



# PARTICLE SIZE AND SHAPE – IMPORTANT FACTORS FOR PACKING DENSITY

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1. Definitions of particle size and shape and bulk density
2. Packing density - state of the art
3. Measuring methods
4. Results

## Particle size

A non spherical particle has many dimensions. Which dimension is measured as particle size depends on:

- the measuring method
- the characteristic particle feature and
- the particle morphology



**Each measuring method has an own definition!**

Basically the particle size is defined as the equivalent diameter of a ball, which has the same measuring effect like the real particle.



## Particle shape

### Fundamental definitions:

- Ratio of particle length to particle width and the resultant fraction

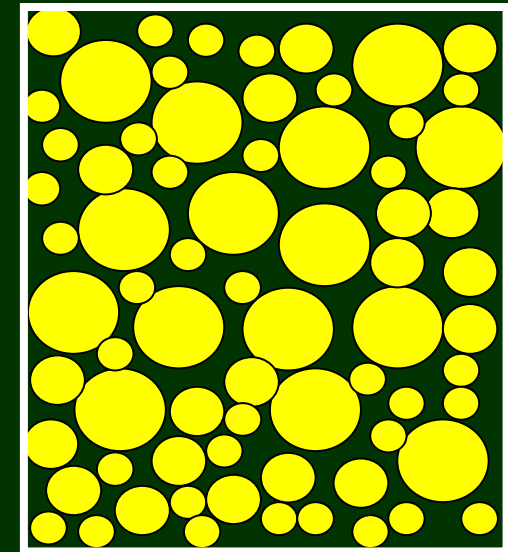
$$\frac{L}{W}$$

- Sphericity as ratio of measured circumference of particle projection and the circumference of a circle of the same area

$$SPHT = \frac{U}{2 \cdot \sqrt{\pi \cdot A}}$$

## Packing density

- Total volume  $V_t =$   
 solid volume  $V_s +$  void volume  $V_v$
- Void porosity  $\varepsilon = V_v / V_t$
- Packing density  $\Phi = V_s / V_t$
- Unit mass  $\rho_{um} = \Phi \cdot \rho_{particle}$  bulk density

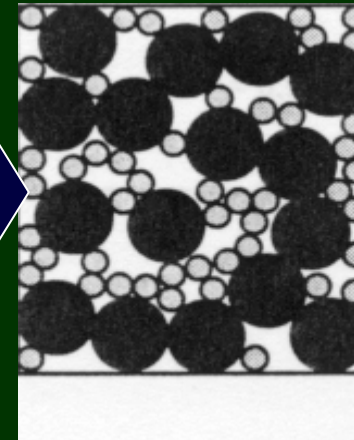
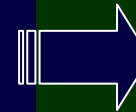


Conditions of measurement influence void porosity and packing density!

## Packing density - conditions of measurement

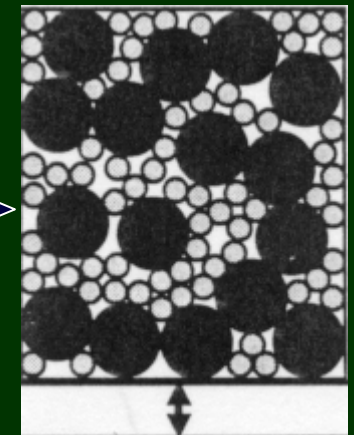
### Loose packing density:

The granular material is filled in a defined volume without any compaction.



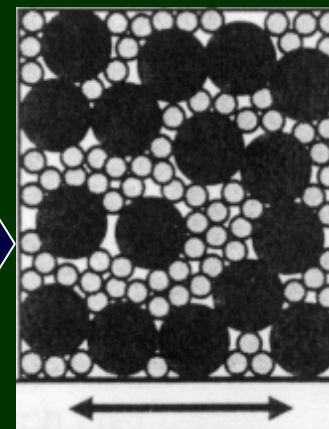
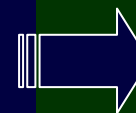
### Tap density:

The granular material is compacted by shocks.



### Vibrating density:

The granular material is compacted by vibration.





### Early works about packing density

Experimental and theoretical approaches to find grading curves of aggregates with a minimum of voids

Fuller and Thomson	1907	Grading curve with a minimum of voids determined by experimental work
Furnas	1931	Packing theory for sphere shaped particles
Andreasen and Andersen	1930	Packing density of ceramic materials
Manegold	1955	Packing density in geotechnical applications



### Early works about packing density

Experimental and theoretical approaches to find grading curves of aggregates with a minimum of voids

Fuller and Thomson	1907	$Q = (x/x_{\max})^m; m = 0,5$
Furnas	1931	Base of model: Small particles fill out the cavities between the big particles without disturbing the packing of the big particles.
Andreasen and Andersen	1930	Packing density of ceramic materials
Manegold	1955	Packing density in geotechnical applications



### Works in the „pre computer“ age

- Broadening and deepening of the earlier approaches
- Development of models for random packing of spheres
- Empirical consideration of influence of shape

Hummel	1959	Experimental determination of void porosity of aggregates. Parameters: Fuller exponent and the particle shape.
Schwanda	1959	Calculation scheme on basis of void porosity of single sized spheres considering interactions between particles. Parameters: Fuller exponent and empirically characterized particle shape.



