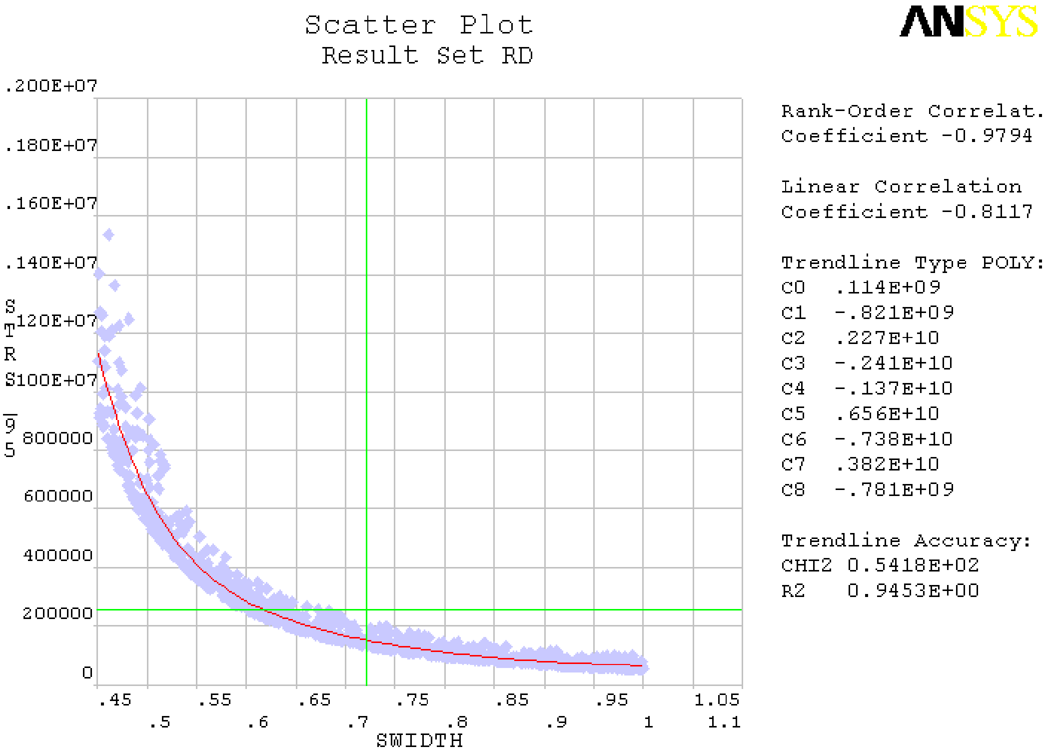
```
Rank-Order Correlat.
Coefficient 0.9955

Linear Correlation
Coefficient 0.9377

Trendline Type POLY:
C0   -3665
C1   1.107
C2   -.000001
C3   .459E-12
C4   .279E-18
C5   0

Trendline Accuracy:
CHI2  0.4564E+02
R2    0.9541E+00
```

Figure 5.59: Nonlinear Stress STRS_95 - Linear Stress LSTRS_95



```
MEAN    0.15414E+03
STDEV   0.63429E+02
SKEW    0.14981E+01
KURT   -0.67820E+02
MIN     0.51005E+02
MAX     0.40887E+03
```

Figure 5.60: Histogram: WT, weight[kN], all samples.



```
Rank-Order Correlat.
Coefficient -0.9794

Linear Correlation
Coefficient -0.8117

Trendline Type POLY:
C0   .114E+09
C1   -.821E+09
C2   .227E+10
C3   -.241E+10
C4   -.137E+10
C5   .656E+10
C6   -.738E+10
C7   .382E+10
C8   -.781E+09

Trendline Accuracy:
CHI2  0.5418E+02
R2    0.9453E+00
```

Figure 5.61: Scatter Plot: SWIDTH - STRS_95, Basic Simulation.