### Works in the „pre computer“ age

Peronius and Sweeting	1985	Empirical equation of void porosity as function of Fuller exponent, particle shape and compaction. Shape characterized by Powers' scale of roundness.
Aberg	1992	Empirical equation of void porosity of any particle size distribution as function of particle shape. Shape characterized empirically.
Yu, Standish, and Zou	1993 1997	Theoretical model of void porosity for continuous or mixed single-sized distributions of different shape. Experimental calibration necessary.
Tsirel	1997	Empirical equation of void porosity of particle size distributions acc. to Fuller as function of particle shape.



### Works in the „computer“ age

- Numerical modeling of packing density of spheres
- First analytical consideration of influence of shape

Stovall et al., Glavind et al.	1986 1999	Model on basis of packing of binary mixtures, extended to multi-sized mixtures. Inclusion of experimentally determined packing densities.
De Larrard	1999	Packing model of multi-sized granular materials considering interactions between the particles and wall effects. Experimentally determined parameters included.



### Works in the „computer“ age

Stroeven and Stroeven	1999	DEM-simulation of dense random packs of spheres with different size distributions.
Stoyan et al.	2001	Spatial-statistical analyses for simulated random packings of spheres with random diameters
Latham et al.	2001	Space filling tetrahedron assembly model and dynamic interaction model for real-shaped particles.

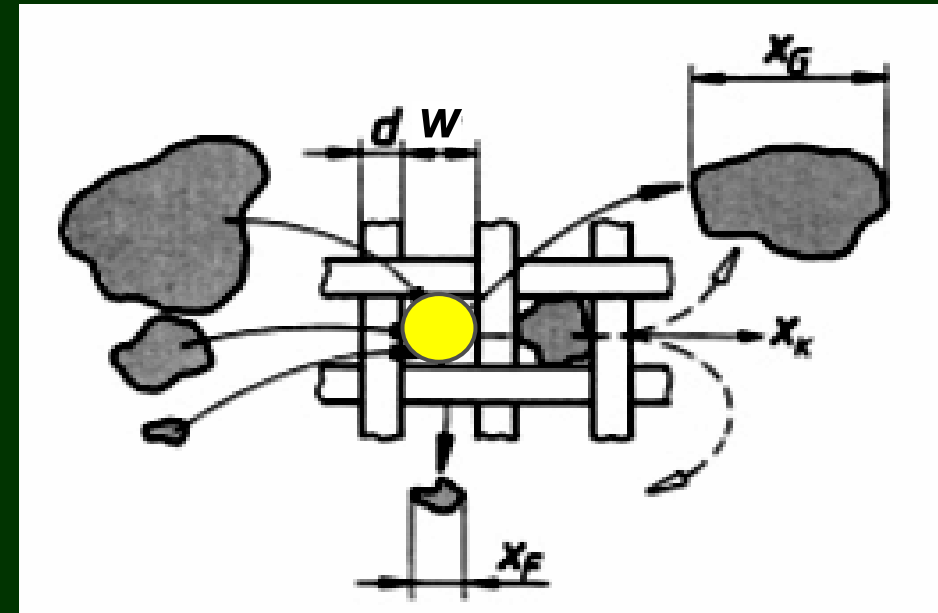
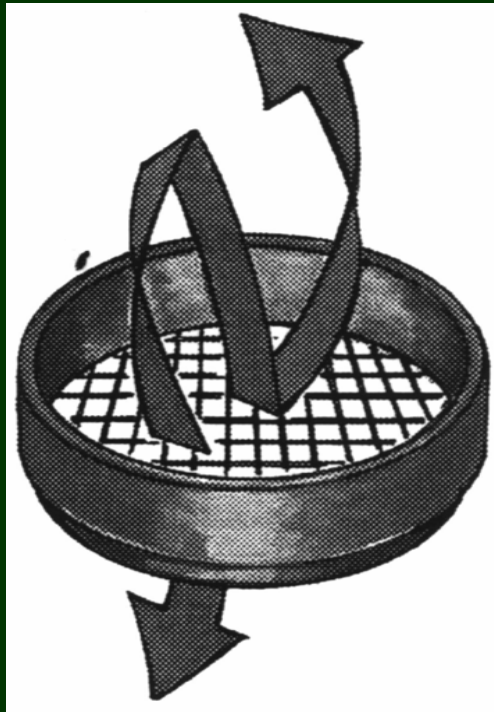


### Comparison of “gravel” and “crushed aggregates” acc. to different measurements and models:

$$\Delta = 100 \cdot (\varepsilon_{\text{gravel}} - \varepsilon_{\text{crushed aggregates}}) / \varepsilon_{\text{crushed aggregates}}$$

	$\varepsilon$		$\Delta$	Compared at
	“gravel”	“crushed Aggr.”		
Hummel	0,188	0,235	22 %	$\varepsilon = \text{min}$
Schwanda	0,132	0,244	46 %	$\varepsilon = \text{min}$
Peronius	0,130	0,235	45 %	$m = 0,5 = \text{const}$
Tsirel	0,176	0,217	19 %	$m = 0,5 = \text{const}$
Latham	0,374	0,568	34 %	$\varepsilon = \text{min}$

## Sieving method



**Geometrical comparison**  
 between particle size  
 and sieve aperture

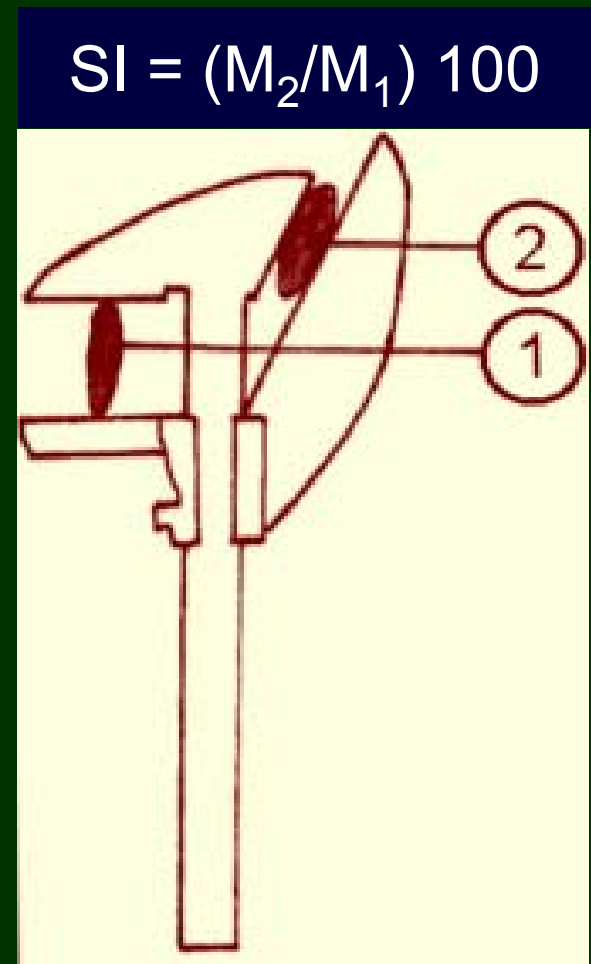
**Particle size** = equivalent  
 ball diameter with size of  
 mesh  $w$

## Particle shape with gauge method – Shape Index SI

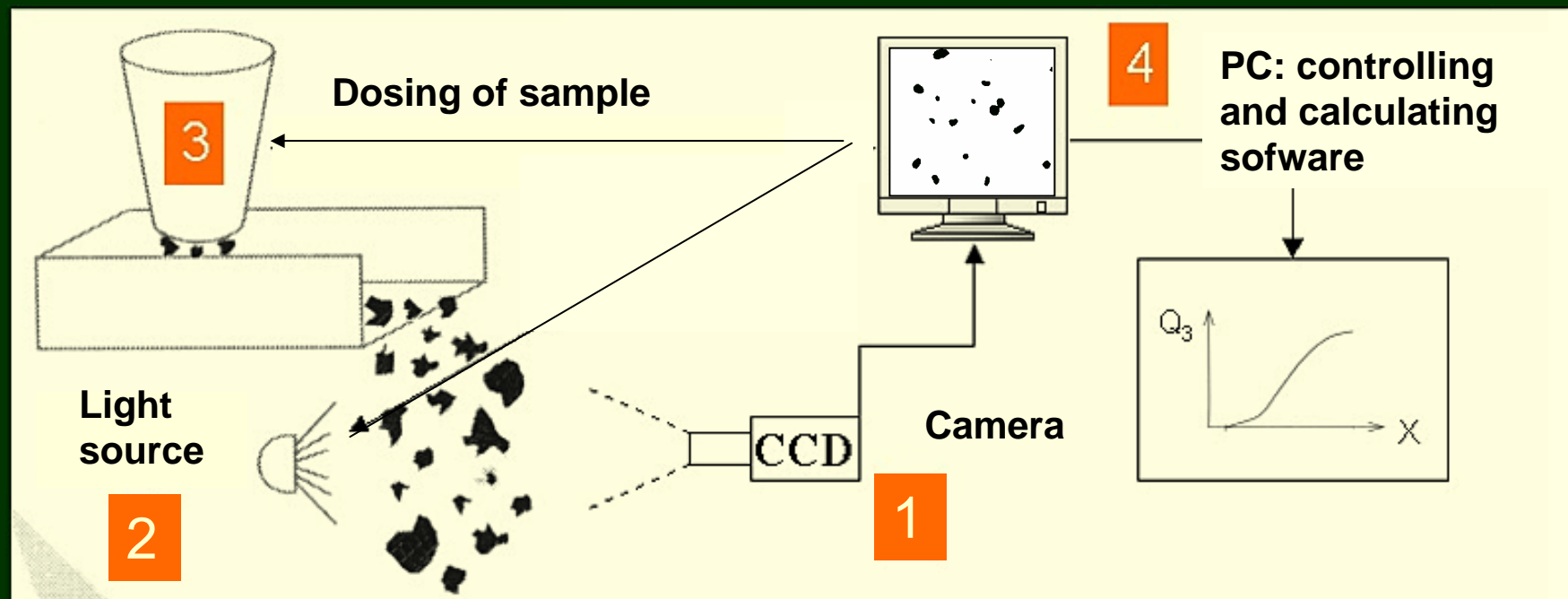
Measurement based on the fundamental  
 definition:  
 ratio Length to Width