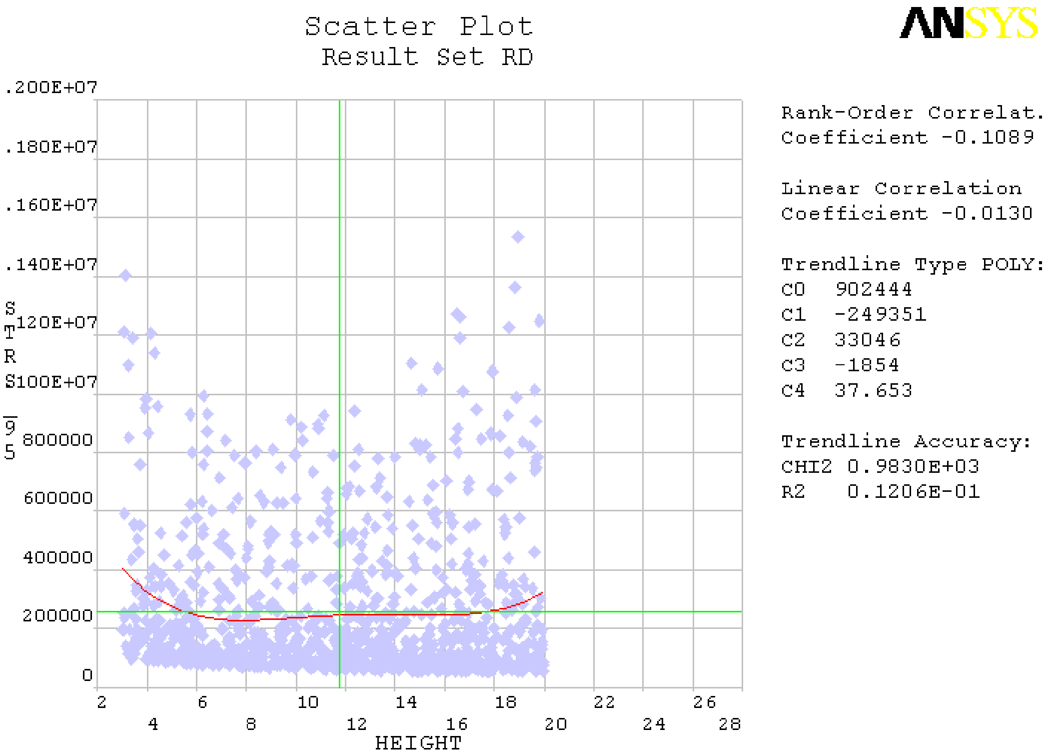


Figure 5.62: Scatter Plot: HEIGHT - STRS_95, Basic Simulation.

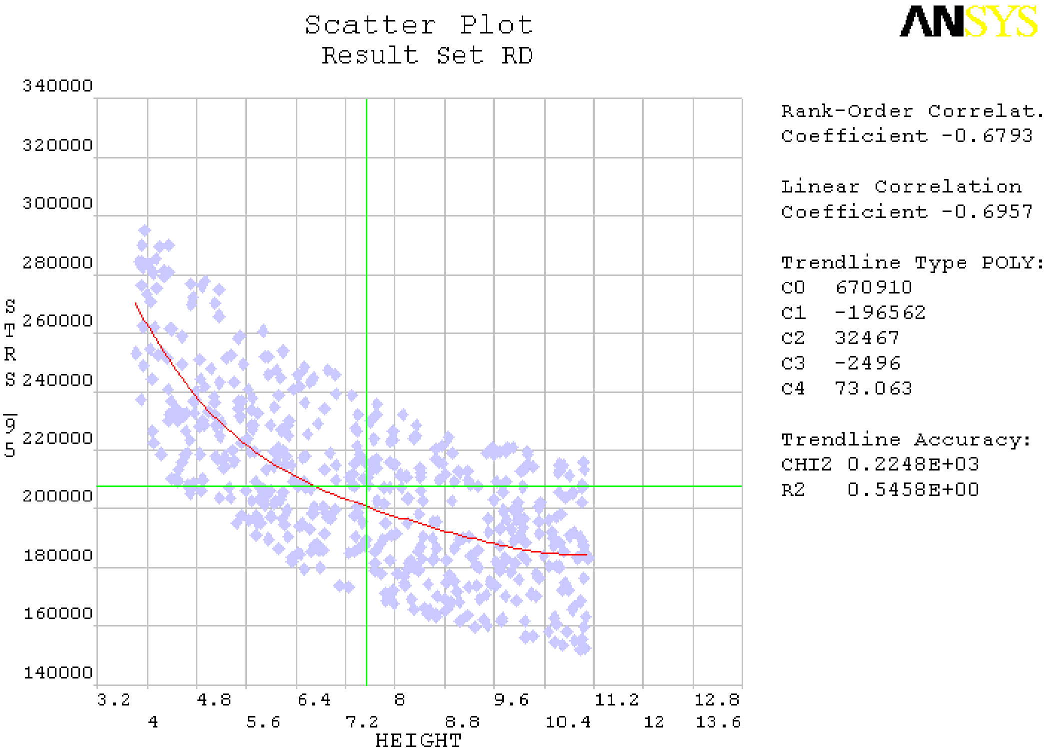


Figure 5.63: Scatter PLot: HEIGHT - STRS_95, SECOND ADAPTION.

Figures 5.50, 5.51	Both Figures show the ranges of selected samples from the basic simulation. Figure 5.50 came into being by unselecting all samples with 95% confidence value of linear stress (LSTRS_95) greater as 214000 $\frac{kN}{m2}$. The corresponding nonlinear stress values still come up to 316000 $\frac{kN}{m2}$. This demonstrates the need to perform a nonlinear analysis and to examine the corresponding nonlinear values. Figure 5.51 shows the appropriate selection for valid samples by using the nonlinear stress value (STRS_95). The serie continues as described in the captions.
Figure 5.58	This scatter plot illustrates the nonlinear character of the structure by comparing the deflection confidence values of the different analysis types. LDEFL_95 $\approx$ 0.11 m corresponds to DEFL_95 $\approx$ 0.20 m. The nonlinear effect intensifies with increasing deflection.
Figure 5.59	Similar to the last figure this scatter plot demonstrates the nonlinear character of the structure by comparing stress values.
Figure 5.60	The histogram of the weight shows a high sample density around 120 kN. This indicates that the adaptions produced many samples near to the optimum.
Figure 5.61	This scatter plot shows the correlation between the width of the arch (SWIDTH) and the 95% confidence value of nonlinear stress (STRS_95). It is remarkable, that the stress values drastically increase below 0.5 m.