1. Ratio of sizes of gauge: Length / Width = 3/1
2. Separating of all grains with  $L / W > 3$
3. Weighing:  
 $M_1$  - Mass of complete sample / Gramm  
 $M_2$  - Mass of non-cuboid grains / Gramm

Amount of total sample:  
 Minimum 300 Grains per fraktion



## Particle size and shape with photo-optical method

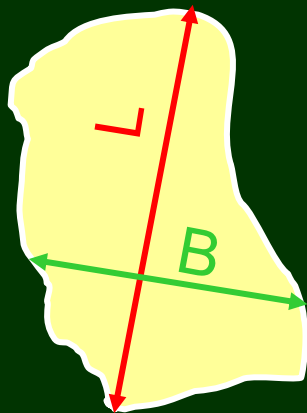


Schematic structure of a photo-optical particle size and shape analyser.

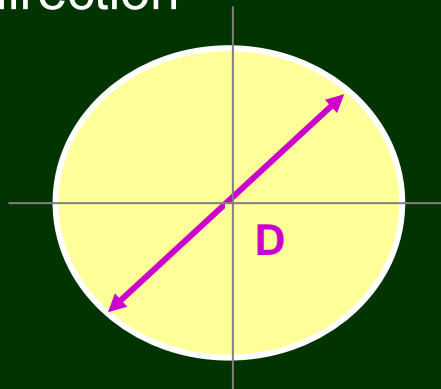
The used device: HAVER CPA 4-2 real time

## Particle size and shape dimensions

### Shadow projection



Measuring direction



### Image analyses



Circumference  $U$



Area  $A$  of projection



Largest Length

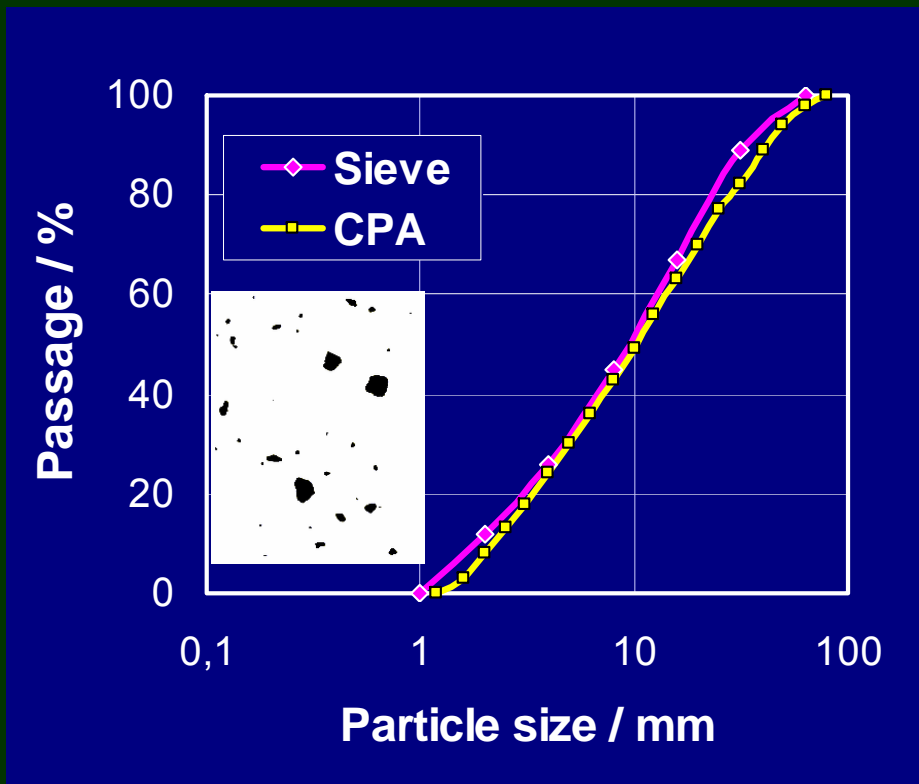


Largest Width

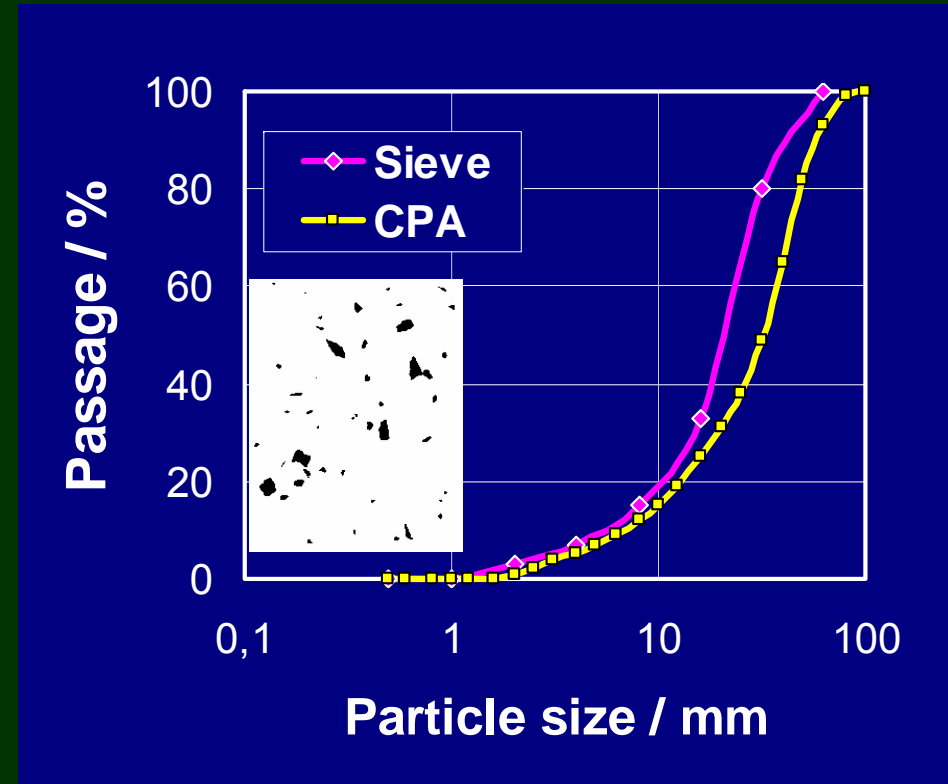


Particle size = Diameter of circle with the same area like the projection area  $A$