Figure 5.62, 5.63

These two scatter plots both illustrate the correlation between the height of the arch (HEIGHT) and STRS_95. The upper diagram shows all samples of the basic simulation and the lower one only includes the samples of the second adaption. The trendline modifies as these two simualtions consider different ranges for the width of the arch. The second scatter plot indicates that increasing the height up to eleven meters results in decreasing stresses.

5.3.6 Best Results

The optimum result of all samples is already shown in Figure 5.57. An overview on the best results of the different blocks is given below:

	SWIDTH $[m]$	HEIGHT $[m]$	WT $[kN]$
Basic Simulation	0.671	5.99	**121.38**
1^{st} Adaption	0.642	8.92	**121.03**
2^{nd} Adaption	0.653	7.61	**120.25**

	STRS_95 $\left[\frac{kN}{m^2}\right]$	DEFL_95 $[m]$	REACTFX $[kN]$	REACTFY $[kN]$
Basic Simulation	212000	0.058	3722	2763
1^{st} Adaption	213000	0.069	2508	2751
2^{nd} Adaption	213000	0.064	2437	2755

The weight reduction from the basic simulation to the second adaption is 1.13kN, which equals a relative decrease of 0.93%. The adaptions have not brought a great improvement. This can be explained as 1000 samples of the basic simulation already thoroughly investigated the two-dimensional control parameter space. However the adaptions focused on the interesting region. This becomes clear when considering following text output, which refers to the selection of Figure 5.56.

```
ROBUST DESIGN SAMPLING -- Result Set RD
 Response Surface Set RS

   Block Nr.   ON/OFF       Samples      Selected          Shown
           0       ON          1000             9              9
           1       ON           500            23             23
           2       ON           500            64             64
   ------------------------------------------------------------
   SUM:                        2000            96             96

   Noise Variations per Sample  :  300
```

Summarized an optimal robust solution was found on the basis of 15 FEM-simulations. Besides the robust design tools enabled to discover various interesting properties of the structure. It has to be added that the scope of the thesis only allowed for creating a demonstration example. For real-life simulation it is proposed to discuss a model with more integrated noises and more control parameters.

5.3.7 Test on FEM Structure

Finally a test on the optimal FEM-structure was performed. This test can not be complete, as Robust Design sampling considered 300 noise settings for each control parameter set. Two truck positions have been picked out for a nonlinear static analysis. On page 107 the deflection and stress intensity for TRUCKPOS=0.5 are shown. The following figures show the results for TRUCKPOS=0.2. In both cases the stresses keep below the limit and the deflections are found to be acceptable. This indicates that Robust Design successfully found an optimal, valid structure. It is remarkable, that TRUCKPOS=0.2 leads to higher stresses and to a double maximum deflection as the symmetrical loadcase.