## Comparison particle size distributions: sieve and CPA analyses

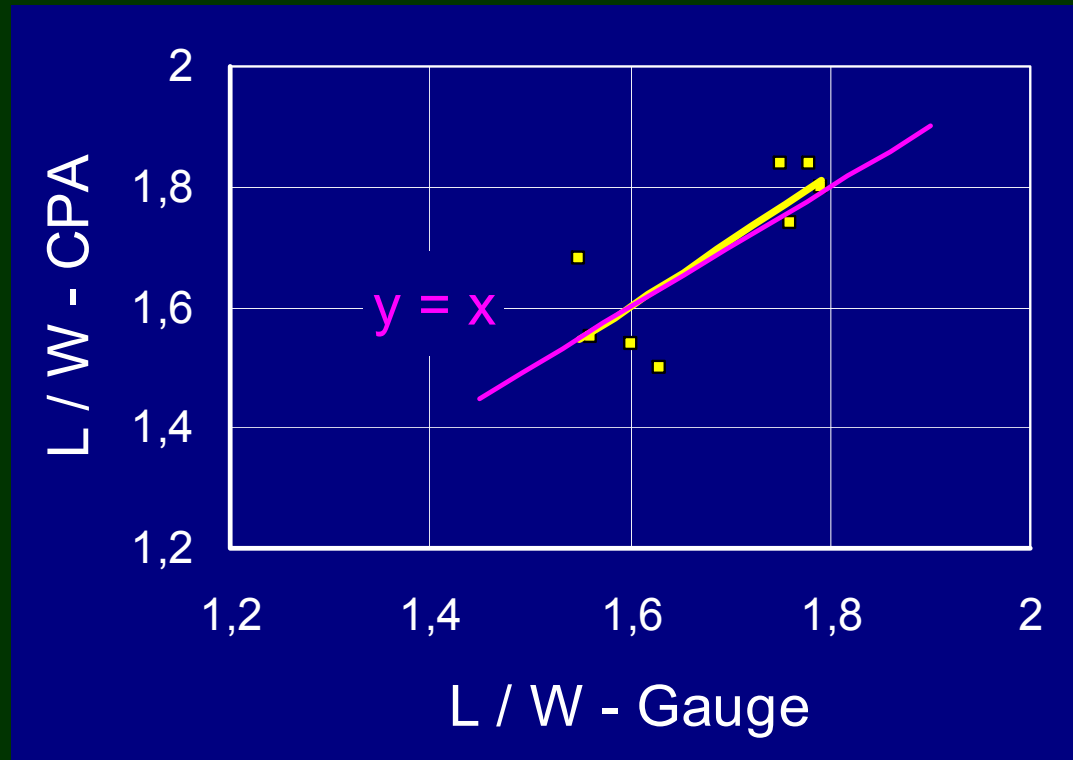


Cuboid particles



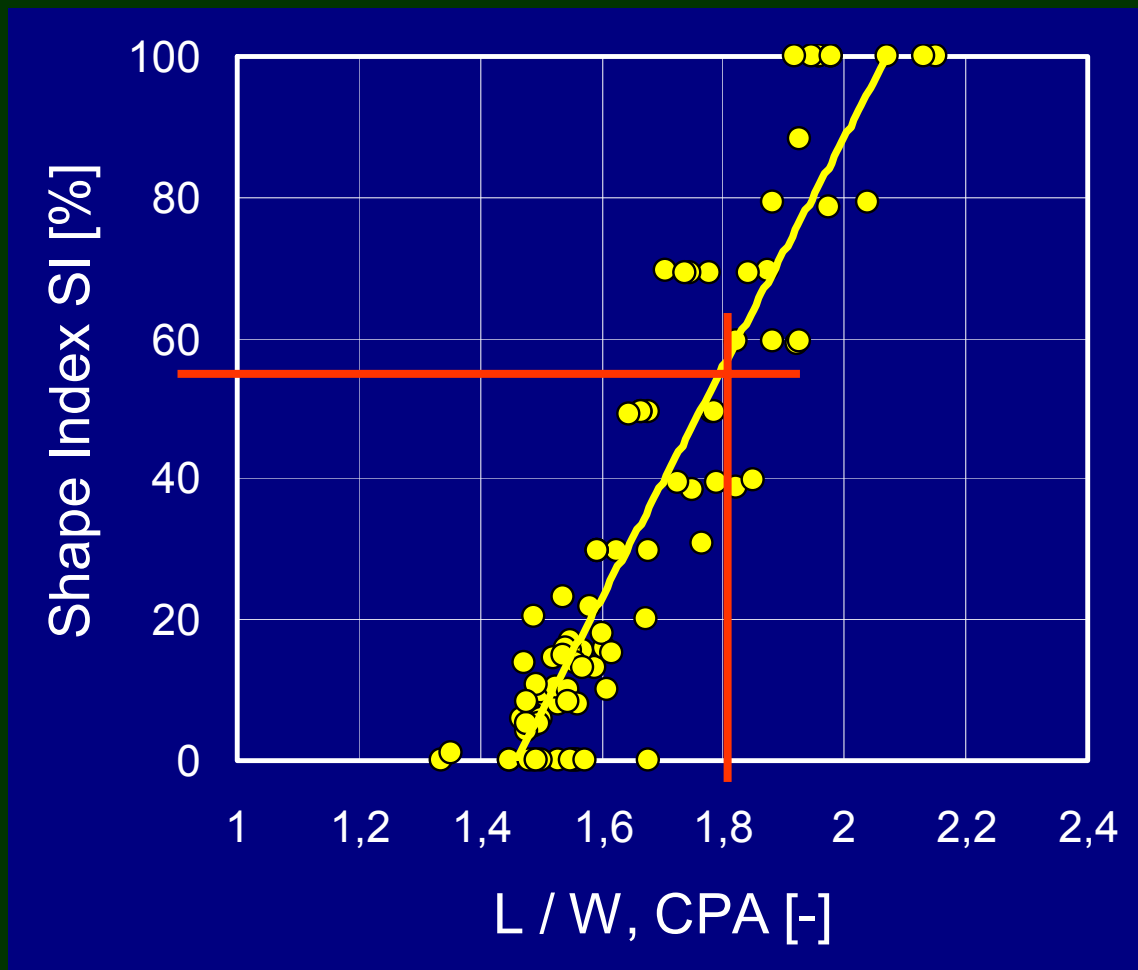
Gritty to flaky particles

## Comparison particle shape results: gauge and CPA analyses



In both cases mean sizes of L/W- Parameters based on number distribution

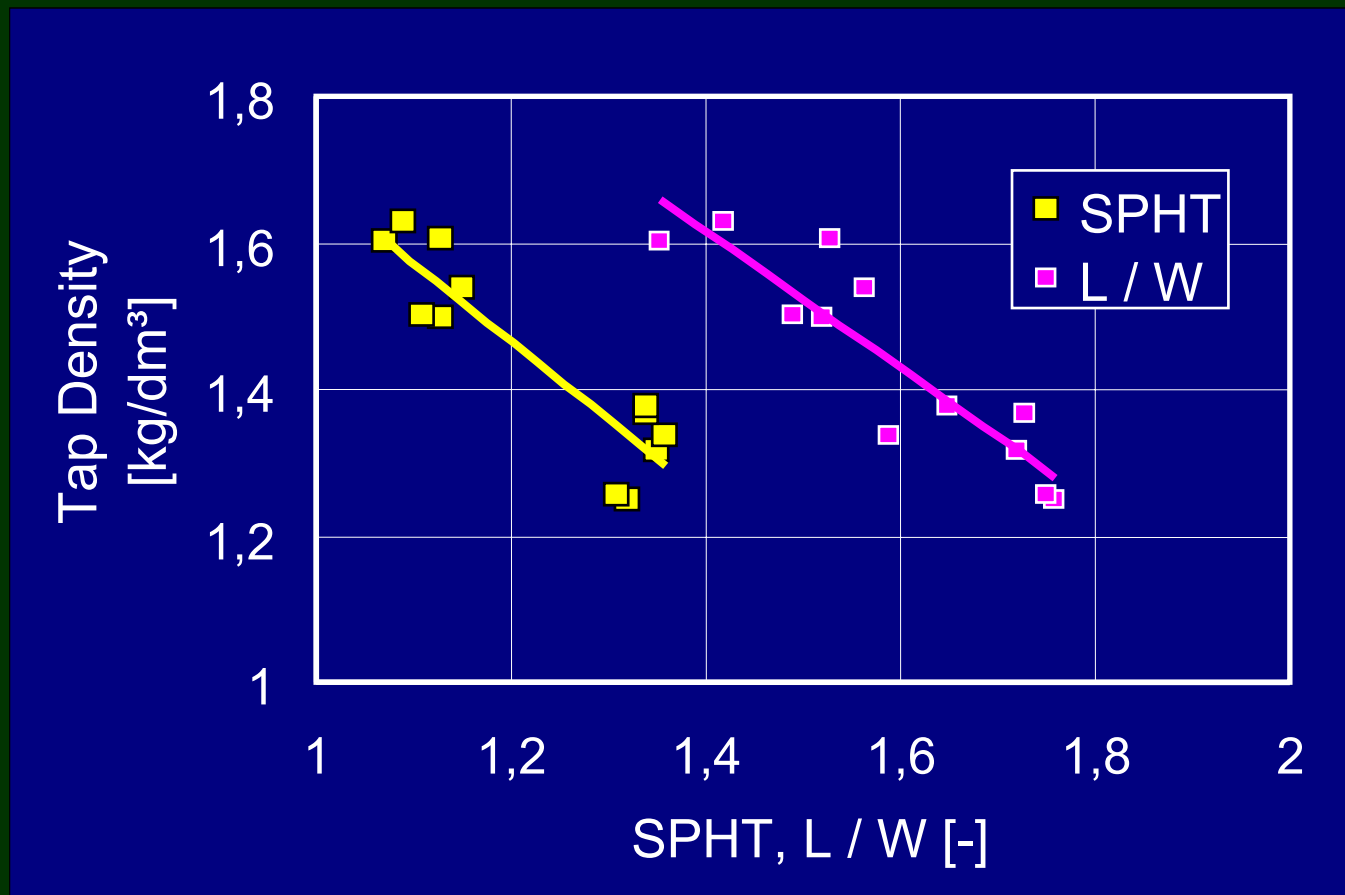
## Correlation - shape parameters $L / W$ and SI