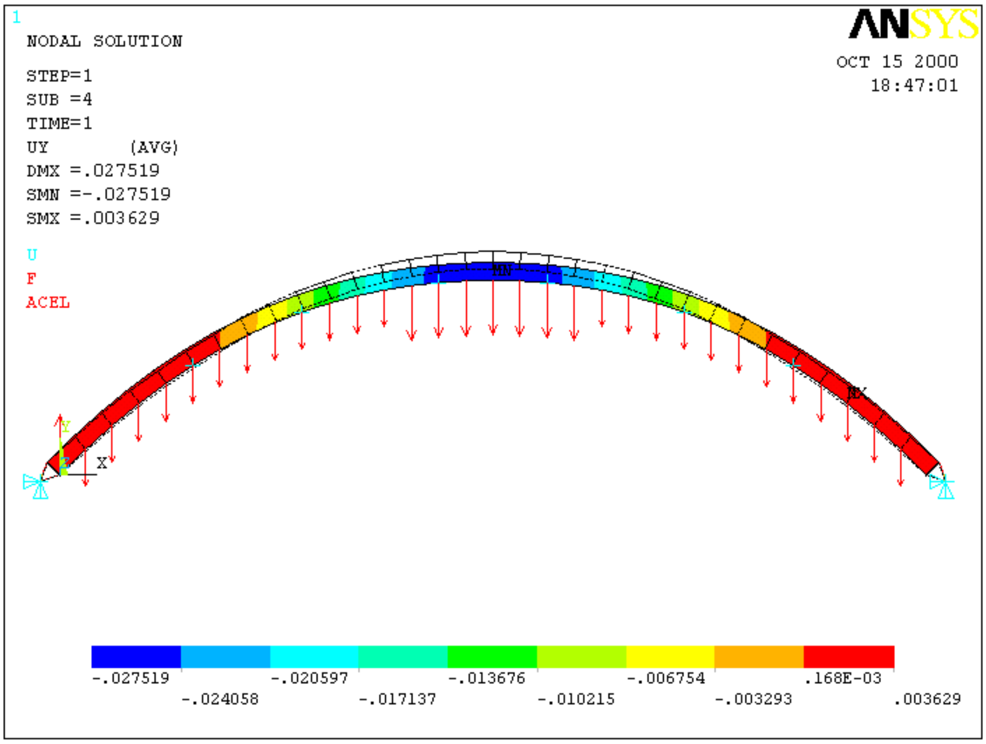


Figure 5.64: Deflection UY, TRUCKPOS=**0.5**, (nonlinear)

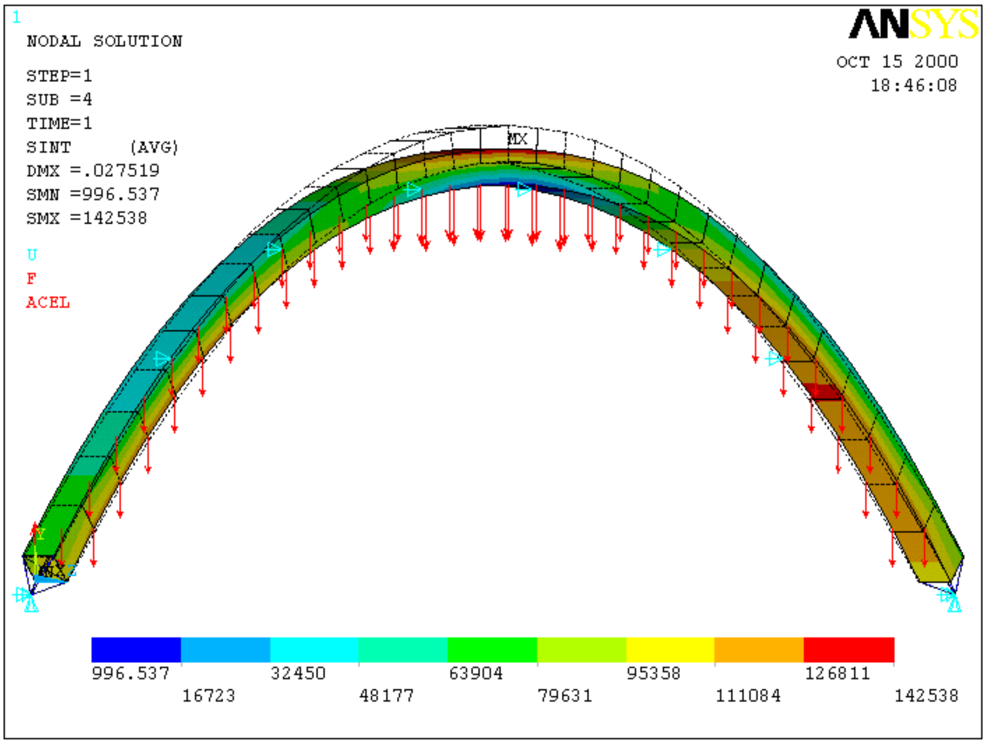


Figure 5.65: Stress Intensity SINT, TRUCKPOS=**0.5**, (nonlinear)

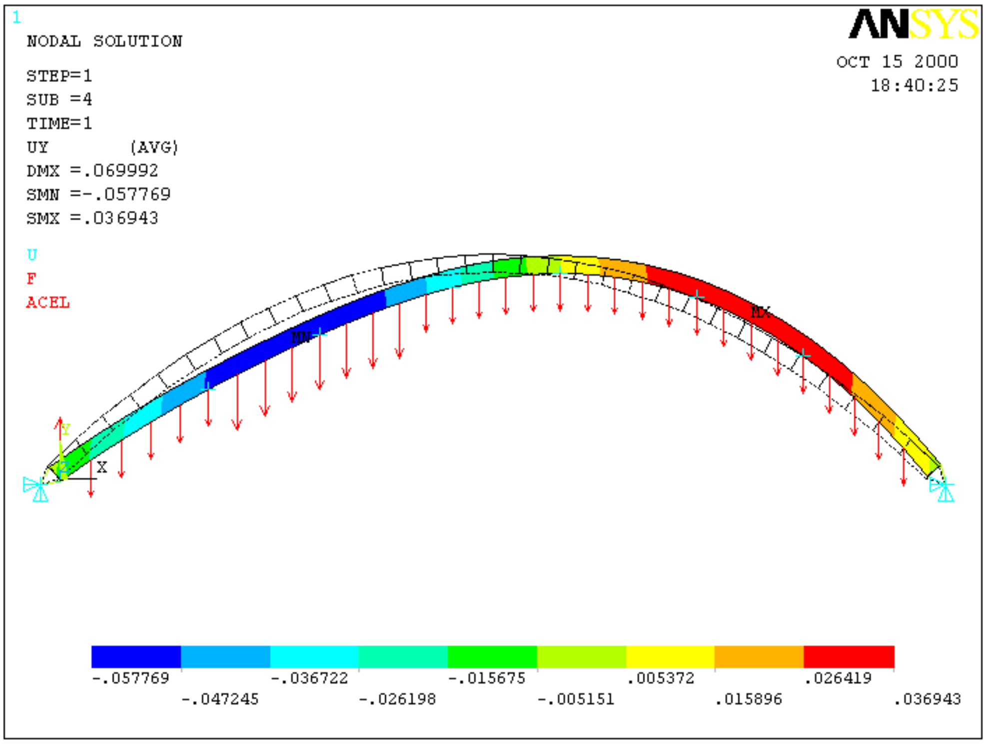


Figure 5.66: Deflection UY, TRUCKPOS=**0.2**, (nonlinear)

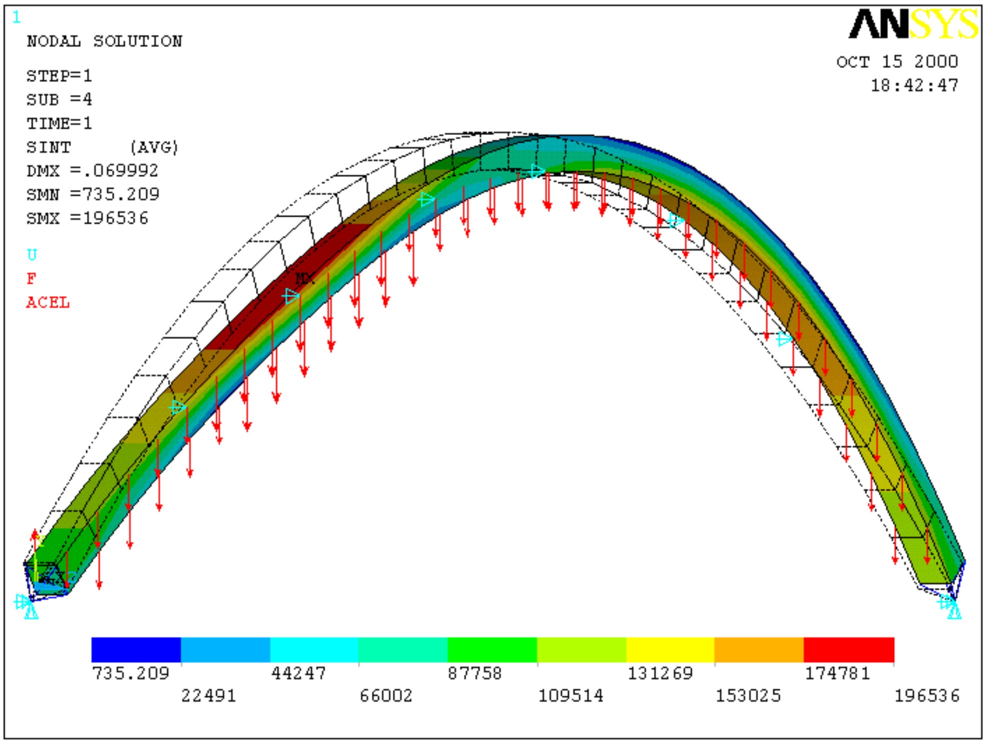


Figure 5.67: Stress Intensity SINT, TRUCKPOS=**0.2**, (nonlinear)

Chapter 6

Summary

The research on Robust Design revealed several inaccuracies and ineffeciences of the original Taguchi method. Taguchi's introduced design of experiments can be inefficient, the S/N-ratios as quality measures can be misleading and the proposed data analysis does not always guarantee proper results. Nevertheless the Taguchi method has been very successful in designing high-quality products and processes of many different fields. Besides Taguchi's algorithms, the effect of his new philosphy should not be underestimated. The design horizon is enlarged to all technical and econonomical properties important to the product. The strategic analysis on the effect of noises in the early design phase is superior to conventional procedures. The numerous combinations of design parameter settings can not efficiently be controlled by human judgement, which results in time and cost consuming prototyping, testing, revising, testing, revising and so on. The Taguchi method offers a strategy for finding optimal, stable results based on a predefined set of analyzed parameter combinations.

Robust Design using response surfaces takes up the concepts of the Taguchi method and offers a standard, homogen procedure based on actual, scientific knowledge. Response surfaces are expected to gain more accurate answers on system behaviour and interaction effects, especially when created on basis of fractional factorial designs. Quick sampling procedures on Response Surfaces allow for rich statistics and for the use of confidence values, which have advantage over S/N-ratios. The best sample directly indicates the optimum solution and the corresponding optimum design parameters. Furthermore several data analysis methods are available for gaining valuable, additional information about the analyzed system. Only limited literature has been found about Robust Design using response surfaces. So the theory of this method is based on the Probabilistic Design System of ANSYS 5.7 and has further been developed for the implementation.