Analysed both  
 natural and crushed  
 aggregate fractions and  
 also modelling fractions  
 by shape

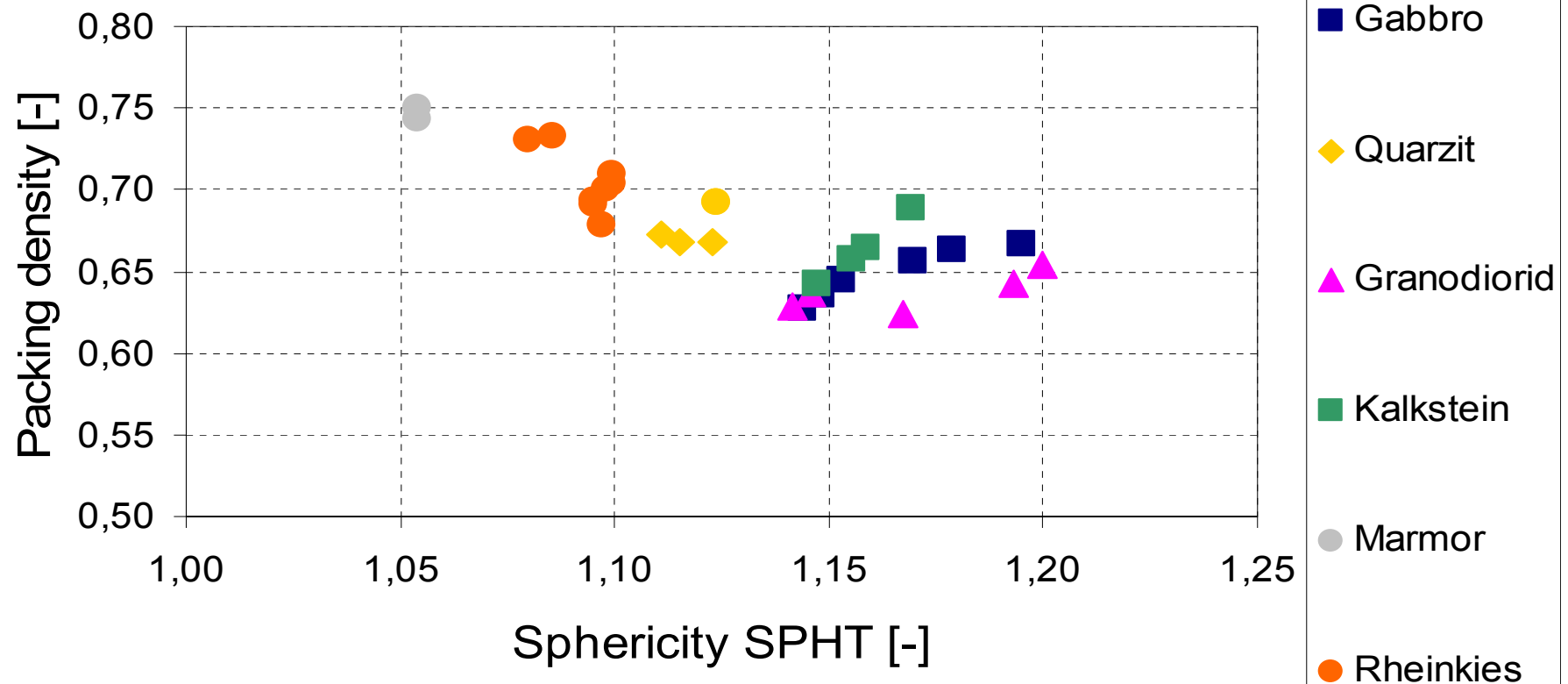
$L / W$  – mean:  
 calculation based on  
 volume distribution