The implementation successfully realizes Robust Design using response surfaces. A rounded, new feature has been created in ANSYS, which includes all main functionalities for the practical use of this method. Appropriate graphical outputs have been adopted from the Probabilistic Design System. For achieving higher efficiency, the functionality of performing adapted simulations has been created. A remarkable approach of this implementation, is the possibility to select adaptions and samples by user-defined criteria in combination with a new graphical tool. This new type of diagram allows to watch the ranges of all input and output parameters for any user-defined selection. So while other graphical tools only allow for analyzing a two- or three-dimensional subspace, this diagram gives an overview on settings of up to fourty parameters.

The implemented code has been verified in connection with a published demonstration example of the Taguchi method using an equation model. For demonstrating the practical usability of the implementation, two examples from the field of civil engineering have been introduced. The first example of a braced bending beam graphically combines economical and technical criteria corresponding to Taguchi's concept of Quality Engineering. A special result is a diagram showing the correlation between cost and stress of all valid structures. The second example is a weight optimization for designing an arch robust to unsymmetrical loads. The special aspect of this example is the combination of linear and nonlinear static analysis. For both examples optimized, robust structures have been determined. The optimal solutions are reasonable and have been selectively checked by performing corresponding FEM-simulations. For both examples certain parameter restrictions had to be considered, which was possible by making selections in combination with the new, implemented graphical tool (see Appendix, pages 113-114). It has to be noted, that these examples are simplified and can not be seen as representative for the FEM-structures, which can be optimized by this tool. The convenient use of Robust Design increases with complexity of the system and besides other possible applications in civil engineering, the variety of analysis methods in ANSYS offers a high potential for optimizing FEM-models of other disciplines (e.g. Thermodynamics, Magnetics).

It is emphasized, that the Taguchi Method and Robust Design using response surfaces are approximation methods, which automatically include a certain error. The deviation from the real simulation should generally not be significant, but it can also be very high in the worst case. For Robust Design using response surfaces the accuracy of the response surfaces is of central importance. Therefore the Probabilistic Design System provides several goodness-of-fit measures, which allow for an estimation of the error. An error source faced within the project, is the possible variation of discretization errors, when creating a flexible, parametric model. This could systematically lead to identyfing the wrong effects and has to be excluded. Additional sources for errors include: inaccurate structural models, inadequate stochastic models, many error types when creating and analyzing FEM-models, wrong use of implemented methods, poor statistics and misinterpretation of results. Summarized Robust Design using response surfaces includes several error sources, which can be minimized by the competent user. The approximation error can roughly be estimated. It is proposed, to develope systematic concepts and algorithms for receiving more detailed answers on the accuracy of the result. Confirmation experiments are always useful, but do not represent a real alternative to an error estimation.

Robust Design using response surfaces can reasonably be used for up to 25-40 input parameters. A higher number results in too many simulations necessary for building the response surfaces. Linear response surfaces afford much less simulations, but might often not be sufficiently adequate. Robust Design using response surfaces can not unlimited be used for any optimization problem. Certain curvatures of simulation response can not adequatly be approximated by a second-order regression model. This would for example have been the case when the unsymmetrical load of the arch-example would have been discussed between position 0.0 and 1.0, instead of 0.0 and 0.5. As the deflection and stress follow the following scheme with increasing the load position: 0.0: low; 0.25: high; 0.5:low; 0.75:high; 1.0:low. Such alterniering effects can often previously be identified and excluded by modifying the model without loosing quality. The accuracy of the response surfaces further depends on the considered ranges of control parameters. If these have been set very wide, the response surfaces might not lead to good answers. But they can still indicate reduced ranges, which include the optimum and allow for better fits. This effect can be used for automatically adapting response surfaces. A user-controlled adaption is demonstrated in the braced bending beam example. Another aspect is that each control parameter set should lead to a feasible simulations or special considerations have to be made if this is might not be the case, like in the arch-example.

In consideration of real optimization problems, the implemented method takes into account that several output parameters might have to be controlled and certain restrictions have to be included. Generally a prototype of Robust Design has been developed, which is widely applicable. The actual optimization procedure is relative quick, easy to use and understand. Most effort was spent on previous considerations and in creating a proper, parametric FEM-model, when using this method. The main characteristic of the implemented method in comparison to other optimization methods is the strategic integration of noise influences in the design process. Engineers have always been interested in finding solutions, which are robust to noise influences. But generally it was not possible to consider the full variety of noises during the design phase. This new concept can be very efficient in finding optimal and stable solutions. The Taguchi method already proved its success and left a high potential for improvments. Robust Design using response surfaces has several advantages over the Taguchi method. But there is still room for improving certain algorithms, simplifying the complete procedure and introducing exacter error estimation.

Appendix A

Range Plots

A.1 Number of Parameters

The implemented graphical tool for plotting the ranges of selected samples automatically adjusts for up to fourty parameters. For demonstration purpose only, the range plot shown in Figure A.1 has been created.

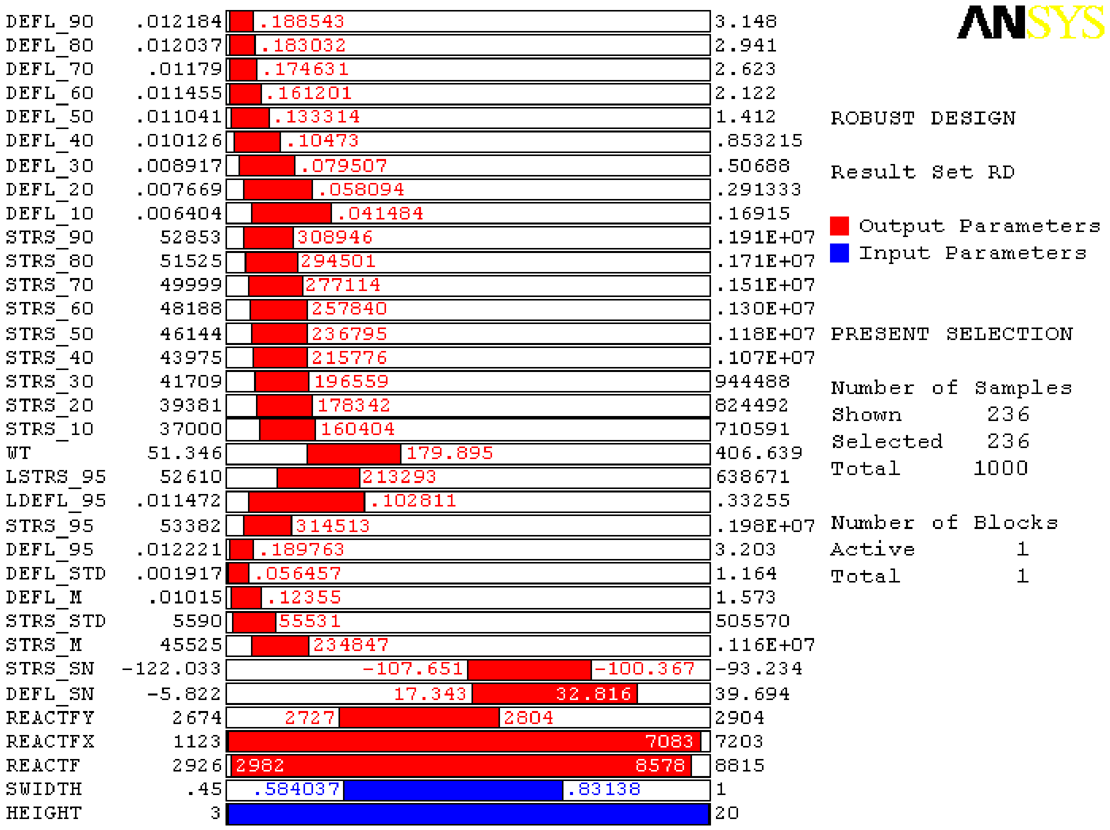


Figure A.1: Demonstration of Range Plot for 34 parameters.

A.2 Animations

A CD is attached to the back of the diploma thesis. The CD contains two animated sequences of Range Plots. The file “beam.avi” refers to the example “Braced Bending Beam” (section 5.2) and the file “arch.avi” refers to the example “Hollow Section Arch” (section 5.3). The results of the animations slightly vary to the documented results, as probably other settings have been used for Robust Design Sampling (e.g. number of noises). But this should not detract from the demonstration character of these animations. An overview on the sequences is given below. For more information it is refered to the corresponding sections 5.2 and 5.3.

beam.avi : Braced Bending Beam

Seconds	
0	All samples are selected.
1	STRESS_95 > 214000 unselected.
2	TENSION_95 > 214000 unselected.
4	H_I < 220 unselected.
5 - 55	COST_95 > c_t unselected, while c_t is decreasing from 3155 to 1925.
55 - 56	Only best valid sample is selected.

arch.avi : Hollow Section Arch

Seconds	
0	All samples are selected.
1	LSTRESS_95 > 214000 unselected.
2	STRESS_95 > 214000 unselected.
3	DEFL_95 > 0.10 unselected.
4 - 31	WT > c_t unselected, while c_t is decreasing from 402.65 to 120.20.
31 - 33	Only best valid sample is selected.

Literature

[1] Annis, J. R., “Application of 3-D magnetostatic analysis and Taguchi methods to sealed switch design”, ANSYS Magnetics Symposium, Swanson Analysis Systems, Inc., May 1994.

[2] Cashman, J.: “Future of engineering simulation”, *ANSYS Solutions*, Vol. II, No. 1, Winter 2000, pp. 3-4.

[3] Chen, W.; Allen, J. K.; Tsui, Kwok-Leung; Mistree; F.: “A procedure for robust design: minimizing variations caused by noise factors and control factors”, *Journal of Mechanical Design*, ASME, Vol. 118, December 1996, pp. 478-485.