## Tap density in comparison with particle shape



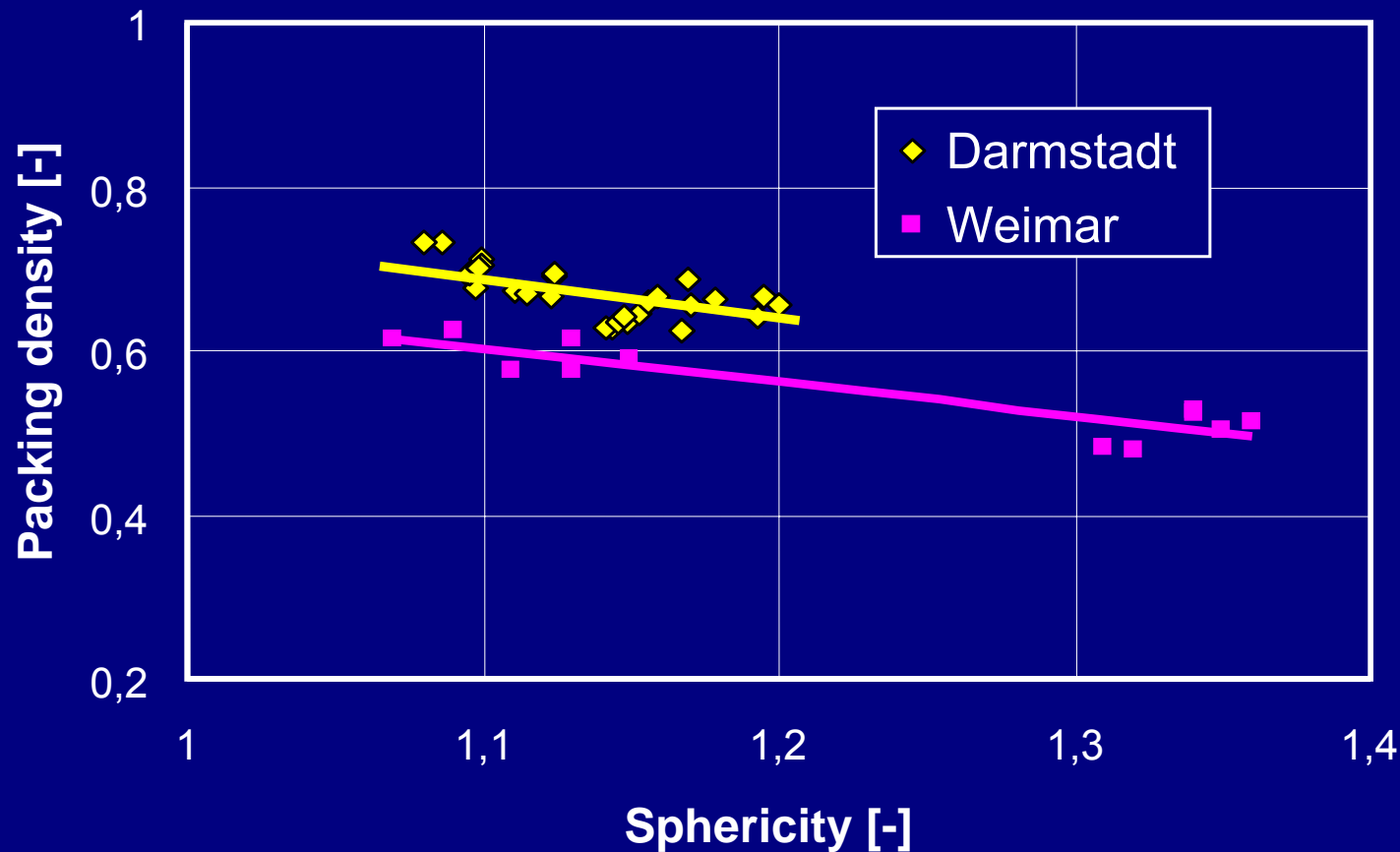
Analysed fractions with different shape and size:  
 8/16 mm  
 16/32 mm  
 8/32 mm.

## Influence of particle shape on packing density



Authors: Graubner / Proske / Ramge, TU Darmstadt

## Influence of particle shape on packing density



Darmstadt:  
 Gabbro  
 Quarzit  
 Granodiorit  
 Kalkstein  
 Marmor  
 Rheinkies  
 Weimar:  
 Kalkstein



## ➤ Simulation models:

Mostly balls are considered. Large differences between the results are obtained.

## ➤ New photo-optical measuring methods:

Particle size and particle shape can be determined quantitatively.

## ➤ Influence of particle shape:

First measuring results of relationship between  
SI-Index and photo-optical measured L / W ratio  
Packing density and L / W ratio and SPHT



## ➤ Future works:

Investigations and experiments about the influence of particle shape on all relevant properties of bulk materials.



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