[4] Eriksson, M.“Using Design-of-Experiments Techniques for an Efficient Finite Element Study of the Influence of Changed Parameters in Design”, ANSYS Conference 1998, August 1998.

[5] Madsen, H. O.; Krenk, S.; Lind N. C.: “Methods of structural safety”, Prentice Hall, Englewood Cliffs, New Jersey (1986).

[6] Matsui, S.,“Design optimization by the design of experiment and response surface method”, ANSYS Conference 2000, August 2000.

[7] Montgomery, D. C.: “Design and analysis of experiments”, 3rd ed., John Wiley & Sons, New York (1991).

[8] Myers, R. H.; Montgomery, D.C.: “Response Surface Methodology”, John Wiley & Sons, New York (1995).

[9] Nellian, S.: “Integration of Taguchi Design of Experiments and Finite Element Method for Robust Design,” Master’s project, Department of Mechanical Engineering, University of Massachusetts, Amherst, MA 1996.
Internet: *http://www.ecs.umass.edu/mie/labs/mda/fea/sankar/layout.html*

[10] Phadke, M. S.: “Quality engineering using robust design”, Prentice Hall, Englewood Cliffs, New Jersey (1989).

[11] Reh, S.: “On the accuracy of various probabilistic methods”, 41^{st} AIAA / ASME / ASCE / AHS / ASC SDM Conference, AIAA 2000-1631, April, (2000).

[12] Roy, R.: “A primer on the taguchi method”, Van Nostrand Reinhold, New York, (1990).

[13] Stoecker, W.F.: “Design of Thermal Systems”, Third Edition, McGraw-Hill, New York, (1989), pp. 149-152.

[14] Taguchi, G.; Chowdhury, S.; Taguchi, S.: “Robust engineering”, McGraw-Hill, New York, (1999).

[15] Unal, R.; Dean, E. B.: "Design for Cost and Quality: The Robust Design Approach",*Journal of Parametrics*, Vol. XI, No. 1, August 1991.
Internet: *http://mijuno.larc.nasa.gov/pap/robdes/robdes.html*

Erklärung

Ich erkläre, daß ich die vorliegende Arbeit selbständig und nur unter Verwendung der angegebenen Quellen und Hilfsmittel angefertigt habe.

Weimar, den 29.10.2000

Theses

1. The term Robust Design arised from the Taguchi method, which has been the starting point of this work. Its success in efficiently designing high quality products results from combining unique algorithms with a new philosophy. A consequent integration of all important design parameters and a strategic study of noise influences in the early design process characterize the Taguchi method.

2. Taguchi proposes, that quality should best be designed into a product, instead of trying to achieve quality by inspection or later revisions. Economical and technical criteria with regard to the complete life cycle of a product should be considered.

3. The procedures of the Taguchi method can be relatively complex and obscure. The method shows several minor inaccuracies, which can lead to ineffiencies and in very few cases to incorrect answers. It is expected to gain improvements in reducing the number of experiments and in achievieng a more accurate data analysis by using Robust Design on response surfaces.

4. In this work confidence values have been introduced as quality measures for achieving stable results at optimum levels. They show somel advantages over Taguchi's S/N-ratios.

5. Robust Design using response surfaces uses continuous distributions and enables a thorough exploration of the design parameter space. The Taguchi method only allows for examining certain levels and margin values. But this might more likely lead to worst case configurations. An additional examination of margin values in Robust Design using response surfaces can be useful.

6. A prototype of Robust Design using response surfaces was created, which includes all main functionalities. A remarkable new feature is the selection of samples in combination with a new graphical tool.

7. Routines for performing adapted simulations have been created. They effectively produce more samples in the optimum regions and lead to the optimum result with less computation effort. Furthermore, by creating this functionality, adaption algorithms can quickly be modified or replaced.
8. Several output parameters can easily be controlled in the optimization process. Restrictions to parameters can be considered. Multiple objectives can be realized by a user-controlled process. Key function is the selection of samples.
9. The implemented routines have been verified in connection with a published example of the Taguchi method. Two demonstration examples from the field of civil engineering have been created. The implemented routines successfully found optimum solutions.
10. Besides the optimum solution, valuable information about the examined structure can be gained by analyzing histograms, sensitivity plots or scatter plots. These tools can be used for any selection of samples and for any parameters. An interesting example of this work is a scatter plot between stress and cost of valid structures.
11. The optimization procedure can be performed relative quickly. The greatest effort lies in previous considerations and in building a proper, parametric FEM-model.
12. The approximation error of Robust Design using response surfaces, mostly depends on the accuracy of the response surfaces. Probabilistic Design System provides several goodness-of-fit measures. Further developments are proposed for a better error estimation of Robust Design results.
13. The fit of response surfaces improves, if the ranges are reduced. This favours the development of adapted response surfaces.
14. An appropriate safety concept for structures of civil engineering is still missing, when working with Robust Design's statistical values. An immense increase of noise sets for specified optimum control parameters principally allows for estimating low failure probabilities of structures. (Approximation error should be included.)